# STUDENTS LEARNING ALGEBRA WITH APPLETS [1]

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*The transition from arithmetic to algebra is a process that involves complex reasoning and it is a topic where the students present many difficulties. Many studies show that the teaching and learning of mathematics may be potentiated by the use of technology. On this paper we intended to show how the use of an electronic tool might help students solve equations. Student's deal formally with this kind of task in 7th grade for the first time and it is possible identify how the tool mediates the learning process. The theoretical framework is based in the activity theory and the formulations of David Tall about the advanced mathematical thinking and the proceptual view of the mathematical concepts. Based in a qualitative approach and using an interpretative methodology, we observed two groups of students working with an applet during the process of solving equations. This work gives us evidences about the procedural and conceptual thinking developed by students and the role of the tool during this process. It is possible observe how the semiotic potential of the artefact mediate the learning process.*

*Keywords: Mediation, Proceptual thinking, Solving equations, Applets.*

## INTRODUCTION

Algebra is one of the topics that are presented across the curriculum of the Basic and Secondary School in Portugal. The transition from arithmetic to algebra is actually formalized in the beginning of the third cycle of the basic school, when students have about 13 years old. The teaching of this topic is a challenge to teachers and the learning process is complex for many students. In this paper we present a group of students that learn about algebraic equations with support of technological tools. The teaching experiment was based mainly with the use of applets for different purposes. Some applets are used to introduce the notion of variable, problem solving with the purpose of developing algebraic reasoning. Other group of tasks are based in the use of applets that act as games, were students may consolidate their previous learning. Other are based in algebra balance scales to develop the notion of equilibrium and consolidate the algebraic principles to be used in the solving equations. In this paper we analyse students' performance with an applet that presents a group of equations to be solved in the algebraic form in which they need to define the strategy to solve it. This task was performed at the end of the study when the students are proficient in the process of instrumental genesis (Rabardel, 1995).

## THEORETICAL FRAMEWORK

Teaching and learning that take place in environments using technology often involve a complex mathematical thinking. Sometimes this kind of thinking is seen from the cognitive point of view and has been designated by advanced mathematical thinking (Dreyfus, 2002; Tall, 2002, 2007;). The processes of representation and abstraction allow students to move from the level of elementary to advanced mathematical thinking and when used in this sense are often mathematical and psychological processes at the same time.

The use of symbols is a key feature in the development of this kind of thinking and can be viewed with a double meaning, introducing some ambiguity between the procedure and the concept. This combination of procedural and conceptual thoughts is called *proceptual thinking* (Gray & Tall, 1994), and it is characterized as the ability to manipulate the symbolism flexibly as process or concept, freely interchanging different symbolisms for the same object. It is this proceptual thinking that gives great power through the flexible, ambiguous use of symbolism that represents the duality of process and concept using the same notation.

Activity Theory initiated by Vygotsky and developed by Leont'ev, assuming its system of collective activity (object oriented and mediated by artefacts) as the unit of analysis, has been developed over three generations. Was initially based on the idea of mediation introduced by Vygotsky in his triangular model that becoming the triad subject - object - mediator artefact, with the separation between the person and the social environment on the background (Engeström, 2001). In the second generation, centred on Leont'ev work, the unit of analysis is no longer individual and now includes links to other areas involved in a collective activity system, focusing now on the interrelationships between individual objects and communities. Based on the collective activity system and giving special attention to the role that the mediator artefacts play in the relationship between subject and object, it becomes crucial to address the concepts of instrumental genesis. The instrumental genesis (Rabardel, 1995) involves two processes, instrumentalization and instrumentation, that enable the development and evolution of the instruments. The idea of mediation has been referred to the potentiality that a specific artefact has to foster the learning process. Rabardel (1995) distinguishes two kinds of mediation; the *epistemic mediation* where the instrument is a medium that allows to know the object and the *pragmatic mediation* where the instrument is the means of a transforming action directed to the object. With the use of technological artifacts it seems to be crucial consider the notion of semiotic mediation (Bussi & Mariotti, 2008) to enhance mathematics teaching and learning. In this context it is important take into account the semiotic potential of the artefact that involves two semiotic links, one between the artefact and the personal signs that emerging from its use and the second between the artefact and the mathematical signs evoked by its use and recognizable as mathematics by an expert.

In summary we intended to understand how technological tools mediate the learning process, based in the semiotic mediation within the activity theory and recognize the proceptual thinking that is potentiated by this tools.

## METHODOLOGY

This study is based in a qualitative approach and uses an interpretative methodology to understand how students develop their algebraic reasoning using technological tools. The topic of algebraic equations was introduced to students using applets and supported by documents produced by the teacher that reproduce the main frames of the applet and help students on the registration of the fundamental procedures presented in the applet. During the learning process students alternate between this kind of tasks and others that are presented in documents produced by the teacher with the goal to consolidate prior learning. In this paper we discuss the work of two groups of students (two students per group) working in computers (one group per computer) with the applet *Solving equations with balance-strategy: game* [2]. This task was performed at the end of the teaching experiment with the goal of consolidate the process of solving equations.

The students involved in this experiment attending the 7th grade. Manuel and Gustavo composed the first group. Manuel is a student that, initially, reveals some difficulties in mathematics and uses mainly a procedural approach to the concepts. Gustavo develops a conceptual understanding of the concepts studied and reveals a proceptual reasoning when solve the tasks proposed. The second group is composed by Isabel and José. Both the students are proficient in mathematics and demonstrate have a proceptual reasoning.

During the teaching experiment, when the uses of applets are sought, these students alternate between the use of the applet and the algebraic resolution through the manipulation of the algebraic expressions. This interchanging between these to ways of representation, potentiated by the tool, help students to develop their proceptual understanding.

## DATA ANALYSIS

The data analysed here is based in the use of the applet "Solving equations with balance-strategy: game" assessed in the site of the University of Utrecht in the connection with the Freudenthal Institute where the WisWeb applets are located. This applet generates an equation that the student must solve by applying the principles of equivalence. Operators are available on the top of the bar (figure 1), and the student must choose it to perform the same operation on both sides of the equation.

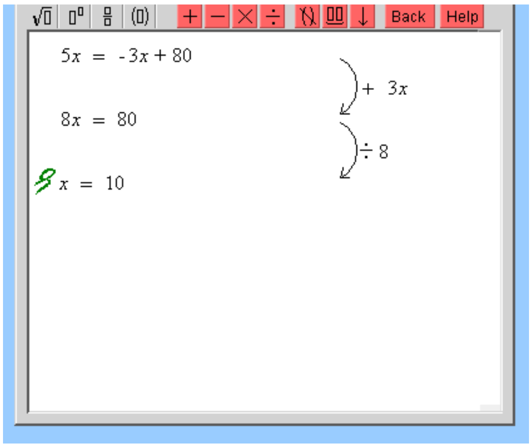


**Figure 1 – Window of the applet "*Solving equations with balance-strategy: game*"**

This applet asks students to solve twenty equations, and the degree of difficulty increases as they work through the task. Our main objective was to consolidate previous knowledge and identify the thinking processes that students develop in the interaction with the tool. It is intended that students submit their resolutions on a sheet. From their solutions students should be able to explain all the procedures performed. At this stage students can start the task by solving the equation using the applet or using first algebraic procedures. If students make a mistake the applet marks it with a red mark. Even if the student change his resolution that mark remains. This feature has made the resolution process to be carried out carefully by both groups.

The didactical methodology used was that of each group member, alternately, solve each of the proposed equations. Students were clarifying some doubts among themselves and had no difficulty in the first thirteen equations and they used the procedures correctly. However we can find different kinds of thinking reflected in their solutions on paper. When solved the equation presented at figure 1, Manuel and Gustavo presented the folloing written answers (figure 2):

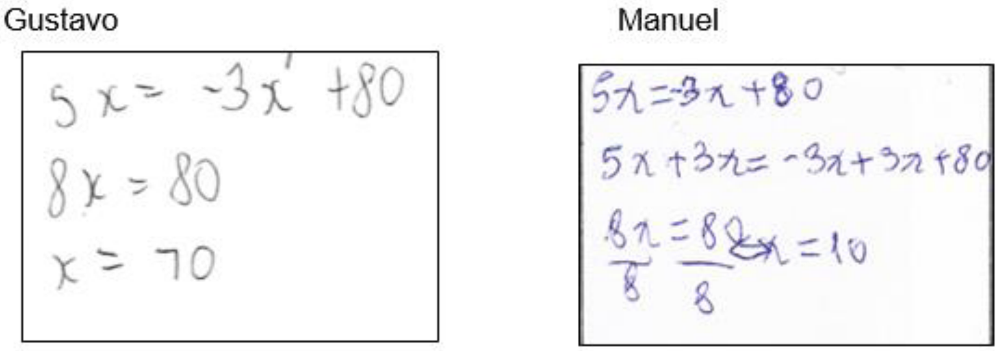


**Figure 2 – Solving the 5th equation.**

While Gustavo did not need to enter all procedures to explain his solution, Manuel choose to indicate all intermediate steps. In this context it is clear that Manuel is still in a transition phase of a procedural thinking for a proceptual thought, while Gustavo has reached this level. For both students the applet worked as a learning mediator helping them to develop the two types of mediation referred by Rabardel: the epistemic mediation because the applet allows them to identify the equations and its solutions and the pragmatic mediation that allows them to transform the equations in order to solve them.

The solution presented in figure 3 was more challenging because it was the first situation where expressions with parentheses appeared.

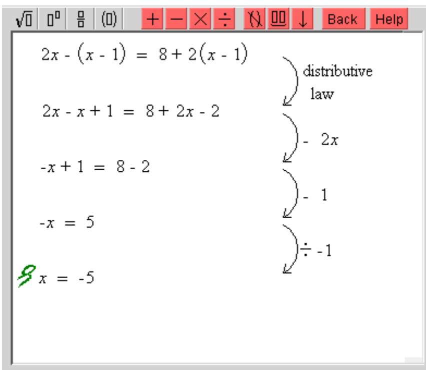


**Figure 3 – Solving an equation with parentheses.**

Students found the operation to be implemented by the applet, but not risked their running as they did not verify its viability algebraically. So they choose to establish the distributive property algebraically and after they advanced in solving the equation with the applet. The group of Isabel and José shows this idea in their solutions (figure 4), presenting all the steps of the process.

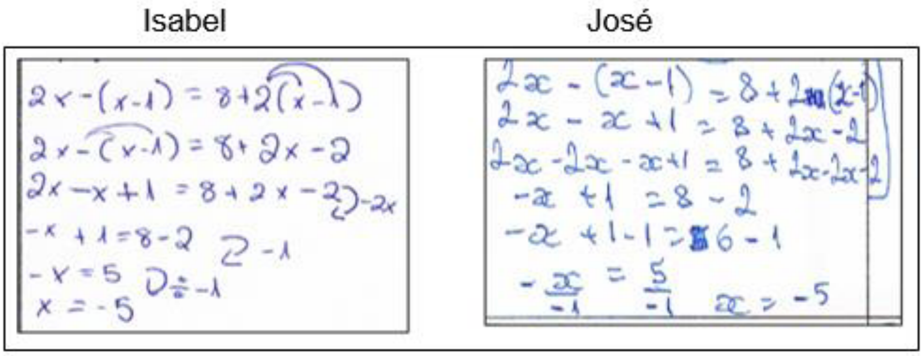


**Figure 4 – Solution made by Isabel and José.**

Non-routine tasks are performed in the interaction between the tool and the algebraic processes, allowing them to achieve the concepts under study without teacher intervention. This solution shows us that to perform more complex tasks students used their procedural thinking as support to develop conceptual thinking. In a written solution Isabel seems to be closer to a procedural thinking by pointing out all the procedures performed without displaying all intermediate calculations. She supported his solution in the representation of the applet that worked as a semiotic mediator of this process. José is limited to perform the various steps involved in the process being able to explain how carried out the various steps of solution.

The last equations presented by the applet involved fractional coefficients (figure 5). This topic was not treated during teaching because it is not part of the curriculum at this level of education.

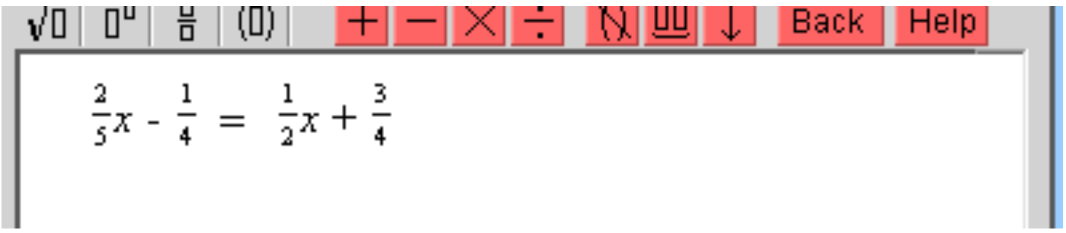


**Figure 5 – Equation presented by the applet.**

The students used their knowledge of operations with fractions and carried out operations in algebraic form. Only later they used the applet to confirm the solution.

José and Isabel decided to solve algebraically the equation using different procedures. José began to use the balance-strategy suggested by the applet. Only after simplifying part of the expression found the same denominator to operate with similar monomials (figure 6).

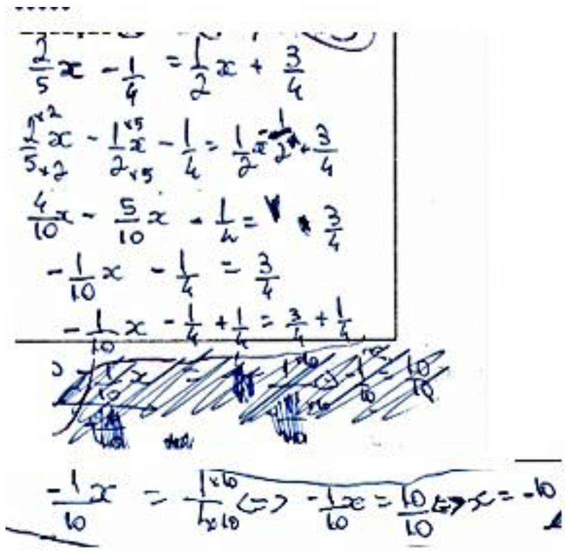

**Figure 6 – Solution presented by José.**

Jose showed a good performance in algebraic calculation, mediated by the applet, while summoning other mathematical knowledge needed to solve the task. So he was able to assemble proceptual knowledge that allows him to solve more complex equations than those that would be expected for this level of education.

Isabel began to use knowledge she had about fractions and reduced all fractions to the same denominator (figure 7) by obtaining all the coefficients with the same denominator used to simplify the balance-strategy suggested by the applet, thus solving the equation.

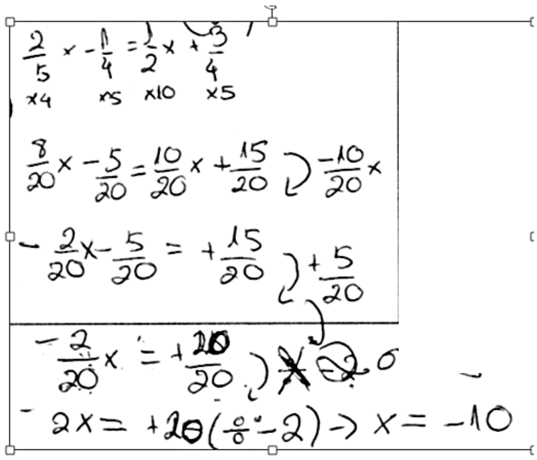

**Figure 7 – Solution presented by Isabel.**

In this way Isabel showed a good algebraic performance mediated by the semiotic representation that the applet provided him. This stage of the study showed that Isabel has a well developed proceptual thought, which allows her to return to the processes that are behind it, whenever this is necessary.

## CONCLUSIONS

In this paper we showed how an applet-based learning environment could help students develop algebraic reasoning. The students analysed made the transition from arithmetic to algebra involved in this technological environment and we discuss here their performance in solving first-degree equations.

The use of the applet appeals to the use of the principles of equivalence in solving equations and students use simultaneously the applet and the algebraic solution of paper and pencil. The applet worked many times as a learning mediator helping students to develop the two kinds of mediation referred by Rabardel (1995): the epistemic mediation once the applet allows them to identify the equations and its solutions and the pragmatic mediation that allows them to transform the equations in order to solve them. Depending on the familiarity with the equation presented, students use a procedural or conceptual thinking. When dealing with a similar equation to others already studied predominates proceptual thought, where students manipulate the equation as a mathematical object. When the equation presented sets up a new situation (eg equations where the coefficients are fractional) predominates initially a procedural thinking. This thought moves rapidly toward a conceptual thinking based on the interaction that students develop with the applet.

We can thus observe learning situations where the semiotic potential of the artefact becomes evident (Bussi & Mariotti, 2008). While the tool provides them with the creation of personal meanings to perform the task, the teacher identifies the mathematical meanings expected by himself in students oral and written productions.

## NOTES

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[2] http://www.fi.uu.nl/wisweb/applets/mainframe_en.html.

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