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Edited by Keith Jones, Christian Bokhove, Geoffrey Howson & Lianghuo Fan

Proceedings of the International Conference on Mathematics Textbook Research and Development (ICMT-2014)

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Thwaites, B. (2012). *The School Mathematics Project 1961-1970: A decade of innovation and its sequel*. Cambridge: Cambridge University Press.

The conference was hosted by the Mathematics and Science Education Research Centre (MaSE) at the University of Southampton; for up-to-date information on MaSE, please see: http://mase.soton.ac.uk/

The support and assistance of the University of Southampton, and the Southampton Education School, was invaluable in enabling the success of the conference.

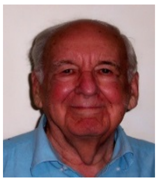
Preface

To many people, especially mathematics educators across the world, Southampton has been historically linked to mathematics textbook reform and development through the *School Mathematics Project* (SMP) since the early 1960s. We are happy that many researchers in mathematics education and mathematics, as well as textbook reformers and developers, policy makers and practitioners from different parts of the world came to attend the International Conference on Mathematics Textbook Research and Development (ICMT-2014) held at the University of Southampton from 29-31 July 2014.

The provision of well-judged, coherent mathematics courses, whether they be textbook-based, hybrid (textbooks with IT), entirely IT or teacher-based, is vital. It is our hope that the proceedings of this conference not only results in improvements to what is currently available, but also promotes further research and helps to assist the propagation of acquired knowledge and experience to administrators, developers and teachers.

Just as we extended a warm welcome to everyone attending the conference, we hope that these conference proceedings prove both valuable and enjoyable.

Geoffrey Howson, Honorary Chair, International Programme Committee (IPC)
Emeritus Professor, University of Southampton, UK



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All members of the Local Organising Committee (LOC) are from the *Mathematics and Science Education Research Centre* (MaSE) of the Southampton Education School at the University of Southampton.

MaSE website: http://mase.soton.ac.uk

MaSE Twitter account: @MASEsoton (conference hashtag #ictm2014)

ICMT 2014 website: http://icmt2014.soton.ac.uk

The official language of the conference was English.

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FROM CLAY TABLET TO COMPUTER TABLET: THE EVOLUTION OF SCHOOL MATHEMATICS TEXTBOOKS

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Over the centuries and around the world, school mathematics textbooks have differed in many ways. In this idiosyncratic survey, I attempt to portray, across time and space, something of what researchers have learned about those textbooks: what they are, what they appear to be, how they are related, and how they have been used. In general, school mathematics textbooks have differed more in approach and form than in function or content. Their principal function has been to serve as repositories of authorized knowledge, although at times they have been enlisted as resources for creative problem solving or as material for self-instruction. In the past, as textbooks took different forms and appeared in different media—clay tablet, papyrus, parchment, bamboo, paper—they also began to take on a wider span of mathematical content and to vary extensively in pedagogical approach. Research on textbooks has examined many of their characteristics, looking at how they have changed over time and, less often, how they differ across communities. Today, school mathematics textbooks seem more similar in mathematical content than they are in appearance, pedagogical outlook, or assistance for the teacher. There does seem to be something of a virtual school mathematics curriculum worldwide, whereas there is little agreement on what features the textbooks enshrining that curriculum should have. Today's textbooks vary along many dimensions. It appears that textbooks are being written as much or more for the teacher as they are for the learner. Tomorrow, computing technology seems likely not only to yield tailor-made textbooks but also to allow authors and publishers to revise the text swiftly in light of how the learner responds.

Keywords: textbook form, textbook function, textbook content, interactive textbook, e-book

INTRODUCTION

Just about every author who attempts to survey the research on school textbooks begins by noting the somewhat surprising scarcity of such research. That scarcity seems due, in part, to uncertainty as how to define *textbook* but also to its somewhat questionable status in school instruction: "Today a number of countries have curricula for primary and lower-secondary schools which literally never mention the word textbook" (Johnsen, 1993, p. 22). As long as educators remain unsure as to whether textbooks actually benefit school instruction, "we hesitate to apply terms like science and research to a field whose existence has not yet been sanctioned" (p. 22). Nonetheless, there appears to be a growing multidisciplinary literature on the nature and content of textbooks as well as how they are seen and used.

Egil Johnsen (1993) categorized international research on textbooks using three categories:

1. Ideology in textbooks.
2. The use of textbooks.

3. The development of textbooks. (p. 28)

These categories—which can be seen as oriented toward *product, use,* and *process,* respectively—are listed in the order in which they have been the focus of research to date, from most to least. In the case of mathematics textbooks, content analyses in the first category have typically given somewhat more attention to the selection and presentation of mathematical content than to philosophical underpinnings of that selection and presentation. But certainly the preponderance of research on mathematics textbooks has examined the contents of textbooks as a product. Research into the ways in which mathematics textbooks are used by teachers and learners has been growing in recent decades, with increased attention to the effectiveness of various uses. In contrast, the process of developing mathematics textbooks—from their conceptualization, writing, and editing to their marketing, selection, and distribution—has received relatively little attention to date.

WHAT IS A TEXTBOOK?

The usual definition of *textbook* is something like the following: a book used for the study of a subject. With that definition, a *mathematics textbook* becomes a book used for the study of mathematics. People who do research on textbooks, however, often observe that coming up with a good definition is not so simple, and they sometimes end up using terms like *teaching media* or *text materials* instead because they do not want to be restricted to a bound collection of printed sheets of paper produced for use in instruction. For example, a 1955 study from the University of Illinois acknowledged that "in recent years . . . text*books* have been supplemented by a variety of other text materials" (Cronbach, 1955, p. 17). Recognizing that devices such as films and recordings could be seen as functioning like texts, the committee conducting the study "restricted its interest to printed text materials of the sort which can be placed in the hands of every pupil" (p. 17).

In recent years, the meaning of *textbook* has expanded to include material produced in an electronic format. For example, in 2011, the state of Indiana defined *textbook* as follows:

> "Textbook" means systematically organized material designed to provide a specific level of instruction in a subject matter category, including: (1) books; (2) hardware that will be consumed, accessed, or used by a single student during a semester or school year; (3) computer software; and (4) digital content.
> (see http://www.in.gov/legislative/bills/2011/HE/HE1429.1.html)

Even before the electronic age, however, textbooks were not necessarily bound collections of pages. Over the centuries, textbooks have appeared in different media—clay tablet, papyrus, parchment, and bamboo—as well as paper. The word *textbook* (a blending of *text* and *book*) apparently did not appear until about 1830 (Stray, 1994, p. 1), with *text* being a word from the 14th century that had the meaning "thing woven" (from the Online Etymology Dictionary at http://www.etymonline.com). Ancient civilizations, however, certainly had equivalents of our modern books. Clay tablets with mathematical content from ancient Mesopotamia were used as school texts (Høyrup, 2002, p. 8), as was the *Nine Chapters on the Mathematical Art* (Shen, Crossley, & Lun, 1999), which "served as a textbook not only in China but also in neighbouring countries and regions until western science was introduced from the Far East around 1600 AD” (p. 1).

Gutenberg’s invention of the modern printing press in the 1450s enabled textbooks to be printed more cheaply than before, but it took many years before they were available to all pupils. Instead, as Nerida Ellerton and Ken Clements (2012) have noted, beginning in Western European nations in the 16th century and moving to North America in the 17th century, pupils prepared and used so-called cyphering books, linking writing and penmanship to the learning of mathematics. It is likely that in North America even by the 18th century

> most students outside of the large cities did not own commercially-printed arithmetic books (Meriwether, 1907; Small, 1914). Some teachers might have owned their own commercially-printed arithmetic texts, but most of them based their methods of instruction on old cyphering books that they had copied or procured from other masters (Littlefield, 1904). Indeed, William Munsell (1882) claimed that well into the 19th century printed arithmetics were so scarce that many scholars never saw one. That appears also to have been true in relation to the use of printed arithmetics in schools in Great Britain in the early 19th century (Williamson, 1928). (Ellerton & Clements, 2012, p. 107)

During the 19th and 20th centuries, bound textbooks printed on paper became increasingly common fixtures of school instruction world-wide. As they became more widespread, such books began to be produced and used in a greater variety of ways. Stray (1994) characterizes the textbook as “a composite cultural commodity which provides an authoritative pedagogic version of received knowledge” (p. 4); that characterization captures some of the ways in which textbooks operate as “the intersection of several relationships involving teachers and pupils, producers and consumers, institutions and the state” (p. 24). In the evolution of mathematics textbooks, there have been changes on the inside (what the textbooks contain) and on the outside (how they are produced and used)—more changes than can be captured in a paper such as this. What follows is a brief sketch of some of the differences those changes have produced.

DIFFERENCES IN CONTENT AND FUNCTION

School mathematics textbooks differ in content, function, approach, and form. In general, they seem to be more similar in the first two dimensions than in the second two. They have certainly grown in the contents they contain and the functions they serve, but they still share many common features in those respects.

Contents of mathematics textbooks

During the so-called new math era—roughly the mid1950s to the mid1970s—one assumption that reformers made was that

> the curriculum would be brought up to date mathematically if they could simply get their new syllabuses and textbooks into the hands of students and teachers. By the end of the era, they had come to see that much more was required. At the crux of any curriculum change is the teacher. The teacher needs to understand the proposed change, agree with it, and be able to enact it with his or her pupils—all situated in a specific educational and cultural context.

> From a distance, school mathematics looks much the same everywhere. Countries include many of the same topics in their syllabuses and expect pupils to solve many of the same sorts of problems. International comparative studies are predicated on having a common framework on which to base the assessment of mathematics achievement. Up close, however, each country has a unique school mathematics. (Kilpatrick, 2012, p. 569)

Research studies conducted during the new math era found that changes made in textbook content did not necessarily translate into changed performance by learners. For example, the results of the National Longitudinal Study of Mathematical Abilities, a 5-year study conducted in the United States that began in 1962,

> showed that, contrary to some expectations, pupils who had used modern textbooks did not achieve superior levels of performance in computation over several years while using textbooks that did not stress computation. . . . To oversimplify, pupils tended to learn what was emphasized in the textbooks they used and not something else. (Howson, Keitel, & Kilpatrick, 1981, p. 193).

International studies such as TIMSS [Trends in International Mathematics and Science Study] depend on mathematics textbooks being roughly equivalent over time and across countries. Analyses such as that by Vilma Mesa (2009) call that equivalence into question when one looks closely at textbook content. But international studies do not look closely at content. They depend on the existence of something akin to an

> idealized international curriculum. No allowance is made for different aims, issue, history and context across the mathematics curricula of the systems being studied. (Keitel & Kilpatrick, 1999, p. 243)

There does seem to be something of a virtual school mathematics curriculum worldwide, whereas there is little agreement on what features the textbooks enshrining that curriculum should have.

Functions of mathematics textbooks

Like every curriculum, every textbook represents a selection from available knowledge. “In the process by which a message gets from the resources of civilization to the student” (Cronbach, 1955, p. 93), a sequence of gatekeepers passes on some messages and blocks others. That sequence can be represented as follows: resources→(writer)→manuscript→(publisher)→printed materials→(distribution & selection)→book in school→(teacher)→messages before learner→(learner)→messages noted and stored (p. 94). Each item in parentheses is a gatekeeper that selects the messages to be passed on. The messages that arrive at the learners, therefore, are a function not only of the text material but also of the teacher, and the function those messages serve in instruction depends very much on how the teacher views the text.

Throughout history, the principal function of mathematics textbooks has been to serve as repositories of authorized knowledge. The content messages they contain have been chosen by their authors to represent the most reliable and important mathematical knowledge that the culture possesses. At times, however, authors have attempted to move away from being formal and comprehensive to enable learners to play a part in developing mathematical ideas on their own. Textbooks have been enlisted as resources for creative problem solving or as material for self-instruction. Those functions can rock the pedestal on which textbooks are placed as authoritative. For example, Hung-Hsi Wu (1997) faulted a precalculus textbook for omitting the formula relating degrees and radians, leaving it as a problem for students to solve.

> The issue here, as any textbook author will recognize, is the tension between textbook as archive and textbook as tool for learning. Once a formula is put into a text for memorization and subsequent reference, there is little point in asking the reader to find it. (Kilpatrick, 1997, pp. 958–959)

Textbooks are often seen as dictating the content of instruction, but that appears to be questionable. In a study of four fourth-grade teachers who kept daily logs of their mathematics lessons for a year, Donald Freeman and Andy Porter (1989) found little overlap between the topics the teachers said they taught and the topics in the textbooks they used. Only one of the teachers followed the textbook closely, and she did not teach every topic in the book, skipping or not reaching almost 40% of the lessons. Further, the time that the teachers spent on topics did not match the emphasis those topics were given in the textbooks. This issue seems to be one deserving more attention by researchers.

Mathematics textbooks typically reflect national or other values, and one of their important functions is to attempt to instil those values (Howson et al., 1981). In the first half of the 19th century, for example, Dutch mathematics textbooks began to include dialogues between teacher and learner, giving advice on the best way to study and holding out eagerness and perseverance as important middle-class values for students to acquire (Beckers, 2000). The problems given to learners to solve were not necessarily realistic, but they were designed to incorporate knowledge from fields such as history and geography. Narratives were given in which good behaviour such as saving money was rewarded and bad behaviour punished. Learners were to become good citizens not only by learning to reason but also by learning right from wrong. It seems that textbooks invariably attempt to transmit their authors' and publishers' beliefs as received knowledge.

DIFFERENCES IN FORM AND APPROACH

School mathematics textbooks seem more similar in mathematical content than they are in appearance, pedagogical outlook, or assistance for the teacher. They appear in a variety of forms and adopt a variety of approaches to the presentation of content.

FORMS OF MATHEMATICS TEXTBOOKS

The earliest textbooks in the Babylonian scribal schools were collections of arithmetic tables or of problems, most of which involved finding a number that satisfied given conditions (Høyrup, 2002; see also Aaboe, 1964; Beery & Swetz, 2012). The problem collections varied in approach: Sometimes the solution was given, and sometimes the text outlined the procedure to be followed in reaching that solution. They were clearly intended to be used in schools, and presumably a teacher was expected to explain the procedure and set the assignment. Today's textbooks vary in a similar fashion but along many more dimensions.

The physical form that a textbook takes has often been seen as part of its value for instruction. In an early comparative study of lower secondary mathematics textbooks, a group of teachers in the Los Angeles, CA, public schools evaluated the books on a variety of criteria. These included not simply the mathematical content but also the number and kind of illustrations; drills, tests, summaries, and reviews; vocabulary; size of numbers; physical features; authors; and matters of organization and presentation (Fuller, 1928). The goal was to determine which textbook ought to be adopted by the district. Although many of the criteria—especially those dealing with the physical form of the books—were easy to apply, it is difficult to see how they should be synthesized to arrive at the optimal choice of textbook.

In a recent related but more sophisticated effort, the Nordic countries—in which textbooks are heavily used—have established a network of researchers to look at issues such as how textbooks are used, hidden messages in the text, and how language and pictures are used (Grevholm, 2011). The network promises to fill a long-standing gap in studies of how the form of a textbook might influence its use and how learners respond to it. A study by Karen Fuson and Yeping Li (2009) provides one example of how the physical forms of textbooks might be analyzed and compared. Fuson and Li examined features of two textbook series—one in China and one in the United States—and found striking differences between the series in the linguistic, visual, and numerical supports provided in the books. The forms that mathematics textbooks are taking should continue to be an active topic of research.

APPROACHES OF MATHEMATICS TEXTBOOKS

Textbooks continue to vary extensively in pedagogical approach: from providing no suggestions at all to including detailed scripts specifying what the teacher should say and do. Research on textbooks has examined many of their characteristic approaches, looking at how they have changed over time and, less often, how they differ across communities.

One of the first, and still very few, studies of changed approaches to the content of mathematics textbooks was conducted by Hobart Heller (1940), who surveyed the topic of factoring in algebra textbooks published in England and the United States from 1631 to 1890. Why did factoring grow in emphasis during that time? Heller proposes that it grew in part because examinations began to include more complex problems dealing with forms used in fractions. “Increased complexity of fraction exercises demanded increased emphasis upon factoring. . . . In brief, the materials of factoring and fractions were in a ‘vicious circle’” (p. 110). Factoring also became more prominent among the exercises students were given to work because textbook authors, failing to distinguish between the functions of “illustrative examples” and “model examples,” began to put increased emphasis on drill exercises set to follow a model in place of broader aims for teaching algebra. Heller’s study shows how tradition appears to govern the persistence of the approach to content in textbooks.

Peter Damerow (1980), who looked at geometry textbooks in Germany during the decade of curriculum reform preceding 1980, made a similar finding. Although the new math reforms were intended to offer a new way of thinking about mathematics, simplifying topics by introducing structural ideas, Damerow’s study showed that the quantity of information grew without being reorganized so that it might be approached differently. “As far as Germany is concerned, [the findings] demonstrate that the changes in the textbooks to a great extent do not correspond to the ideas of the reform movement" (p. 281). After examining the nature and number of theorems related to the topic of similarity in the available textbooks, Damerow found the following:

> Instead of exchanging the old conceptual structures against the new ones there were simply added theorems formulated with new concepts. Moreover the added conceptual structures correspond rather to the ideas of Felix Klein introduced at the beginning of the 20th century than to the ideas of Bourbaki as they are defining features of the intended new math curriculum. (p. 301)

In addition to studies of changes in the approach to mathematical topics over time, there have been some studies of differences across communities—in particular, national differences in the way in which certain topics are treated. For example, Tony Harries and Ros Sutherland (2000) examined how primary mathematics textbooks represent multiplication in England, France, Hungary, Singapore, and the United States. (The last country is not discussed in the paper.) They found that the Hungarian, French, and Singaporean textbooks tried, in different ways, to support the development of mathematical meaning by moving from a real situation to a mathematical representation, whereas the English textbooks did not follow a coherent progression.

Mesa (2009) looked at the treatment of the concept of function in textbooks from 18 countries. She found that within and across the countries, there were four clusters of textbooks: rule oriented, abstract oriented, abstract oriented with applications, and applications oriented. These clusters were not aligned with cultural similarities across countries, nor were they necessarily characteristic of individual countries. She concluded:

> When it comes to functions, there may be no such thing as a canonical curriculum in school mathematics. It seems to be false—and this result is also supported by the TIMSS curriculum analysis—that mathematical content is expressed in the same way across the globe. (p. 116)

The approaches to instruction taken in mathematics textbooks continue to pose open questions. Constance Dooley (1960) analyzed the content and approach of textbooks and concluded that the market for U.S. arithmetic textbooks is reasonably sensitive to the results of recommendations from research. That raises the question of how research results are being used, or not used, today.

Another question is for whom are textbooks written? Johnsen (1993) claims that "in principle, today's textbooks are not written for teachers" (p. 323), and he cites a survey of Norwegian textbook authors responding to a questionnaire who said that they write for the pupils. Peggy Kidwell, Amy Ackerberg-Hastings, and Dave Roberts (2008), in contrast, trace the transformation of American textbooks "into a common tool and lucrative product" (p. 4), suggesting that over the years, the marketplace has done much to determine the nature of school mathematics textbooks. It appears that, despite what textbook authors might claim, textbooks continue to be written as much or more for the teacher as they are for the learner.

TOMORROW'S TEXTBOOKS

Koeno Gravemeijer (2014) discusses the need to transform mathematics education in the 21st century but simultaneously notes the limitations of current textbooks as means of making that transformation. He says,

> The scripted character and the ready-made tasks of conventional textbooks limit teachers in adapting to their students' reasoning. Instead, textbooks will have to inform teachers about local instruction theories, and explicate what mental activities hypothetical learning trajectories have to focus on (p. 166).

He also argues that textbooks will need to be more explicit about the theories being used and will need to contain exemplary instructional activities. He notes that such textbooks may be accompanied by computers and other tools. New textbooks will be far from enough, however, if school mathematics is to be transformed. Teacher education and professional development should accompany any changes in text materials. Gravemeijer claims that "textbook design and [teacher] professionalization will have to be carefully aligned to be successful."

Tomorrow, computing technology seems likely not only to yield tailor-made textbooks but also to allow authors and publishers to revise the text swiftly in light of how the learner responds. That change promises to open up new vistas in the contents, functions, forms, and approaches of mathematics textbooks. In particular, interactive computer textbooks will allow learners—just like those Babylonian learners using clay tablets several millennia ago and those Western European and North American learners using cyphering books several centuries ago—to create their own text. Tomorrow's learners will be able to get instant feedback on their mathematical work from a human teacher or a sophisticated computer program, and instruction can then be tailored to their abilities and interests.

We cannot, however, count on the change to be as rapid as we might like:

> It's typical for new technologies initially to mimic an existing one: Gutenberg's forty-two-line Bible is not easy to distinguish from a manuscript copy. It takes time to figure out what a new medium can do besides the same thing bigger, faster, or cheaper, and for its particular strengths and weaknesses to emerge. Fifty years after Gutenberg, printing had shown itself vastly superior for Bibles and legal texts, a cheap substitute for deluxe books of hours, and no replacement at all for wills, inventories, and personal letters. (Hays, 2014, p. 20)

The strengths and weaknesses of the computer tablet as mathematics textbook remain to be discovered.

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CHALLENGING THE AUTHORITARIAN ROLE OF TEXTBOOKS[i]

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A textbook is a special type of book that is part of institutionalized schooling, assumed to present accumulation of knowledge and usually used in a particular way. Digital books offer new kinds of participation, flexibility and personalization – properties that are in contrast to the traditionally authoritative structure of the textbook and the passiveness of the reader (the teacher or the student). Current proposed pedagogical changes, especially those directly touching upon teaching and learning from open interactive educational resources seem to be the appropriate ones to support constructivist pedagogies but challenge the accepted and still dominant functions of the textbook and textbook culture. We will review the accepted norms of textbook authority in mathematics classrooms and address the challenges interactive eTextbook post to the foundational ideas of textbooks: authority and stability. Three concerns will be in focus; the interactive multimodality designed to further authorize students' ideas, teaching with non-sequential resources that challenge ideas of textbook authority, and the nature of communal creation.

Keywords: digital textbook, e-textbook, textbook authority, teaching resources

RETHINKING READING: ENGAGEMENT WITH E-BOOKS

A textbook is a special type of book that is part of institutionalized schooling, usually used in a particular way. Kuhn (1962) considers textbooks to present accumulation of knowledge rather than promoting shifts of paradigms. As it often happens with the introduction of new technologies, the relatively close familiarity with digital books, eBook readers, and interactive books[1] of the last decade amplified broader thinking about structure, functions, and authoring processes, resulting in the realization that a book can either be printed and bound, an electronic device, or a digital file (Young, 2007). These developments confirm the claim that the *object* called "book" has undergone a complete change. Young claims that the book looks as if it were dead because the dramatic change of the object is what concerns publishers, for whom the book trade is about selling objects. Others claim that the book is dead because reading habits have noticeably changed. The art and craft of authoring an engaging book involves a long process, as does the reading of it: self-reflection that demands what Young (p31) "*internalactivity*": slow immersion and sinking into ideas, carefully crafted by the author. But eBooks are often assumed to offer just the opposite sense of reading: rapid browsing of short writing. The new digital media has produced a change not

[1] The term *eReader* refers to a device on which it is possible to read electronic or digitized books. *Digital book* or *Virtual book* refers to several different formats of books that are not paper-based. *Interactive eBooks* require physical engagement from the reader such as recording, reordering, operating animations or videos, etc.)

only in the *object* but also in the *processes of reading* and authoring, in the engagement of the reader, in the duration of the reading and writing activities, and in the investment in them. Roughly, there are two formats of interactive reading that demand the readers' engagement. The first applies to books that call on the reader to participate in portions of the authoring. These are known as hypertext books. Their main characteristic is the *non-linear, multiple path* that can be established through the content according to different links associated with key characters or topics (the role-playing model is described by Spielvogel et al., 2010). The second format involves readers serving as writers in *networked*, socially developed, *evolving books*. Textbooks are different from fictional prose, but the question of how to encourage readers to enter into the world of the textbook with reference to their own experience is a focal question of this paper.

School mathematics textbooks

Textbooks are supposed to provide guidance and present opportunities for students to learn, making the objectives and ideas of the curriculum more readily apparent. For teachers it also provides guidance in bringing their teaching in line with the expectations of the external authority which may be the school, the syllabus, or central assessment. In this function, the textbook serves as syllabus and timekeeper, and its author is considered to be the authorized entity charged with delivering content and pedagogy. Note the direct etymological link between "author" and "authority" (as further described by Herbel-Eisenmann (2009)), underscoring the authoritarian position of the textbook as written by a recognized expert author. Teachers use various practices to confer authority onto the text and simultaneously onto themselves (Remillard 2005). Drawing a direct link between the author, the central authority that authorized the textbook, and the way teachers teach and students learn appears to be negotiable. Cuban (1993) argues that attempts to change the formal textbooks hoping to change what and how students learn is an illusion that does not take into account the complexities of education as a socially and personally dependent system. Analyzing newly developed textbooks, Reys et al. (2006) indicate that half of the teachers they studied were influenced more by the state-determined curriculum and assessment materials than by the textbook. At the same time, studies that document changes occurring in classrooms and learning environments, where teachers witness the reform, suggest that textbook culture is becoming more flexible and is characterized by a multiplicity of approaches manifest in different situations (Chazan et al. 2007). Studies on the use of the textbook by students are rare and as Rezat (2012) argues, although the textbook is directed to the student and instructions' voice and tone are written for engaging students, we know only a little about how students take advantage of various designed opportunities. In sum, the textbook culture in current math teaching suggests the following: (a) the textbook remains a key object that acts as authoritative pedagogic guideline for what should be learned and for how it should be taught and assessed; (b) although the textbook is assumed to provide devices for actively engaging the students, the studies focus on engagements with textbooks that are reserved mostly for the teachers; (c) teaching does not depend on a single textbook: approximately 30% is accomplished by teachers using other teaching materials; and (d) technological resources are reported to be considered as enrichment while the textbook remains the core authority.

Virtual textbooks

Digital textbooks were developed and marketed primarily for higher education courses, but are making their appearance in schools as well. An increasing number of school textbooks are now supplemented by continuously upgraded digital resources. Publishers allow teachers to personalize digital textbooks for their courses, emphasizing flexibility and inexpensive dynamic changes. Hardware manufacturers have encouraged developers to use authoring tools to develop digital textbooks, as, for example, in the iBook Apple model (http://www.apple.com/ibooks-author/), and central authorities in countries with emerging economies – India and some African countries – found the digital textbook to be a unique opportunity for delivering textbooks to distant rural schools. The Israeli education system requires that each digital textbook appear in at least one of three formats: a digitized textbook, a digitized textbook that is enriched with external links to multimodal materials, or a textbook that is especially designed to work in a digital environment and includes online tools for authoring, learning, and management of student work. Korea, considered to be one of the leading countries in math and science achievements, became a leading innovator in the area of eTextbooks. Korea holds an integrative view on which textbooks remain the central learning resource, surrounded by other types of facilitating media. Other educational systems are adopting a similar view of the new textbook (Taizan et al., 2012). In the integrative view the digital textbook assumes the traditional functionality of the textbook. But alongside these national trends, usually supported by commercial production, another type of virtual textbooks is being developed, most often by non-commercial foundations, academia, and teachers' associations. These textbooks preserve facets of the integrative view but are designed anew to function only as digital environments. They attempt to explore and exhaust the opportunities the media offers for modes of authoring and engagements for mathematics students and teachers. A few noticeable examples have recently been described in Gueudet and Trouche (2012), Drijvers et al. (2013), and Yerushalmy (2013). Analysis of the design and functionality of such eTextbooks, in which the conventional characteristics of textbooks are not apparent, require new thinking.

DESIGN THAT AUTHORIZES STUDENTS' PERSPECTIVES

External authority invoked by the expert author in the textbook is often at odds with and the readers' engagement. Authorizing students' perspectives (as, for example, discussed by Cook-Sather (2002)) is a challenge to any educational system, and textbook authors have long been seeking less formal control structures that would better reflect student-centered teaching and support teaching as guided scientific inquiry. The different formats of multimodal interactive textbooks in mathematics have yet to be thoroughly studied. To explain the challenge and to exemplify design decisions, I would analyze the design of the VisualMath interactive textbook. Yerushalmy, Shternberg, and Katriel (2002) designed a textbook characterized by the extensive roles assigned to visual semiotic means, to interactivity between the reader and the visual mathematical objects and processes, and to a conceptual order of digital pages that could be rearranged to serve a variety of instructional paths. The design of the VisualMath (further detailed in Yerushalmy (2013)) is situated in the larger view field of mathematical guided inquiry. It contains the foundation of the “what” and the means for the “how” to teach and learn a full school-algebra course. Traditionally, textbooks are orderly structured and balanced along Expositions, Examples, and Exercises (Love and Pimm 1996). Although the organization of each unit resembles a traditional set of textbook tasks, the principle that guided the design of tasks and characterization of activities is rooted in the interactivity of tools and diagrams. Each type of interactive element has its semiotic and pedagogical meaning. An interactive diagram (ID) is a relatively small and simple software application (applet) built around a pre-constructed example. Interactive elements are designed to support the systematic generation of examples in linked multiple representations, to accommodate various entry points, and to provide non-judgmental mirror feedback that should be interpreted subjectively and support conjecturing and argumentation. Indeed, the challenge in constructing a task around an ID is to design opportunities for action.

Below is an attempt to map traditional textbook components (exposition, exercise, problem) onto another set of multimodal signs (*illustrating, elaborating, and guiding interactive diagrams and tools*). This mapping is inherent to the visual-semiotic functions that drive the VisualMath design.

Exposition: Freisen (2013) questioned the role of the exposition in traditional textbooks, and the attempt to present the universal truths and to expect students to later recite them back to the teacher. Our assumption was that learners need to construct the concept image and definition by creating their own example space and image. The challenge is to design expositions that can be worked on and personalized by controlling interactive illustrations. Illustrating Ids are designed to present the objective of the activity to the reader.

Tools: The assumption in the design of the VisualMath eTextbook was that tools become a way of thinking and knowing, and have an epistemological role as they change the traditional assumptions of what we mean by knowing mathematics. Toolbox and unit tools are integral part of the textbook, sometimes offered and at other times are linked to a task. Tools are artefacts designed to signify specific elements of mathematics. A Toolbox of 10 tools for doing mathematics is part of the eTextbook. The unit tools (or activity tools) are special cases of the tools in the Toolbox, designed to explore specific concepts mainly by limiting the generated types of example spaces through supporting a smaller range of actions.

Exercises: The primary means used to design exercises are Elaborating IDs, which include a wide range of representations and controls within the representations. The exercise (each consists of an infinite number of exercises generated by restricted randomness) is a traditional task that requires an answer. It is designed as an example presented through software tool. Input is free, trials can be attempted, and feedback is given by the linked representations. Specific tools such as the transformation tools, which could have been helpful here, are missing on purpose. It is assumed that students can solve the exercise on paper without the tool, and use the tool for checking, or that they are exploring the exercise as beginners and need, at least at first, to find their own way of solving the task with a tool that does not limit such exploration.

Problem: In the pedagogical endeavor for which the VisualMath book is designed, each activity was intended to grant opportunities for students to explore in order to formulate ideas. The traditional order of direct teaching of procedures followed by drilling, practice, and word problems (applications of the taught algorithms) was not a relevant consideration. The tasks require making sense of problems, and as the new core standards state, “persevere in solving them,” spending longer on analyzing givens, constraints, relationships, and goals. Although problem solving can always be helped by use of appropriate tools, it should be carried out strategically, constructing viable arguments and critiquing the reasoning of others. The primary means of designing problems are IDs. Especially useful are the Guiding IDs, designed to be the principal delivery channel of the message of the activity.

NAVIGATION OF NON-SEQUENTIAL RESOURCES

It is commonly believed that changing the authored order of a digital textbook is the most relevant and easy manipulation that serves teaching needs. But the authored order and sequence in digital resources are not trivial. Gueudet and Trouche (2010) argued that the notion of author and authorship is often less transparent in online sources than in printed material. Usiskin (2013), discussing the disappearance of transparency in integrative multilinked and multimodal textbooks, agreed. Improving textbook use is often a question of developing the right human-machine interface. Object-oriented user interfaces, hierarchical content trees, and other new navigation tools are being studied in this connection (Liang-yi et al. 2013). Teaching in which a principal challenge is conceptual navigation imposes a new type of professional responsibility. I therefore argue that in order to consider a collection of non-sequential, multimodal digital “pages,” which to a certain extent stand on their own, a textbook, the structure of the concepts and the interrelations between them must made visible by the design and must be conceptually “clean.” Two principles guided the design of the VisualMath eTextbook (described in detail in Yerushalmy (2013)) to achieve this conceptual transparency. Our first decision was to organize the content along a single view of the algebra, focusing on the algebra of functions. The second decision was to organize the materials around a relatively small number of mathematical objects and operations that can mathematically and pedagogically support a variety of progressions and sequences. This is not the common structure of mathematics textbooks, which usually represent a progression along various themes and views of algebra. In its current form, the VisualMath eBook accommodates two objects (functions), the linear and the quadratic, and six operations on the objects and with them. The six operations included do not form an exhaustive list. Rather, they are what Schwartz had called the “interesting middle” (Schwartz 1995) – operations that represent important mathematical concepts, and are appropriate and useful to learn as part of function-based school algebra. The operations are: represent (a function), modify (reforming the view or structure without changing the function), transform (using operations to transform a function into families of functions), analyze the changing of a function, operate with two functions (synthesizing new functions out of two different or identical functions), and compare two functions. The two lists, of the objects and the operations, are distinct and were therefore placed in an orthogonal organization in a 2D matrix map, where each cell represents the opportunities for learning resulting from the corresponding operation and object. Each operation with an object can take place in symbolic, graphic, or numeric representations. The design model, borrowed from a museum setting and used to form our guided-inquiry vision, is consistent with the distinction Kress and van Leeuwen (1996) made to describe linear and nonlinear texts. They compared linear texts to "an exhibition in which the paintings are hung in long corridors through which the visitors must move," and non-linear texts to an "exhibition in a large room which visitors can traverse any way they like… It will not be random that... a particular major painting has been hung on the wall opposite the entrance, to be noticed first by all visitors entering the room" (p. 223).

THE INTERNAL AUTHORITY OF COMMUNAL AUTHORING

Technological advancements, mainly of Web2 participatory tools, authoring tools for production of learning objects, and societal norms of collaboration consistent with constructivist pedagogies, pose a challenge to the accepted function of the textbook as a message completed in the past by a recognized external authority that must be unpacked by learners and teachers. Teachers everywhere create teaching materials, search through professional materials available on the Web, and share their repositories and ideas through online social networks. It is unclear whether and how teachers and schools could assume an important role in designing and developing curriculum materials, and how this would change the way they use textbooks in a sustained way. A major challenge to evolving collaborative textbooks designed by teachers is sustainability. Would teachers commit themselves to long-term participation in an evolving project? What form would the sustained collaboration take and from where would the required leadership come? In this regard, the French mathematics teachers' online association, known as Sesamath, is an important example to be followed. The association (described by Gueudet and Trouche (2012)), dedicated to the design and sharing of teaching resources, was created over a decade ago and has rapidly grown into an online community of practice. Leadership, which is considered to be a critical factor shaping the lifecycle of successful open-source projects (Bonaccorsi and Rossi 2003), seems to grow voluntarily and consists of teachers who assume the obligation of spending extensive time every day engaged in the virtual life of the association. Sesamath textbooks exist in both digital and printed versions, and most of the interactive resources are part of the Sesamath environment, external to the textbook. If we regard the textbook as a message about which content should be taught and how, coherence becomes an important requirement. Quality and quality control are important challenges to any collaboratively created open source. On the assumption that the quality of internal authority is improving as a result of the participation of large communities from multiple contexts, as proclaimed in the Wikipedia project, and that a crucial part of what makes for high quality open source products is the size of the authoring and participating community, we must acknowledge that the so far relatively small size of the open educational communities casts doubt on current measures of quality being acceptable.

CONCLUDING REMARKS

To conclude, we re-examine whether with the disappearance of the traditional object, the change of modes and means of the delivered authority adequately supports inquiry-based, student-centered learning that authorizes creative ideas. I argued that introducing new perspective of innovative design and inventing new ways to achieve transparency and clarity of non-ordered resources are two central challenges that must be met for the future textbook to preserve its basic educational functionalities. But technological innovation by itself has never produced a significant educational change. Some societal forces or other technology-related changes play an important role. It is likely that textbooks will become less central because common standards and assessments will play some of the roles that textbooks used to play. Standardization followed by high-stake, centralized digital assessment, together with curriculum documents that are growing to the size of a "slim textbook," may help the authority deliver the fundamental requirements and substantially reduce the authority of the textbook. Adapted teaching may gain strength in a world without textbooks, and personal tutoring sites (e.g., the Khan Academy) may become recognized authorities. Such tools, organized along core national standards, can provide learning and coaching opportunities, which new social norms can elevate to a status of authority. The question that is more difficult for me to answer is whether teachers will take the lead in the collaborative authoring of textbooks. Networked communities have been proven to bring about dramatic societal changes, and teachers worldwide have called for such communities to assume the role of textbook publishers, but it is difficult to imagine how teachers would become the responsible sustained authority for producing textbooks without a significant change in their professional worklife.

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MESSAGES CONVEYED IN TEXTBOOKS: A STUDY OF MATHEMATICS TEXTBOOKS DURING THE CULTURAL REVOLUTION IN CHINA

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The potential of textbooks as a source of learning differs among countries according to the different cultures and practices of the countries concerned. In China, the Great Proletarian Cultural Revolution (commonly known as the Cultural Revolution) was explicit in articulating communist values and criticizing the capitalist and traditional Chinese cultural elements and values. Selected topics in the mathematics textbooks from the Cultural Revolution period in China are analysed and compared to the textbooks published in Hong Kong roughly at the same time and with contemporary mathematics textbooks where the ideological values are not as explicitly propounded as those during the Chinese Cultural Revolution. It is argued that even contemporary mathematics textbooks still convey the values of the dominant authorities, albeit in a more implicit manner. Messages about the nature of mathematics, the nature and goal of education, as well as the political ideology of the time, are delineated.

Keywords: nature of mathematics, nature of education, political ideology, China, Hong Kong

SUMMARY

Textbooks are an important source of potential learning (Mesa, 2004), but the potential differs among countries according to the different cultures and practices of the countries concerned. So it is essential for textbook studies to be situated within the cultural contexts in which the textbooks are produced and utilized, since “mathematics textbooks ….. (are) historically developed, culturally formed, produced for certain ends and used with particular intentions”(Rezat, 2006, 482). No textbooks have illustrated Rezat’s claim more vividly than the textbooks produced during the time of the Cultural Revolution in China.

The Great Proletarian Cultural Revolution, commonly known as the Cultural Revolution, was a social-political movement that took place in China roughly between 1966 and 1976. It was a time of great political and social turmoil, and its espoused goal was to remove capitalist as well as traditional cultural elements from the country in order to achieve an idealistic form of communism. As such it was explicit in articulating communist values and criticizing the capitalist and traditional Chinese cultural elements and values. One important arena for this revolution is education, and this arena provides an excellent opportunity for studying the interplay between the cultural context and educational goals, through examining the messages conveyed in the textbooks.

In this plenary, selected topics in the mathematics textbooks from the Cultural Revolution period in China were analysed and compared to the textbooks published in Hong Kong roughly at the same time. Messages about the nature of mathematics, the nature and goal of education, as well as the political ideology of the time, were delineated. A comparison was also made with contemporary mathematics textbooks, where the ideological values are not as explicitly propounded as those during the Cultural Revolution. Yet, it is argued that even these contemporary mathematics textbooks that were not produced at a time of political upheaval still convey the values of the dominant authorities, albeit in a more implicit manner. This testifies to the intricate relation between the mathematics textbook and the underlying values of the education systems concerned, and corroborates the assertion that the mathematics textbooks can be regarded as a cultural artifact. Implications of the analysis for curriculum developers and classroom teachers were discussed.

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PLENARY PANEL

BACK TO THE FUTURE OF TEXTBOOKS IN MATHEMATICS TEACHING

Ken Ruthven (Chair, UK)

Jere Confrey (USA)

John Ling (UK)

Binyan Xu (China)

During the plenary panel, the invited experts drew on their extensive experience in conducting research and/or development relating to textbooks (a term which is taken as also covering cognate curriculum materials) to discuss factors and considerations which have influenced their own work on textbooks, and to offer key points of advice to the textbook developers of the future.

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THE TEXTBOOK IS DEAD: LONG LIVE THE TEXTBOOK

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This conference takes place during a period of considerable change in the form of textbooks and challenge to their place in mathematics teaching. This situation has many aspects, including shifts in the types of educational materials used in schools; the emergence of new and multiple modalities for representing mathematics and mediating learning; criticism of the pedagogically malign influence of narrowly examination-oriented textbooks; the rise of re-sourcing and a trend away from use of a single textbook towards combining varied resources; and renewed ideas about the use and functioning of textbooks. These are discussed with particular reference to mathematics teaching in English secondary schools.

Keywords: teaching resources, examination-oriented textbooks, England

INTRODUCTION

This conference takes place during a period of considerable change in the form of textbooks and challenge to their place in mathematics teaching. This situation has many aspects, five of which I shall review here. Since we are meeting in England and that is the system with which I am most familiar, I will employ it as a convenient example to illustrate these issues.

SHIFTS IN THE TYPES OF EDUCATIONAL MATERIALS IN USE

To examine trends over time in the use of – and so the demand for – textbooks and other competing types of educational materials, I will draw on evidence from the TIMSS surveys (Beaton et al., 1996, p. 158; Mullis et al., 2012, p. 394). Between 1995 and 2011, the proportion of mathematics teachers in England who reported using a textbook, either as the “basis for instruction” or “as a supplement”, in teaching their Year 9 (Grade 8) class fell from 100% to 86%. Moreover, in 2011, only 29% of teachers affirmed that textbooks served as the “basis for instruction” with their class, little more than the 21% for workbooks or worksheets, or the 21% for “computer software for mathematics instruction” (which seems likely to have been interpreted by respondents as including digital resources such as projected and online resources). The proportion of teachers who reported that these types of resource served “as a supplement” in teaching their class was 57% for textbooks, 74% for workbooks or worksheets, and 76% for digital resources. This testifies to a diminishing and more marginal use of textbooks and suggests an increasing use of digital resources (bearing in mind, too, that TIMSS did not choose to ask an equivalent question about these in the earlier survey). Fuller examination of the TIMSS data suggests that, while England is setting the pace in this shift, the trend is much wider (Ruthven, 2013).

NEW AND MULTIPLE MODALITIES FOR REPRESENTING MATHEMATICS AND MEDIATING LEARNING

Recent reports give some insight into the main ways in which digital resources are being used in mathematics teaching in English schools. A state-of-the-system review of mathematics teaching based on school inspections by the Office for Standards in Education [OfStEd] indicates that it had become the norm for classrooms to be equipped either with an interactive whiteboard or with a data projector to which a classroom or teacher computer is attached. This review reported that:

> The interactive whiteboard featured in many (but not all) primary and secondary classrooms... Good practice included the use of high-quality diagrams and relevant software to support learning through, for example, construction of graphs or visualisation of transformations. Pupils enjoyed quick-fire games on them. However, many of the curricular and guidance documents seen did not draw sufficient attention to the potential of interactive whiteboards. Additionally, too often teachers used them simply for PowerPoint presentations with no interaction by the pupils. (OfStEd, 2008, p. 27)

A survey of mathematics teachers in London secondary schools found that PowerPoint presentations and online curriculum materials were the most frequently used digital teaching resources (Bretscher, 2011). Such presentations may be locally prepared or purchased commercially, and online materials are typically accessed on websites providing banks of resources, often reproduced or adapted from conventional textbook or worksheet exercises or activities, but some are more dynamic and interactive in their conception and operation.

There is evidence, then, that some digital resources are associated with more far-reaching shifts in the representation of mathematics and mediation of learning. Indeed, the next most frequently used type of resource reported in the London survey was graphing software which, like the geometry software – reported as being less frequently used – supports a more visual, dynamic and interactive style of engagement with mathematical ideas than conventional media afford. The interest in these novel media is indicated by initiatives such as the recent pan-European Intergeo project which established the i2geo platform to serve as a repository of dynamic mathematical resources (Trgalova & Jahn, 2013). The response of many publishers has been to seek to enhance their textbooks by providing supplementary materials – via one digital medium or another– which are similarly visual, dynamic and interactive. In the longer term, then, we can expect to see the emergence of digital textbooks incorporating such new and multiple modalities for representing mathematics and mediating learning (Center for the Study of Mathematics Curriculum, 2010).

CRITICISM OF THE PEDAGOGICALLY MALIGN INFLUENCE OF NARROWLY EXAMINATION-ORIENTED TEXTBOOKS

Over the last 20 years, however, the intensification of high-stakes assessment and evaluation has shaped the style of textbook widely used in English schools in a much less progressive manner. One study compared the use of mathematics textbooks in English, French and German schools: it reported that the textbooks in widespread use in England tend to present mathematics as a set of unrelated rules and facts; they serve mainly to provide exercises for repeated practice of routine skills by pupils (Haggarty & Pepin, 2002). Another study compared the mathematics textbook series most widely used in the 1970s and in the 2000s: it found that the more recent series made few demands on pupils for deeper thinking; and that this was compounded by the advice given in the teacher's guide where little account was taken of research on pupils' learning of mathematics (Hodgen et al., 2010). Such use of currently popular textbooks is reported by OfStEd (2008) as being typical of many English schools, and this has led to widespread depreciation of the role of textbooks in favour of a re-sourcing approach.

THE RISE OF RE-SOURCING AND A TREND AWAY FROM USING A SINGLE TEXTBOOK TOWARDS COMBINING VARIED RESOURCES

The idea of re-sourcing teaching is rooted in a broader critique of standard classroom materials designed for system-wide dissemination. Advocating a more devolved approach to the creation, diffusion and adaptation of resources, the re-sourcing movement seeks to bring a more diverse range of sources for subject teaching and learning into play, and to involve teachers themselves in matching these to local working contexts and educational values. Its proponents suggest that more localised design efforts have important beneficial effects, in generating resources better tailored to specific school contexts, and in deepening teacher understanding of the basis for their effective use. Indeed, some educational systems – including the English one – have a long history of re-sourcing activity often organised around local or national professional networks.

Such ideas have been influential among many teacher educators and professional developers in England. The result has been professional opinion and national policy which has encouraged each school to developing its own "scheme of work" in mathematics. The official guidance accompanying the introduction of a national curriculum in mathematics encouraged all schools to develop such a scheme; the subsequent introduction of a national framework for mathematics teaching effectively mandated teachers in each school to develop a collective "scheme of work" to guide their teaching of mathematics (Ruthven 2013). While such schemes can, of course, make reference to standard curriculum materials, notably textbooks, the expectation is that the use of such materials will be locally customised and complemented by a more diverse range of resources. This movement, then, has made an important contribution to the shifting pattern of usage of educational resources noted earlier amongst lower secondary mathematics teachers in England where all types of educational material were much more frequently used "as a supplement" than as "basis for instruction", and, for a substantial minority of teachers, no type of educational material provided the "basis for instruction" with their class.

RENEWED IDEAS ABOUT THE USE AND FUNCTIONING OF TEXTBOOKS

Nevertheless, it is worth recalling that some textbooks have been informed both by research insights and professional wisdom; they have been refined through extensive trialling and feedback from users; and they provide a coherent system of high-quality pedagogical resources. Recently, indeed, attention has focused on the issue of how to design textbooks so that they are not just educational for pupils but "educative" for teachers through the support they provide for development of their mathematico-pedagogical expertise (Davis & Krajcik, 2005). Historically, indeed, innovative programmatic materials have been an important vehicle for renewing the school curriculum, developing the knowledge of teachers, and improving the quality of teaching and learning. For example, the School Mathematics Project [SMP] textbooks of the 1970s that Hodgen and colleagues found superior to the commercially developed textbooks of the 2000s were of this innovative type. Of course, those later examples demonstrate also that not all textbooks achieve high quality. However, a shift from the diffusion of standard curriculum schemes and materials towards more localised re-sourcing by teachers themselves raises the same important questions about quality and coherence, about viability and economy. Recent thinking, then, has emphasised the potential role of textbooks in supporting quality of teaching and in developing pedagogical expertise, particularly in the many education systems which continue to struggle to recruit and retain confident and accomplished mathematics teachers.

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BUILDING DIGITAL CURRICULUM FOR MIDDLE SCHOOL USING CHALLENGES, PROJECTS AND TOOLS

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In my talk, I discussed the activity of designing digital curriculum for middle grades mathematics. My starting point was the implications of placing students into a mathematical workspace that supports collaboration, presenting, sharing, and working on dynamic tools. From this, I discussed two kinds of mathematical work: engaging in mathematical projects and using challenges to spur productive struggle. For mathematical projects, I illustrated how to use media, context, tools and presentations. For productive struggle, I illustrated how to sequence and design challenges built around learning trajectories for major topics. In each kind of mathematical work, I focused on what features are made possible by digital affordances of the tablet.

Keywords: middle grade mathematics, mathematical workspace, mathematical projects, digital affordances

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THE SCHOOL MATHEMATICS PROJECT: MAPPING ITS CHANGES OF DIRECTION

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This presentation focused on aspects of the work of the School Mathematics Project (SMP), one of a number of initiatives originally set up in the 1960s to modernise the secondary mathematics curriculum and foster in students a deeper understanding of the subject. By the mid-1970s, pedagogical thinking had embraced mixed-attainment classes and the SMP responded with a new set of materials designed to facilitate this in the first two years of secondary school. The model of individualised learning was adopted, with a pedagogy heavily influenced by research that was largely Piagetian in approach. Mathematical content was presented through concrete situations in profusely-illustrated and informally-written booklets. Over time the weaknesses of individualised learning became apparent and the title of the SMP's next, and current, initiative – SMP Interact – *indicates a recognition of the important element missing from the individualised approach. Two external developments had serious repercussions on this project. First, the introduction of the National Curriculum for England curtailed the freedom that SMP had hitherto enjoyed to decide on curriculum content. Second, examination boards began publishing their own textbooks matched to the modules of their assessment schemes. This practice has grown and makes it increasingly difficult for independent organisations to compete in the textbook market.*

Keywords: School Mathematics Project (SMP), individualised learning, National Curriculum, textbook market

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COMPETENCE-ORIENTED DEVELOPMENT OF MATHEMATICS TEXTBOOKS IN THE TWENTY-FIRST CENTURY IN CHINA

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In the twenty-first century the development of mathematics textbooks in China is emphasizing the cultivation of mathematical capabilities. The core capabilities, originally including operations, logical reasoning, and spatial imagination, have been extended. The cultivation of mathematical capabilities highlights basic capabilities, including the sense of numbers and symbols, spatial concepts, geometric intuition, consciousness of data analysis, capabilities of calculation and reasoning, and the idea of modelling. While carrying out the national project Cross-national comparison of senior high school mathematics textbooks, we developed an analytic index framework to investigate the organization and representation of inquiry content in mathematics textbooks. The preliminary findings indicated that Chinese textbooks used in Shanghai, as well as used in other provinces, paid attention to incorporating open-ended questions and representing problems with interrogative sentences. Concrete findings were reported in this panel session.

The new development of mathematics textbooks in China attaches importance to the different roles of mathematics textbooks in relation to teaching and learning. From reviewing the literature and doing our own interviews with teachers, we found that teachers changed their understanding of the roles of mathematics textbooks and made efforts to put the understanding into practices. In this panel session we discussed the successes and failures in dealing with textbooks in China.

Keywords: inquiry content, mathematical capabilities, Shanghai, China

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SYMPOSIUM 1

TEACHERS EDITING TEXTBOOKS

Teachers editing textbooks: Transforming conventional connections among teachers, curriculum developers, mathematicians, and researchers
Ruhama Even & Michal Ayalon

Teachers editing textbooks: Changes suggested by teachers to the mathematics textbook they use in class
Shai Olsher & Ruhama Even

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TEACHERS EDITING TEXTBOOKS: TRANSFORMING CONVENTIONAL CONNECTIONS AMONG TEACHERS, CURRICULUM DEVELOPERS, MATHEMATICIANS, AND RESEARCHERS

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*The relationships between teachers and textbooks are generally associated with curriculum enactment and teachers' use of curriculum materials. This paper examines what might be gained and what is entailed in providing teachers with the opportunity to edit the textbooks they use in class, using the M-TET (**M**athematics **T**eachers **E**dit **T**extbooks) project as the focus of investigation and illustration. The paper first presents the rationale and structure of the M-TET project, which invites teachers to collaborate in editing the textbooks they use in their classes as a means to transform the conventional connections among teachers, curriculum developers, mathematicians, and researchers in mathematics education into multi-directional and more productive ones. Next, the paper illustrates: (1) the nature of the interactions of the participating teachers with the textbook authors and the mathematician that were made available to them for consultation, and (2) the contribution of these interactions to shaping the teachers' editing processes.*

Keywords: teacher-editors, mathematicians, curriculum developers, M-TET Project

INTRODUCTION

Conventional connections between teachers and curriculum developers are limited and mainly unidirectional – stemming from curriculum developers to teachers. In contrast to their central role in curriculum enactment and the use of curriculum materials prepared by curriculum developers (e.g., textbooks), teachers usually play a rather insignificant role in the development of textbooks. Teachers' aspirations about desired textbooks and adjustments that they make in textbooks – based on their experiences, their knowledge and beliefs about mathematics and its teaching and learning, as well as their acquaintance with the system in which they teach and with their own students – often remain unknown to curriculum developers.

Conventional connections between teachers and mathematicians are also limited. They occur mainly during the teacher preparation stage, when prospective teachers study advanced mathematics in courses taught by mathematicians. However, teachers rarely have opportunities to consult with mathematicians about the mathematics they teach in class throughout the years of their teaching career.

The M-TET (Mathematics Teachers Edit Textbooks) project[1]examines how the conventional connections among teachers, curriculum developers, mathematicians, and researchers in mathematics education might be transformed into multi-directional and more productive ones, while contributing to professional development and building of a professional community of teachers. Below we first present a general description of the project. Then we illustrate the nature of the interactions of the participating teachers with the textbook authors and the mathematician that were made available to them for consultation, and the contribution of these interactions to shaping teachers' editing.

THE M-TET PROJECT

As a country with a centralized educational system, the Israeli school curriculum is developed and regulated by the Ministry of Education. In 2009 the Ministry of Education launched a new national junior-high school mathematics curriculum (Ministry of Education, 2009). In response to the introduction of the new curriculum, the mathematics group in the Department of Science Teaching at the Weizmann Institute of Science began developing a new comprehensive junior-high school mathematics curriculum program entitled *Integrated Mathematics* (*Matematica Meshulevet*). The textbooks are used in more than 250 schools throughout Israel. The M-TET project, now in its fourth year, uses the *Integrated Mathematics* textbooks as a point of departure. It invites teachers to collaborate in editing the textbooks they use in their classes and to produce, as group products, revised versions of these textbooks that are suitable for a broad student population, while consulting with mathematicians, textbook authors, and researchers in mathematics education.

To enable collaborative textbook editing and the production of a joint revised textbook, we use, with some modifications, the MediaWiki platform and Wikibook templates for constructing the project website. This website serves as an online platform for collaborative work on a common database (i.e., a textbook) and for discussions in a forum-like fashion (for more information, see Even and Olsher, 2014).

Participation in this project consists of (1) on-going distance work, and (2) monthly face-to-face whole-group meetings. The on-going distance work includes textbook editing of various types, reacting to other participants' suggestions, and discussions of mathematical and pedagogical issues (for more information, see Even and Olsher, 2014). The monthly whole-group face-to-face meetings are built on the preceding teachers' distance work of textbook editing, and they also serve as departing points for subsequent distance work. These meetings consist of collaborative work on textbook editing, instruction on the technological tool, discussions of mathematical and pedagogical issues, and discussions of community working norms. From the second year on the face-to-face meetings include also semi-structured discussions with a mathematician, the textbook authors, and other experts, as well as carefully planned address of important issues related to the teachers' work.

[1] The M-TET project is part of the Rothschild-Weizmann Program for Excellence in Science Teaching, supported in part by the Caesarea Edmond Benjamin de Rothschild Foundation.

Participating teachers are provided with two kinds of support that accompany both the distance work and the face-to-face meetings. One is technical support in using the technological platform for textbook editing. The aim of this support is to provide a smooth running work environment that enables teachers to perform desired editing without having to deal with, or be constrained by, technological difficulties. The other kind of support is related to conceptual issues that emerge as part of the editing work. To address that, participating teachers are offered the opportunity to consult with various professionals throughout their ongoing distance work and the monthly face-to-face meetings. The professionals made available for consultation include the authors of the textbooks, a research mathematician, and researchers in the field of mathematics education.

During the first year of project the project team purposely avoided any intervention with, commenting on, or evaluation of the teachers' work, besides instructing the teachers on how to use the project website. The role of the project team during that year was to provide a smooth running work environment and to moderate, but not direct, the monthly face-to-face meetings. Similarly, during that year, the consultants associated with the project were explicitly instructed not to initiate any intervention with, comment on, or evaluate the teachers' work. Instead, the consultants were directed to respond only when explicitly approached by the teachers, and to address only queries related to the following areas: reasons for specific choices made in the textbook by the textbook authors, the mathematics in the curriculum, and research in mathematics education. Findings from a study that examined the changes that the first year participants in the project suggested to make in the 7th grade textbook they were using in class are reported in Olsher and Even (2014).

From the second year on the participating teachers continued to receive an autonomous work environment wherein they could freely edit the textbooks as they wished. However, the consultants associated with the project were allowed to freely comment on the teachers' editing suggestions and could freely address any query raised by the teachers. Also, a sizable part of the monthly face-to-face meetings was devoted to semi-structured discussions with the textbook authors and with the mathematician. In the following we present two examples from the work on editing the 8th grade textbook during the second year of the project to illustrate: (1) the nature of the interactions of the participating teachers with the textbook authors and the mathematician, and (2) the contribution of these interactions to shaping the teachers' editing processes.

ILLUSTRATIONS OF THE INTERACTIVE EDITING WORK

The Pythagorean Theorem

When introducing the topic of Pythagorean Theorem, the 8th year textbook asks students to find by measuring the lengths of the short leg *a*, the long leg *b*, and the hypotenuse *c*, of four drawn right triangles, to fill in the answers in a given table, and to look for connections between the lengths of the legs and the hypotenuse. Then the students are asked to evaluate whether a claim offered by a fictional student in the textbook "$a \cdot a = b + c$" (satisfying several of the triangles in the table but not all of them) is correct. This activity aims to highlight the idea that examples may serve as a useful tool for making generalizations, but not for making definite conclusions. Later in that lesson, the textbook presents the Pythagorean Theorem, illustrating it with several examples, and then states that the theorem is always true for right triangles.

A lively discussion developed among the teachers around two central issues regarding the way the textbook introduces the Pythagorean Theorem. The first issue concerned the fact that the lesson opens with a false statement. Two contrasting approaches were raised: One was to revise the textbook and start with a correct phrasing of the Pythagorean Theorem, and only later give examples that suit different relationships as well. The other approach was not to revise the textbook, because one way to deal with mistakes is to purposely start with experiencing an examination of a false statement that appears to be true. The second issue regarding the way the textbook introduces the Pythagorean Theorem concerned the fact that the textbook presents the idea that it is impossible to reach conclusions based on examples while simultaneously justifies the Pythagorean Theorem relying on a few examples only. This issue bothered the teachers, but they did not have a clear idea of how to deal with it.

The teachers decided to consult with the author of the textbook unit regarding the two issues. When meeting with her, the teachers first raised their dilemma regarding the opening of the lesson with a false statement. The author responded by explaining the potential of such an introduction to create a feeling of surprise that the Pythagorean Theorem is true, and the need to find a way to prove it. Below is an excerpt from the discussion:

Teacher A: I'm afraid that the error [the erroneous formula] will stick with the students.

Teacher B: Why? … We need to put the mistakes on the table. This is overwhelming. It creates a conflict. It requires them to use critical thinking.

Teacher C: It's not good to start a new topic with a mistake. I think that we need to change this.

…

Teacher A: It is simply similar to the Pythagorean formula and it's confusing.

Author: The idea is to illustrate that you can't depend on examples in order to generalize and to reach conclusions. The Pythagorean Theorem is a surprising theorem. But it won't be surprising if we just introduce it in class. Therefore, this is a golden opportunity to bring students to evaluate another formula that works in some cases and suddenly does not work, to build the need for a different sort of justification, not generalization from examples.

Teacher C: I agree. This is a wonderful opportunity.

This conversation followed by the teachers' suggestion to add a proof to the Pythagorean Theorem, which gained the support of the author:

Teacher D: I still have a problem. The goal is to prevent reaching conclusions from examples. We are showing them first that it's prohibited and then that it's okay. What are we showing them?

Teacher E: Why, then, isn't a proof added, even a visual one?

Teacher F: Good. I have a physical proof.

…

Author: Concerning what you said (turning to Teacher D), you’re right…. I think we should include a proof…. Perhaps if we revise the textbook we will include a proof – maybe the visual side of the proof. To have a justification for why the theorem is true.

Follow this conversation, the teachers decided to leave the introduction to the topic as is, and to add a proof for the Pythagorean Theorem. Several ideas for proof were raised (e.g., based on geometrical statements, visual) and the teachers found it difficult to make decision which proof would better fit the current learning stage. They decided to consult with the mathematician. When consulting with him later in that day, the mathematician supported the teachers’ suggestion to add a proof to the textbook: "More and more examples do not convince or prove… It is against mathematical thinking”. He emphasized the importance of building coherently the idea of proving in mathematics, beginning already in this stage. Yet, he added that he leaves the decision whether to add a proof, and of which kind, to the teachers, who are "the experts on pedagogy". However, in case the decision is not to add a proof or to add a partial proof, he strongly recommended adding a comment in the textbook that a complete proof will be presented in future learning.

Eventually, the teachers added a geometrical proof, adding also a link to an applet that concretizes the Pythagorean Theorem (by dragging and filling in the squares) directed at classes where the presentation of a proof is not appropriate.

Mathematical theorems and proofs

At the beginning of a later unit on mathematical theorems and proofs, the textbook introduces the inference statement “if p then q”, using statements related to everyday life, such as: “If it is raining then there are clouds in the sky”, “If it is snowing then the temperature is less than 2 degrees”. A stormy discussion developed among the teachers. Two opposite approaches were raised: One was to revise the textbook and omit this introduction, as inference in everyday life is based on experience and intuition, and thus does not correspond to the inference processes we want to teach in mathematics. Another approach was not to revise the textbook, since examples from everyday life are close to students’ heart and intuitive for them, especially for those that encounter difficulties. Teachers holding the first approach suggested replacing the examples taken from daily life with examples from the field of mathematics, such as, "If a polygon is a square then the sum of the angles in the polygon is 360^0".

After a vote was taken, the teachers decided (1) to revise the introduction of the topic to focus on mathematics, and (2) to add a unit on logic to strengthen understanding of mathematical statements. In a conversation with the mathematician during that day, he emphasized the importance of dealing with aspects of inference in mathematics. However, he was not sure whether there was one pedagogically sound way to do it. Later on, during a meeting with the author of the textbook unit, the author, who in principle supported the integration of everyday life into the examples in the textbook, was fond with the teachers' idea of constructing an additional unit. She asked to see the unit when ready in order to examine the possibility of including it in a revised version of the textbook.

Finally, the teachers began to plan the unit on logic. However, the work did not progress much. According to the teachers, the main difficulty was in building a new unit that was not based on already written work. As one teacher said:

> We were very excited about building a new unit that would deal with logic. We really thought this would be an important contribution to the textbook and to the students. But we were overwhelmed by the amount of work. This is not an easy task. We didn't know where to begin. It was too much to start from scratch, from zero.

FINAL NOTES

Several characteristics of the work environment offered by the M-TET project are not usually part of teachers' practice. They include, designing a textbook for a broad student population instead of focusing on the specific student population taught, generating a textbook by making changes to a textbook designed by expert curriculum developers, and consulting with professionals that are not part of the teachers' usual milieu (mathematician, curriculum developers, mathematics education researchers). Initial findings suggest that these characteristics have the potential to transform the conventional connections among teachers, curriculum developers, mathematicians, and researchers in mathematics education into multi-directional and more productive ones, while contributing to professional development and building of a professional community of teachers. This is illustrated in the following statement made by one of the teachers:

> The talks, the collaboration with the authors and the mathematician, there are not such things anywhere. It makes me feel important, that they want to listen to me and to work with me. They talk to me eye-to-eye… It changed the way I see myself and the way I use the curriculum in class. I ask myself questions now: What is the aim of this task? What would the author say about this part of the lesson? Is the mathematical concept in this lesson used correctly?

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TEACHERS EDITING TEXTBOOKS: CHANGES SUGGESTED BY TEACHERS TO THE MATHEMATICS TEXTBOOK THEY USE IN CLASS

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This study focuses on the changes teachers suggest making in math textbooks. The study addresses this issue by investigating the changes the first year participants in the M-TET project suggested to make in the math textbook they used in class, adopting Activity Theory as a theoretical framework. The participants, nine 7th-grade teachers, worked in an online environment combined with monthly face-to face group meetings and on-going consultation. Four types of teachers' changes to the textbook were identified: (1) Creating organizers to improve teacher work and accessibility to parents, (2) Integrating technological tools for improving mathematics teaching and learning, (3) Re-structuring textbook content presentation to better suit student learning, and (4) Adding materials for students with low achievements. This study contributes to gaining insights into teachers' needs, desires, and aspirations about textbooks and the mathematics curriculum, and also revealing areas that require professional development.

Keywords: teacher-editors, activity theory, textbook development, M-TET Project

INTRODUCTION

This study investigates the changes teachers suggest to make in the textbook they use in class. The study is situated in the M-TET Project, which invites teachers to collaborate in editing the textbooks they use in class and to produce, as group products, revised versions of these textbooks (Even & Ayalon, 2014; Even & Olsher, 2014). This unique setting enables an examination of the changes teachers suggest to make in the math textbooks from an angle that has not been used in other studies on this aspect.

BACKGROUND

Research studies that examine the enacted curriculum suggest that in many countries textbooks considerably influence classroom instruction: teachers often follow teaching sequences suggested by textbooks, and base classwork mainly on tasks included in textbooks (e.g., Eisenmann & Even, 2011; Haggarty & Pepin, 2002). Yet, this line of research also reveals that whereas textbooks affect classroom instruction teachers do not necessarily faithfully follow suggested textbook content, lessons, activities, pedagogical strategies and teachers' classroom practices.

For example, numerous studies show that teachers often do not cover in instruction all textbook parts (e.g., Drake & Sherin, 2006; Eisenmann & Even, 2011; Tarr et al., 2006), omitting parts that appeared less central. For instance, parts related to topics such as data analysis and probability (Tarr et al., 2006), or advanced or computer-based units (Eisenmann & Even, 2011). Research also shows that teachers do not essentially follow suggested work settings – individual, group or whole class work (Drake & Sherin, 2006; Eisenmann & Even, 2011).

Studies that focused on deeper characteristics of the enacted curriculum produced intriguing findings. For example, several studies revealed that teachers tend to lower the level of cognitive demand in mathematical tasks (Brouseau, 1997; Stein et al., 2000). Moreover, studies that examined complex aspects of the mathematics taught in class found discrepancies between the mathematics in the textbook and the mathematics teachers addressed in class, for instance, in the use of different representations (Even & Kvatinsky, 2010) or the type of algebraic activity enacted (Eisenmann & Even, 2011).

As portrayed above, the literature about curriculum enactment is a rich and important source of information about the changes that teachers make in the textbooks they use in class. Yet, this information is restricted by the focus on curriculum enactment. Our study approaches the question of what changes teachers make in textbooks from a different angle. The study is situated in the M-TET Project, which invites teachers to collaborate in editing the textbooks they use and to produce, as group products, revised versions of these textbooks (Even & Ayalon, 2014; Even & Olsher, 2014). The study investigates the changes that the teachers who participated in the first year of the project suggested to make in the math textbook they used in class, adopting Activity Theory (Engeström, 1987; Leont'ev, 1974) as a theoretical framework. Activity Theory was chosen because it attends to developmental aspects of human practices and links between individual and social levels, thus acknowledges the environmental conditions under which these practices take place.

METHODS

Participants in the study were the nine 7th grade teachers who participated in the first year of the M-TET project, and the project team. The teachers worked in an online environment combined with monthly face-to face group meetings and on-going consultation. Data sources included the project website (the edited wiki-book version of the textbook with its editing history and discussion pages), video-documentation and field-notes of the monthly whole-group meetings, interviews with the teachers at the end of that year, individual papers written by the teachers as a final assignment, and a research journal written by the first author. Analysis of the textbook editing activity focused on the changes suggested by the teachers. First we identified the changes by analysing actions and their goals (in terms of Activity Theory). Next, we characterized the dynamic work process, the engagement of the participants, the tensions, and the product, drawing on the Expanded Mediating Triangle (Engeström, 1987).

RESULTS

Our analysis revealed four types of change that the teachers suggested making in the textbook. In the following we describe, for each type of change, the goal, characteristics, and challenges that the teachers faced.

Creating organizers

The goal of creating organizers was to improve teachers' work and accessibility to parents. It is demonstrated by a teacher's response when asked at the end of the year to describe an important change made:

> A need that rises strongly from this book is to make it easier for orientation, to ease the work with it. … For me it was very, very important. I also know that other teachers find this as the main problem of this book. That it is flooded and teachers that don't prepare themselves in advance drown in it and get lost (Bar, Final interview, 22.08.2011).

Creating organizers involved: (a) improving accessibility to content highlighted in the textbook, (b) marking the textbook core, (c) adding meaningful unit and lesson titles, and (d) creating a table of contents for practice exercises. When working on creating organizers the teachers faced several challenges. For example, whereas all teachers agreed with the suggestion to mark the textbook core, there was not an agreement on how to go about doing that. Several teachers wanted to set criteria before marking the core. Yet, other teachers advocated marking the core without any prior constraints. This disagreement was not resolved during the whole year, and the debate on how to work on marking the core continued to influence the means by which this action had been carried out.

Integrating technological tools

The participating teachers integrated technological tools into the textbook in order to convey the unique advantages technology could bring to the teaching and learning of mathematics. Among these advantages the teachers indicated: independent work of students, expanding the variety of tools that could be used for calculation and illustration, and enhancing student interest. For example, when describing a change she incorporated, one of the teachers wrote: "The educational applets I integrated bring the student to focus on the learning assignment, meaning that the student will think and the computer will calculate" (Tal, Final assignment, 3.7.2011).

Integrating technological tools involved: (a) integrating tasks with online feedback, (b) integrating and creating online applets, (c) adding presentations, and (d) adding links to games. Most of the challenges that the teachers faced when working on integrating technological tools into the textbook were related to technical issues. For example, dealing with off-the-shelf products that could not be modified to fit teachers' specific aims. Such a product was, for instance, a quiz applet that the teachers wanted to integrate into the textbook in order to provide students with automatic online feedback. Yet, the applet they chose gave inappropriate feedback for zero and negative numbers. In order to use such tools and avoid inappropriate responses, the teachers altered textbook tasks.

Re-structuring textbook content presentation

The teachers made changes in the structure of textbook content presentation in order for it to better suit the way they believed students learn. For example,

> Assignment 3… it seemed very complicated and not easy for students… assignment 4 seemed easier. This is why I changed the order of assignments 3 and 4. What stood behind

> my change in this lesson… was arranging the assignments by the level of difficulty… Another participant thought there was no need for a change as the assignments were already ordered by the level of difficulty (Dana, Final interview, 22.08.2011).

Re-structuring textbook content presentation involved: (a) arranging tasks by the level of difficulty, (b) placing practice exercises immediately following the related lesson, (c) grouping exercises by content, and (d) changing the numbers in examples to be different than those in tasks. Unlike the case of creating organizers, there was not a collective agreement on some of these suggestions for re-structuring textbook content presentation. For example, grouping exercises by content revealed a fundamental difference of opinions among the teachers. Some teachers suggested grouping exercises by content, in a number of places in the textbook, to enable students to anticipate what is expected of them. This view was not shared by some of the other teachers, who thought that students should encounter mixed types of exercises, as one of the teachers said: “I don't think that it is necessary to group tasks of a certain type in a special place... Tests and quizzes also have mixed assignments of different types” (Hadas, wiki discussion webpage, 18.12.2010). These discrepancies of approaches continued to manifest throughout the year.

Adding materials for students with low achievements

The teachers felt that the textbook did not adequately address the needs of students with low achievements. This feeling is demonstrated in what one of the teachers said during one of the monthly meetings: “It is very difficult in a large class that almost all of it is comprised of students with low achievements... I need to thin out a lot, to filter and to take out the core” (Bar, monthly meeting, 24.5.2011).

The changes teachers suggested making in the textbook to better suit the needs of students with low achievements involved: (a) adding support in selected assignments, (b) adding preparatory exercises before starting a new topic, and (c) editing textbook units and offering them as an alternative parallel track for students with low achievements. Initially, the teachers started adding support in textbook assignments and preparatory exercises to address the needs of students with low achievements. These actions encountered an opposition, led mainly by one teacher. This teacher claimed that not all of the assignments in the textbook need to be addressed in class if they are too difficult for the students and thus, there was no need to change anything in the textbook. One of the teachers, who saw great importance in adding materials for students with low achievements, continued to look for potential solutions. Eventually, an accepted solution was developed after several months. The teachers decided that materials for students with low achievements would appear in separate pages, not altering the original pages of the textbook. This led to the editing of special units, based on original textbook units, which were added as alternatives to the original textbook for students with low achievements.

CONCLUSION

The unique setting of this study – teachers editing the textbook they use in class – contributed to gaining new insights regarding the changes teachers suggest to make in textbooks. Focusing on the changes teachers suggest when they edit the textbook instead of using it in class revealed the special attention the participating teachers gave to increasing the accessibility of the textbook contents. Yet, increasing the textbook accessibility was not the sole focus of the changes that the teachers in this study incorporated into the textbook. In addition to working on the textbook accessibility the teachers attended to several other aspects they thought were important. They invested a great amount of effort to integrate technology into the textbook, re-structured the textbook content to better suit student learning (as they conceive it), and worked on making the textbook fit for a broad population of students, in this case by adding materials for students with low achievements.

This study contributes to improving the ability of the research community to study from mathematics teachers' wisdom of practice, and also reveals areas that require professional development (e.g., student learning). In addition, this study could assist professional curriculum developers and policy makers learn about teachers' needs, desires, and aspirations about textbooks and the mathematics curriculum.

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SYMPOSIUM 2

LESSONS LEARNED FROM THREE DECADES OF TEXTBOOK RESEARCH

Lessons learned from three decades of textbook research
Denisse Thompson & Sharon Senk

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LESSONS LEARNED FROM THREE DECADES OF TEXTBOOK RESEARCH

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The University of Chicago School Mathematics Project (UCSMP) is a K-12 curriculum development and research project that has generated materials for U.S. classes consistent with national recommendations. As part of the curriculum development for grades 6-12, the Secondary Component has conducted research on the effectiveness of its textbooks. Research studies typically involve a combination of formative and summative elements conducted simultaneously. Formative elements inform author teams about content, sequence, and other aspects of the materials that need revision prior to commercial publication. Summative aspects investigate the effectiveness of the materials in typical classroom settings in comparison to textbooks already in place in schools. In research over three decades, numerous issues have arisen and lessons have been learned about dealing with these issues. This paper shares six lessons we have learned in conducting textbook research that we believe transcend national contexts and may be of benefit to international scholars.

Keywords: University of Chicago School Mathematics Project (UCSMP), textbook development, classroom enactment, curricular effectiveness

INTRODUCTION

The University of Chicago School Mathematics Project (UCSMP) is a K-12 curriculum research and development project that began in the U.S. in 1983 with funding from private foundations. At that time, national recommendations for curriculum reform were being discussed (e.g., National Commission on Excellence in Education, 1983; National Council of Teachers of Mathematics, 1980), and UCSMP attempted to develop curricula to implement these recommendations. Over the last three decades, the textbooks have been updated several times to be on the leading edge of reform initiatives.

In this paper, we focus on work of the Secondary Component, which has developed textbooks for grades 7-12, and beginning in 2005, for grades 6-12. All secondary textbooks have common features: wide mathematical scope, integrating algebra, geometry, discrete mathematics, and statistics/probability as appropriate; an expectation that students will read and write about mathematics; the use of applications to show how mathematics is applied in the real-world; a multi-dimensional approach to understanding, balancing Skills, Properties, Uses, and Representations; and integration of appropriate technology, including graphing technology, computer algebra systems, and dynamic geometry software.

RESEARCHING TEXTBOOK EFFECTIVENESS

Since its inception, UCSMP has conducted research on the effectiveness of its textbooks as part of the development process (e.g., Mathison et al., 1989; Senk, 2003; Thompson, 1992;

Thompson & Senk, 2001; Thompson, Senk, & Yu, 2012; Thompson et al., 2003, 2005; Usiskin, 2003). These studies contain both formative and summative components. Formative elements inform authors about aspects of the materials that need revision prior to commercial publication. Summative aspects investigate the effectiveness of the materials in comparison to the textbooks already in place at schools.[1]

We have faced a number of issues and learned many lessons while conducting this research. Although some issues and lessons may be unique to the U.S. context, we believe many are generalizable and applicable across countries. These are the ones we share in this paper.

Lesson 1: When studying the effectiveness of a textbook, the classroom is the appropriate unit of analysis.

Mathematics teachers in U.S. secondary schools typically have classes of 20-40 students. Although part of a typical class period may involve students working in small groups, instruction is generally teacher-led, not self-paced by students. Consequently, the class rather than the student is the appropriate unit of analysis for assessing the effectiveness of one textbook compared to another.

UCSMP research studies use a matched-pair quasi-experimental design, matching classes on the basis of one or more pretests of prerequisite knowledge (Campbell & Stanley, 1963). Each pair of classes within a school is then a mini-study, with patterns of achievement differences considered across all schools to determine the overall effectiveness of a textbook. Such a design ensures comparability of groups of students, regardless of whether random assignment to classes may have been possible. This design also avoids the methodological difficulties that ensue from matching individual students or adjusting for differences in students' backgrounds using ANCOVA (Kilpatrick, 2003).

Lesson 2: Compare classes within the same school when possible.

We have found it essential to match classes within the same school rather than across schools in different districts or even in the same district. Why might this be important? Schools are mini-communities with cultures of their own. Some have a culture of high expectations with high achievement and good behaviour; all students are expected to continue studying mathematics and pursue education after high school. Others have lower expectations, with students having attendance and behaviour problems that result in lower achievement. Even if pretest scores suggest that students' prerequisite mathematics knowledge is comparable across classes in two schools, the unseen environmental factors related to culture in the two schools have the potential to influence the effectiveness of a particular textbook with students. In addition, external and internal influences on teachers that support or inhibit their teaching are likely to be the same within a school but are often different across schools. By comparing classes in the same school, these environmental or school culture factors should have the same effect on students using different textbooks.

In several studies, we observed that *posttest* scores for classes in one school, while an improvement over their pretest scores, were below the *pretest* scores for classes in another school (e.g., Thompson & Senk, 2001). Regularly, there are larger achievement effects between schools than there are within schools.

Lesson 3: Select more classes for study initially than you think you may need.

Personnel and financial resources often determine how many sites might be selected for participation in a research study. UCSMP studies investigate the use of textbooks over an entire school year because developers want to assess the effectiveness of the textbook in typical school settings. Consequently, UCSMP provides textbooks to all classes participating in its research studies. In exchange, schools agree to identify classes of students that should be similar in achievement based on previous coursework; maintain the integrity of classes throughout the year, without having students change class periods or teachers; have teachers use the UCSMP materials without heavy supplementation from other sources; facilitate the use of provided technology in the UCSMP classes; permit several days of testing at both the beginning and end of the school year; expect teachers to complete regular chapter evaluation forms; and provide access for classroom observations at some point during the school year (Thompson, Senk, & Yu, 2012).

Despite these agreements, we have experienced numerous unanticipated situations. For instance, teachers have failed to give all the pretests, so it became impossible to determine comparability of classes. Or, schools have changed teachers' assignments, so the teacher using the new UCSMP textbook had not originally agreed to be involved; during the school year, some of these teachers dropped out of our studies without informing the research team. In addition, some schools, particularly high schools, permit students to change class periods and teachers at the semester break. Because the final sample in our studies consists of only those students who have taken all project-administered tests and stayed in the same class all year, students who changed class periods were lost from the study. These are just some of the unanticipated issues we have faced, even when we have been judicious in school selection.

Lesson 4: Collect data from teachers about the opportunities they have provided students to learn the mathematics in the textbook.

Curriculum researchers generally recognize the importance of collecting information about the extent to which teachers have actually implemented the textbook, what some call *fidelity of implementation* (e.g., National Research Council, 2004). Throughout our studies, UCSMP teachers complete an evaluation form for each chapter on which they rate the lessons taught and homework questions assigned to assist developers in making revisions, and comment on particular features of a chapter that might be unique to UCSMP. Comparison teachers complete a simplified form to indicate what lessons they taught and questions they assigned.

Information from these forms provides broad insight into students' opportunity to learn mathematics. For instance, among six teachers using the UCSMP *Algebra* textbook, the percent of lessons taught ranged from 47-100% (Thompson & Senk, 2010). However, there was only 1 of 13 chapters for which these UCSMP teachers taught all lessons. Within this chapter, the percent of questions assigned ranged from 25-97%. Hence, these results suggest major differences in opportunity to learn, even among teachers using the same textbook.

We have also found it critical to have teachers report specifically on whether they taught or reviewed the content needed for their students to answer *each* posttest item. This is true for assessments constructed by UCSMP staff, where we might be particularly interested in whether items are perceived as appropriate for comparison students, as well as standardized instruments. Among the six UCSMP *Algebra* teachers mentioned above, there were only 16 of 32 items (50%) from the standardized *Terra Nova Algebra Test* that all teachers reported as having taught or reviewed the content needed for their students to answer the items.

When considering teachers using different textbooks, these opportunity-to-learn (OTL) results can vary widely. For instance, in the study of *Transition Mathematics* at grade 7, we had five pairs of matched classes in four schools, with one teacher in each pair using the UCSMP textbook and the other teacher using the textbook already in place at the school. Three posttests were given: for the standardized *Iowa Algebra Aptitude Test*, the reported OTL ranged from 70-92% of the items; on the UCSMP developed multiple-choice test, the reported OTL ranged from 55-100%; and on the UCSMP constructed-response test, the reported OTL ranged from 54-100% (Thompson, Senk, & Yu, 2012).

We use these differences in OTL to analyse and report achievement in three distinct ways. First, we report achievement by matched pair for the entire test, and indicate teachers' OTL percent. Second, for each matched pair of classes, we consider just those items for which both teachers in the pair reported having taught or reviewed the content; we call the results on just these items the Fair Test, because OTL is controlled at the matched pair level. Third, we consider those items for which all teachers in the study, both UCSMP and comparison, reported teaching or reviewing the content; we call the results on just these items the Conservative Test because OTL is controlled at the overall study level (see, for example, Senk, Thompson & Wernet, 2013; Thompson & Senk, 2001; Thompson, Senk, & Yu, 2012).

By using the OTL measures as reported by the teacher, we consider achievement differences in ways we believe are transparent. Some achievement differences that are significant for the entire test are not significant when OTL is controlled. Additionally, when we report achievement on individual items, we indicate when teachers have reported not teaching the mathematics needed for that item to aid in interpretation of results.

Lesson 5: Collect multiple measures of implementation of the textbook, and when possible, collect implementation data from both teachers and students.

Because we typically have 6-12 schools in a field-study in various geographic locations, we rely extensively on teacher self-report data. Although some might criticize such data, we believe it is possible to have faith in teacher reported data when collected from several perspectives. UCSMP teachers complete a form for each chapter they teach and questionnaires at the beginning and end of the school year. They participate in an interview in conjunction with a classroom visit and attend two focus group meetings with other UCSMP study teachers. Thus, data collection at different points provides an opportunity to triangulate, ensuring faith in the integrity of data. These data provide empirical evidence that even teachers using the same textbook provide dramatically different opportunities to their students to learn mathematics (Thompson & Senk, 2014).

Implementation is not just dependent on the teacher, but also depends on students. Do they complete as much homework as teachers intend? Are they using technology as frequently and in ways that teachers expect? Do they perceive the importance of reading and writing about mathematics, and recognize when teachers engage them in these activities? Thus, we solicit data from both teachers and students to determine the extent to which they have similar perspectives about classroom instruction with the textbook. When reporting these data, we indicate how teachers and students responded to identify any anomalies that might help interpret achievement. Student perspectives on instructional strategies, such as reading or use of technology, provide another triangulation tool to gauge the accuracy of teacher reports on such strategies. Although we do not expect perfect agreement, we have typically found that students and teachers report similar frequencies with which given strategies are used in class.

Lesson 6: Pilot everything, including items, instruments, and procedures.

Piloting is essential to the research process. If a new instrument is needed, items are often constructed to be similar to those in either the UCSMP or comparison textbook. Even when knowledgeable individuals create such items, they may fail to anticipate issues. For instance, in the study of the first edition of *Precalculus and Discrete Mathematics*, one form of a posttest originally had an item for which students were given a statement and then asked to identify the assumption to begin a proof of that statement by contradiction. Later, when asked to write a statement that could be proved by contradiction and prove it, many students simply wrote the statement used earlier on the form. Thus, this item failed to assess the intended understanding. Because piloting identified this failure, we were able to revise the posttest.

However, at other times, we have failed to catch an incorrect graph or a problem whose numbers enable students to obtain a correct answer with incorrect processes. We often use portions of assessments from previous studies and do not always pilot the revised instrument; however, piloting is critical to the process. We often pay the price when we shortcut this step.

CONCLUSION

Since the 1980s, UCSMP researchers have studied the effectiveness of the UCSMP textbooks prior to their final commercial publication. Along the way, we have faced issues about study design, documenting implementation, and analysing results in ways that are fair to both UCSMP and comparison classes. In dealing with these issues, we have learned numerous lessons, six of which we have shared here. Further discussion of ways we have assessed textbook implementation can be found in Thompson and Senk (2012).

Endnote

[1] All UCSMP evaluation reports are listed at http://ucsmp.uchicago.edu/resources/materials/.

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SYMPOSIUM 3

US MATHEMATICS TEXTBOOKS IN THE COMMON CORE ERA

US mathematics textbooks in the Common Core era: A first look
William H. Schmidt & Richard T. Houang

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US MATHEMATICS TEXTBOOKS IN THE COMMON CORE ERA: A FIRST LOOK

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Since 2010, 45 states and the District of Columbia in the US have initiated the implementation of the Common Core State Standards for Mathematics (CCM). Two key implementation issues for mathematics were examined: teacher coverage and the coverage in available textbooks. Popular textbook series were analysed following the textbook analysis procedures developed in TIMSS. During 2011, participating K-8 teachers recorded their daily coverage of the different CCM standards. In addition, a group of experts - mathematicians, math educators and practitioners – were asked to assign instructional time to the standards according to their complexities and relative importance for that grade's instructional content. The results indicated that US textbooks generally did not cover all of the on grade CCM standards. On average, 70% of the CCM standards were covered at each grade. In terms of content coverage, only an average of 43% of the book covered the on grade standards. Time spent by teachers was more aligned with the textbook suggested coverage than with the experts. The policy implications of these results on the implementation of CCM standards are also presented.

Keywords: CCSSM (Common Core State Standards for Mathematics), teacher coverage of CCSSM, textbook coverage of CCSSM

INTRODUCTION

The Centre for the Study of Curriculum (CSC) has focused on studying the implementation of the Common Core State Standards for Mathematics (CCM) since they were adopted by 45 U. S. states and the District of Columbia. During 2011, CSC conducted surveys of teachers, curriculum directors, and parents in those states that had adopted the standards. The vast majority reported supporting the idea of common standards for all students and, more specifically, for the Common Core in mathematics (Cogan, Schmidt, & Houang, 2013a; 2013b; 2013c). Nonetheless, results also suggested three major impediments to the successful implementation of the CCM: 1) teachers use of the standards in planning and conducting their classroom instruction; 2) the presence of instructional materials that can support instruction of all the CCM standards; and, 3) teachers' understanding of and use of the CCM's mathematical practices in their classroom instruction.

This paper focuses on the available instructional materials. It includes some findings from a related study on how teachers were responding to the CCM in their classroom instruction. The goal of this work is to learn how the CCM Standards are being interpreted and used to support and guide classroom instruction. Ultimately, we hope these insights would be included in the design of professional development for teachers that would improve their understanding and use of the CCM Standards so as to improve students' understanding of mathematics.

The conceptual and theoretical foundation of the present work – that coherent, focused, and rigorous standards would lead to improved student learning – was one of the main conclusions from PROM/SE (see http://promse.msu.edu) and of the U.S. participation in the Third International Mathematics and Science Study (TIMSS) (Schmidt, et. al., 2001).

ANALYSIS OF INSTRUCTIONAL MATERIALS

In PROM/SE, teachers and mathematics experts examined the textbook series used by participating districts and identified what should be covered at each grade according to their district standards as well as providing links to the relevant portions of their textbooks, kits, and other instructional materials in use. The mapping process enabled districts to make modifications in terms of the sequence and amount of coverage, fill any gaps while at the same time making clear those standards that are not adequately addressed with their current curricular materials. The final product provided a coherent road map for each district in mathematics that described the order in which the district's current textbook chapters and/or kits may be best organised, both within a given grade as well as across grades.

At the advent of CCM Adoption, CSC took on mapping/encoding additional mathematics textbook series currently in use. Without new materials, districts must continue to rely on what they have on hand. The procedure we followed was originally developed for TIMSS. It concentrates on the coverage of CCM Standards in the textbook materials. It is a low-inference approach in that the coder determines if a lesson in a textbook covers one or more CCM Standards. As expected, textbooks published prior to the publication of the CCM include content that address CCM Standards and thus embody them to some extent. We present here some of our findings from this examination of textbooks.

Series	Pub. Year	Grades	K	1	2	3	4	5	6	7	8	Ave. per Grade	Average
1	2005	K-6	80%	88%	71%	78%	51%	55%	70%			71%	
2	2007	K-6	76%	75%	86%	68%	76%	63%	57%			71%	
3	2007	K-5	72%	79%	71%	65%	78%	55%				70%	
4	2008	K-5	84%	54%	75%	59%	57%	50%				63%	69%
5	2009	K-5	80%	92%	75%	76%	81%	75%				80%	
6	2009	K-5	80%	79%	61%	51%	70%	53%				66%	
7	2012	K-5	84%	54%	75%	59%	57%	50%				63%	
8	2012	K-5	96%	100%	93%	89%	92%	73%				90%	93%
9	2013	K-5	88%	100%	96%	97%	100%	93%				96%	
10	2006	6-8							34%	56%	67%	52%	
11	2006	6-8							62%	77%	61%	67%	
12	2007	6-8							57%	63%	69%	63%	54%
13	2009	6-8							19%	58%	50%	42%	
14	2009	6-8							40%	47%	56%	47%	

Figure 1. Percent of on grade CCM standards covered in 14 textbook series

Critical issues have emerged in evaluating the extent to which a textbook "embodies" the CCM: 1) the extent to which the CCM content is found in the textbook; 2) the extent to which the CCM content in a textbook matches the grade level assignment of topics/standards of the

CCM; 3) the extent to which the emphasis provided in the CCM standards reflects the coherence and focus of mathematics envisioned by the CCM: and 4) the extent to which the on-grade CCM coverage is dominate in the book.

The textbook series we coded are among the most widely used across the US and reflect tremendous variation in the percent of the appropriate CCM Standards covered for all of the grades included in the series. The analysed books were used by more than 60% of the students in the 41 states that responded to the survey. Figure 1 makes clear the challenge teachers, schools, and districts currently face. Very few textbooks for any given grade address all the topics identified by the CCM standards for that grade. Seventy percent of the on-grade CCM standards were covered. This is the average series by grade coverage. K-5 and K-6 series are better than middle school series (74% vs 54%). And series published most recently ("CC Edition") did much better (69% vs 93%).

Another key finding is that these textbook series are not organized in a manner that can easily support instruction according to the CCM. The instruction for each CCM Standard is distributed throughout a textbook in two, three, or more places and embedded in many disjointed lessons. This diffuse or scattered treatment of the standards makes it difficult for teachers to construct focused and coherent instruction. Consequently textbooks do not reflect the mathematical coherence embedded in the CCM.

Figure 2 illustrates in greater detail how each grade's textbook lines up with the CCM's grade level assignment of topics/standards for one textbook series. The data is from one of the better grade level match-ups between textbooks and the CCM. Textbooks for many series do not have a majority of the textbook addressing the CCM framework for the relevant grade.

Grade Level of Standards	K			1			2			3			4			5			6		
	EG	OG	LG	EG	OG	LG	EG	OG	LG	EG	OG	LG	EG	OG	LG	EG	OG	LG	EG	OG	LG
Total Number of On-Grade Standards Intended		**25**			**24**			**28**			**37**			**37**			**40**			**47**	
K		**19**		11			3			2			2								
1			14		**18**		10			3			3								
2			7			17		**24**		9			4			2			1		
3			1			8			20		**25**		22			9			5		
4			2			2			10			24		**28**		19			13		
5									2			13			20		**25**		14		
6						1			1			4			12			27		**27**	
7			1									6			10			19			26
8												2			3			6			11
HS																					2
Number Standards		**19**	25	11	**18**	28	13	**24**	33	14	**25**	49	31	**28**	45	30	**25**	52	33	**27**	39
Total Number Standards		**44**			**57**			**70**			**88**			**104**			**107**			**99**	
% of On-Grade Standards Covered		76%			75%			86%			68%			76%			63%			57%	
% of On-Grade Standards NOT Covered		24%			25%			14%			32%			24%			38%			43%	
% of Covered Standards That are On-Grade		43%			32%			34%			28%			27%			23%			27%	
Topics in Textbook but Not in CCSSM		1									1			1			1			1	

Legend	
EG	Earlier Grades
OG	On-Grade
LG	Later Grades

Figure 2. Grade alignment for CCM standards covered in one textbook series

Virtually all the encoded textbooks cover CCM standards at grades other than those identified in the CCM. Textbooks' content was not limited to that of the CCM standards appropriate to that grade. What this means is that the apparent focus on the CCM in Figure 1 for textbooks that addressed more than 75 percent of that grade's standards may not be as good as it appears; the textbook may address many topics extraneous to the appropriate CCM's standards and devote much more space to these topics than to the grade-appropriate CCM.

Textbooks for many series do not have a majority of the textbook addressing the CCM framework for the relevant grade.

Figure 3 focuses on the instructional coherence provided by one textbook series that lines up rather well with the CCM. Few non-CCM topics are included and none of these are a major focus of instruction. Nonetheless, a few CCM topics receive the same scant emphasis as the non-CCM topics, all of which are intended to be taught for a single day or less. Recall from Figure 1, 70% of the on-grade CCM Standards are mentioned for each grade. If the coverage of the CCM Standards is weighted by the amount of time recommended, the same average will be much lower: 43%.

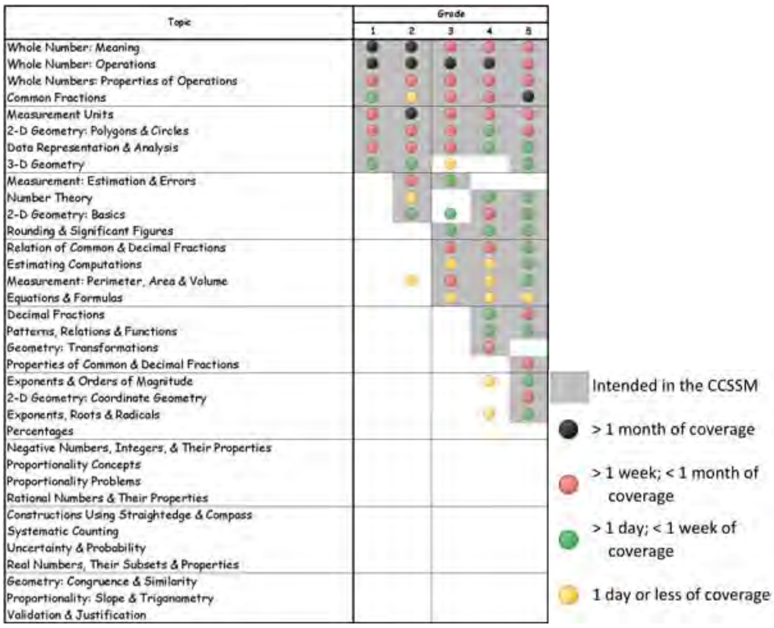

Figure 3. Instructional coherence of CCM standards in one textbook series

TEACHERS' CLASSROOM COVERAGE OF CONTENT

A case could be made that what is in the textbooks would not have much impact on student learning of mathematics as they are a supplement toward supporting classroom instruction. Therefore, classroom instructional coverage could substantially deviate from that presented in the textbooks.

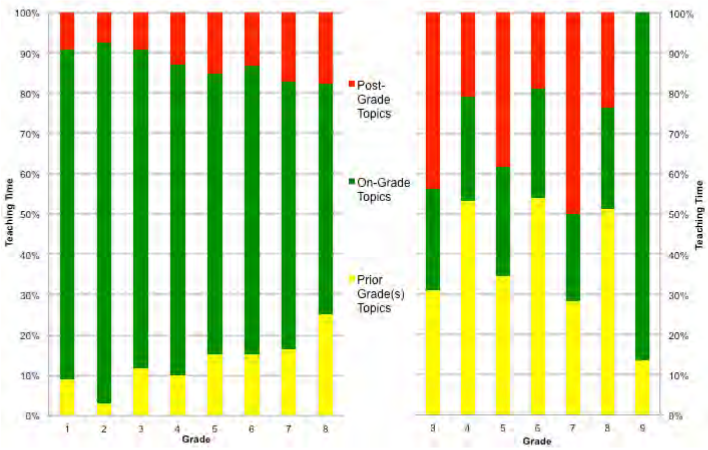

Figure 4. Average percent of on-grade CCM Standards instruction time for 2 districts

To test this assertion, CSC developed an easy to use, web-based reporting tool for teachers to record daily the content of their mathematics instruction using the standards and mathematics practices of the CCM. In 2011-2012, approximately 400 K-8 teachers participated. Although the majority of teachers' instruction focused on grade content according to the CCM significant proportions of time were spent on content intended either for a prior grade or a later grade. Figure 4 provides a snapshot from two different districts of the average percent of teachers' instruction that focused on content that was, according to the CCM, content for a prior grade, the grade taught by the teacher, or content intended for a later grade. Although the two districts look different they both have a significant amount of instructional time being devoted to off-grade content.

How much time?

Instructional time is the most important classroom-level resource. One of the decisions teachers wrestle with almost on a daily basis is how much time to devote to instruction on any given topic or standard. Currently, the CCM lists standards to be covered in classroom instruction for a given grade without, however, any indication as to how much instructional time to allocate to each one. One would expect that the standards are to be treated differently by their complexity and the breadth of the mathematics embedded in them and thus some require more time than others. What should be the expected allocation of time to the different CCM Standards by current teacher instructional practice for their grade level's CCM standards?

To address this question, a group of mathematicians, mathematics educators and partitioners were asked to assign instructional time to each CCM standard according to the complexity of the standard and it's relative importance for that year's instructional content. In Figure 5, we contrast the instructional emphasis provided CCM standards according to teachers' current practice, mathematics experts' evaluation, and that evidenced by textbooks. The teachers are generally more aligned with the textbook coverage than with the experts' expectation.

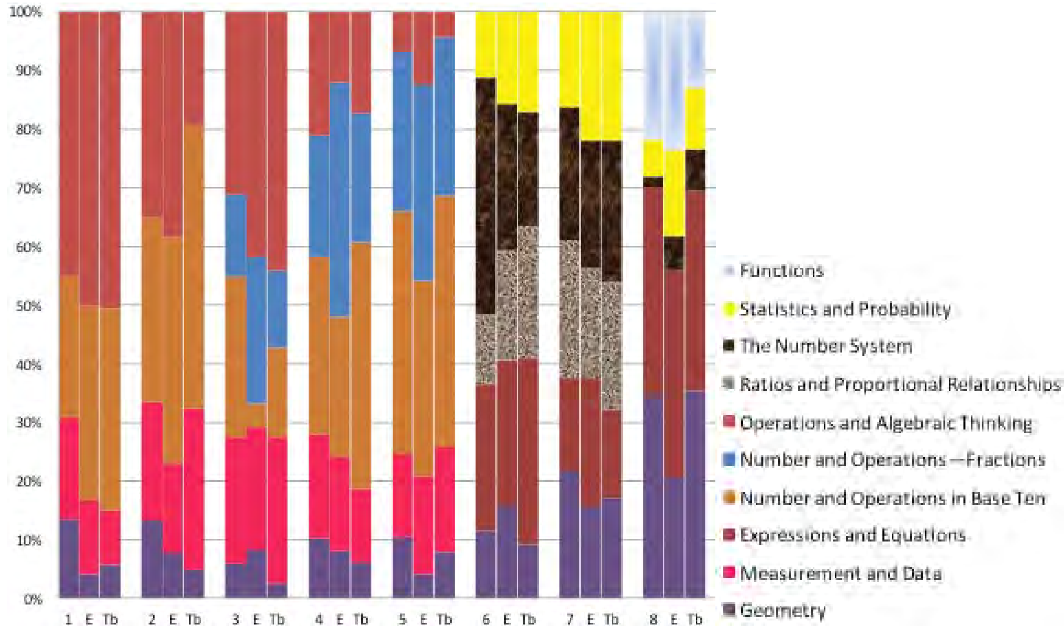

Figure 5. Comparing CCM coverage by teachers, mathematics experts, and textbooks

IMPLICATIONS FOR POLICY AND PRACTICE

In this paper, we briefly summarised our work surrounding the implementation of CCM focusing on the available instructional materials. While the research methodology was demanding, it did provide important insights into the impediments of successful implementation. Teachers were relying on textbooks that were not CCM ready. For newly available textbooks, it is critical to determine the extent to which CCM standards are not merely present but are prominent in the implicit instructional development embodied in them.

Another danger at this time is the lack of clarity of the CCM mathematics practices. Teachers are likely to understand them in light of their current practice and conclude, “I do this” and miss the intended in-depth mathematical thinking they’re meant to engender. The practices are ripe for simplistic reduction to algorithmic methods and a re-ignition of the “math wars” of previous decades. This would undermine the integrity of the entire CCM Framework and threaten meaningful implementation of it. Thus it is critical to understand teachers’ current practices around the CCM mathematics practices and to refine the definition of them so teachers can make sense of them in the context of the mathematics they teach. This further highlights the need for clarifying both theoretical and operational definitions related to any possible comparisons involving instructional materials and the coverage of classroom content and practices evaluated with respect to the CCM.

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SYMPOSIUM 4
TRANSITION TO COLLEGE MATHEMATICS AND STATISTICS

Transition to college mathematics and statistics: A problem-based, technology-rich capstone course for non-STEM students
Christian Hirsch

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TRANSITION TO COLLEGE MATHEMATICS AND STATISTICS A PROBLEM-BASED, TECHNOLOGY-RICH CAPSTONE COURSE FOR NON-STEM STUDENTS

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This paper describes a four-year project in the U.S., funded by the National Science Foundation, to design, develop, and evaluate Transition to College Mathematics and Statistics (TCMS), an innovative senior-level course to help meet the diverse quantitative needs of students whose intended undergraduate programs do not require calculus (e.g., business; management; economics; the information, life, health, and social sciences; and many teacher preparation programs). For students intending to enrol in non-STEM undergraduate programs, many schools in the U.S. have little to offer as a transition to college-level mathematics and statistics other than Precalculus or narrow Advanced Placement courses. Consequently, many students opt out of mathematics their senior year or study mathematics that is inappropriate for their undergraduate and career aspirations. TCMS focuses on contemporary topics, including mathematical modelling, data analysis and inference, informatics, financial mathematics, decision-making under constraints, mathematical visualization and representations, and important mathematical habits of mind. TCMS is accompanied by TCMS-Tools, a concurrently developed suite of curriculum-embedded Java-based software, including a spreadsheet, a CAS, dynamic geometry, data analysis, simulation, and discrete mathematics tools together with specialized apps. The focus of the paper is on the TCMS content and its organization, pedagogical design, affordances of TCMS-Tools, and preliminary evaluation results from classroom trials of TCMS use as a capstone course in diverse settings.

Keywords: mathematics curriculum, statistics, mathematical modelling, curriculum-embedded software, technology

INTRODUCTION

Transition to College Mathematics and Statistics (TCMS) is a problem-based, inquiry-oriented, and technology-rich fourth-year high school mathematics course. It was developed to help ensure student success in college and careers in an increasingly technological, information-laden, and data-driven global society. *TCMS* was specifically designed for the large numbers of non-STEM oriented students whose undergraduate programs of study do not require calculus—such as business; management; economics; the information, life, health, and social sciences; and many teacher preparation programs. See Figure 1 for the top 10 undergraduate majors in the U.S., as reported in the *Princeton Review* (2014). Students intending to complete one of these majors or other non-STEM majors in college are often ill-prepared by the lack of appropriate fourth-year high school mathematics courses. Research has repeatedly shown that students who are not enrolled in an appropriate mathematics course their senior year are much more likely to be placed in a remedial (non-credit bearing) course in college (cf. Key & O'Malley, 2014). It is in this context that TCMS was designed, developed, and evaluated.

Top 10 Undergraduate Majors in the U.S.
Based on Enrolment

1. Business Administration and Management/Commerce
2. Psychology
3. Nursing
4. Biology/Biological Sciences
5. Education
6. English Language and Literature
7. Economics
8. Communications Studies/Speech Communication and Rhetoric
9. Political Science and Government
10. Computer and Information Sciences

Source: http://www.princetonreview.com/college/top-ten-majors.aspx

Figure 1. The case for an alternative to Precalculus as a fourth-year course.

OVERVIEW

Transition to College Mathematics and Statistics is designed to be used as a fourth-year capstone course for students who have successfully completed a conventional single-subject sequence of algebra, geometry, and advanced algebra or a three-year international-like integrated mathematics sequence. The course has been carefully field tested in high schools with students using conventional mathematics curricula and with students using an integrated mathematics program.

TCMS builds upon the theme of mathematics as sense-making. Through investigations of real-life contexts and problems, students develop a rich understanding of important mathematics that makes sense to them and that, in turn, enables them to make sense out of new situations and problems. This theme of sense-making as well as the pervasive expectation that students reason about mathematics align well with the recent National Council of Teachers of Mathematics (NCTM, 2014) recommendations for high school mathematics.

Key themes and instructional features as outlined below have been informed by research on student learning (cf. National Research Council, 2005; NCTM, 2014) and recommendations from client disciplines on the focus of undergraduate non-calculus-based mathematics and statistics courses (cf. Ganter & Barker, 2004).

Balanced Content—*Transition to College Mathematics and Statistics* reviews and extends students' understanding of important and broadly useful concepts and methods from algebra and functions, statistics and probability, discrete mathematics, and geometry. These branches of mathematics are connected by the central theme of modeling our world and by mathematical habits of mind such as visual thinking, recursive thinking, searching for and explaining patterns, making and checking conjectures, exploiting use of multiple representations, providing convincing explanations, and a disposition towards strategic use of technological tools.

Flexibility—*TCMS* consists of eight focused and coherent units, each of which is generally self-contained with attention to content prerequisites provided by "Just-in-Time" review tasks in lesson homework sets. The course has been organized to be as flexible as possible. The organization permits teachers to tailor courses that best meet the needs and interests of their students. For example, some teachers choose to use the unit on *Mathematics of Democratic Decision-Making* as the second or third unit of the course to parallel state or national elections in the U.S.

Mathematical Modeling—*TCMS* emphasizes mathematical modeling including the processes of problem formulation, data collection, representation, interpretation, prediction, and simulation. Problem situations and phenomena modelled in this course involve discrete and continuous variables and entail deterministic as well as stochastic processes.

Technology—Numeric, graphic, and symbolic manipulation capabilities such as those found in *TCMS-Tools®* and on many graphing calculators are assumed and appropriately used throughout the course. *TCMS-Tools* is a suite of software tools that provide powerful aids to learning mathematics and solving mathematical problems. This use of technology permits the curriculum and instruction to emphasize multiple linked representations (verbal, numerical, graphical, and symbolic) and to focus on goals in which mathematical thinking and problem solving are central. Figure 2 provides three sample screens from the geometric linear programming (LP) custom app in which, in this case, students formulate and then enter the constraint inequalities and the objective function for two-variable linear programming problems. The LP app also supports solutions of three-variable linear programming problems. The software enables the student to dynamically control the line (plane) representing the objective function in search of an optimal solution.

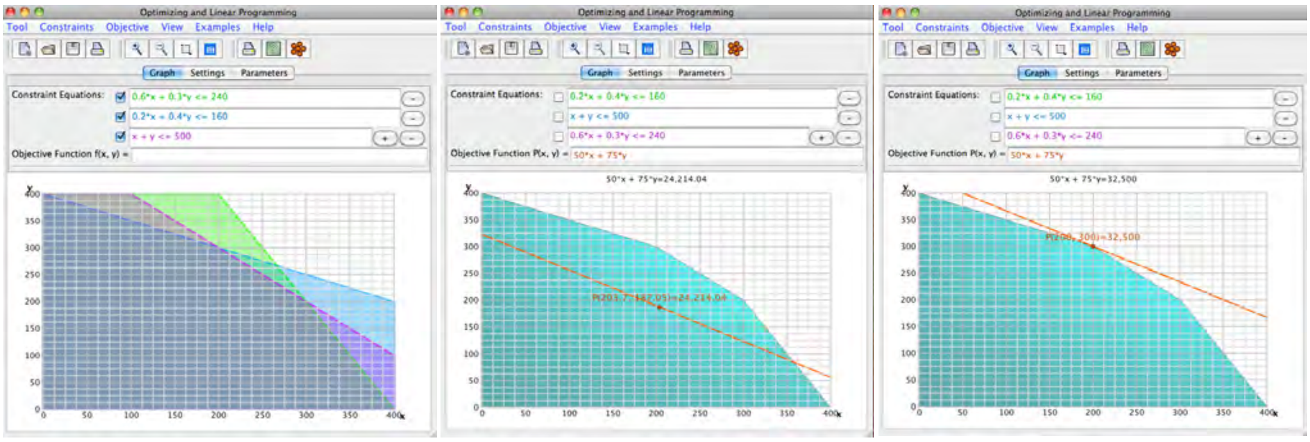

Figure 2. Searching for optimal profit for video game systems production.

Active Learning—The instructional materials are designed to promote active learning and teaching centered around collaborative investigations of problem situations followed by teacher-led whole-class summarizing activities that lead to analysis, abstraction, and further application of underlying mathematical ideas and principles. Students are actively engaged in exploring, conjecturing, verifying, generalizing, applying, proving, evaluating, and communicating mathematical ideas.

Multi-dimensional Assessment—Comprehensive assessment of student understanding and progress through both curriculum-embedded formative assessment opportunities and summative assessment tasks support instruction and enable monitoring and evaluation of each student's performance in terms of mathematical practices, content, and dispositions.

CONTENT FOCAL POINTS

TCMS features a coherent and connected development of important ideas drawn from four major branches of the mathematical sciences as reflected in Figure 3.

Transition to College Mathematics and Statistics

Unit 1	**Interpreting Categorical Data** develops student understanding of two-way frequency tables, conditional probability and independence, and using data from a randomized experiment to compare two treatments.
Unit 2	**Functions Modeling Change** extends student understanding of linear, exponential, quadratic, power, circular, and logarithmic functions to model quantitative relationships and data patterns whose graphs are transformations of basic patterns.
Unit 3	**Counting Methods** extends student ability to count systematically and solve enumeration problems using permutations and combinations.
Unit 4	**Mathematics of Financial Decision-Making** extends student facility with the use of linear, exponential, and logarithmic functions, expressions, and equations in representing and reasoning about quantitative relationships, especially those involving financial mathematical models.
Unit 5	**Binomial Distributions and Statistical Inference** develops student understanding of the rules of probability; binomial distributions; expected value; testing a model; simulation; making inferences about the population based on a random sample; margin of error; and comparison of sample surveys, experiments, and observational studies and how randomization relates to each.
Unit 6	**Informatics** develops student understanding of the mathematical concepts and methods related to information processing, particularly on the Internet, focusing on the key issues of access, security, accuracy, and efficiency.
Unit 7	**Spatial Visualization and Representations** extends student ability to visualize and represent three-dimensional shapes using contour diagrams, cross sections, and relief maps; to use coordinate methods for representing and analyzing three-dimensional shapes and their properties; and to use graphical and algebraic reasoning to solve systems of linear equations and inequalities in three variables and linear programming problems.
Unit 8	**Mathematics of Democratic Decision-Making** develops student understanding of the mathematical concepts and methods useful in making decisions in a democratic society, as related to voting and fair division.

Figure 3. TCMS units and their sequence.

ACTIVE LEARNING AND TEACHING

Each lesson includes 2–5 mathematical investigations that engage students in a four-phase cycle of classroom activities, as described below—*Launch, Explore, Share and Summarize*, and *Check Your Understanding*.

The Launch phase of a lesson promotes a teacher-led class discussion of a problem situation often supported by a video clip and of related questions to think about, setting the context for the student work to follow and providing important information about students' prior knowledge. For example, "How can you assess the risks of behaviour such as using tanning beds?" or "How is a credit card number sent securely when you busy music online?"

In the second or Explore phase, students collaboratively investigate more focused problems and questions related to the launch situation. This investigative work is followed by a teacher-led class discussion in which students summarize mathematical ideas developed in their groups, providing an opportunity to construct a shared understanding of important concepts, methods, and supporting justifications. Finally, students complete a formative assessment task related to their work.

Each lesson includes homework tasks to engage students in applying, connecting, reflecting on, extending the concepts and methods of the lesson, and reviewing previously learned mathematics and statistics and refining their skills in using that content.

FORMATIVE AND SUMMATIVE ASSESSMENT

Assessing what students know and are able to do is an integral part of *Transition to College Mathematics and Statistics*. There are opportunities for formative assessment in each phase of the instructional cycle. Quizzes, in-class tests, take-home assessment tasks, and extended projects are included in the teacher resource materials for summative assessments.

PRELIMINARY EVALUATION FINDINGS

The following findings are based on multiple beginning and end-of-year measures in six high schools located in Colorado, Kentucky, Michigan, New York, and Texas:

- Three schools in which students had completed a single-subject sequence of algebra, geometry, and advanced algebra from different publishers;
- Three schools in which students had completed a three-year integrated mathematics program.

TCMS end-of-year attitude survey

Among the major findings revealed by the end-of-year attitude survey:

- Students across all six field-test schools generally found the statistics and coding/cryptography units to be most interesting, noted the mathematics was reasonable to understand, and found the contexts very interesting and something they could relate to.
- Students from schools using conventional single-subject courses couched many of their comments in terms of the TCMS real-world problems in contrast to the non-context experiences they generally encountered in their previous courses.

ITED highest level quantitative thinking test

The ITED Quantitative Thinking Test for end of grade 12 consists of 40 items focusing on thinking, reasoning, and problem-solving skills.

- All schools showed pre-post gains greater than expected (national) norms.
- One school using the integrated program made significant gains at the $p = .05$ level, as did one school using a conventional curriculum.
- Across all schools, gains were particularly notable for students in the third and fourth quartiles.

Teacher interviews

Field-test teachers were interviewed throughout the TCMS field test. At the end of the school year, they were asked: “What do you see as the major strengths of TCMS?” Responses included:

- TCMS reaches out to a select population of students that we previously had nothing to offer them.
- TCMS is the perfect class for collaborative learning. Students learn to actually read in a math class, they learn how to make mistakes and learn from them as opposed to being discouraged by them, and they also develop a deeper understanding of the mathematical material since the topics are all in a real-world context.
- This course really made both the teacher and students *think* about the mathematics being taught and learned. It gave a lot of students who were unsuccessful in Algebra 2 an opportunity to be successful. They enjoyed most topics and the contexts were very engaging. Many of my students left at the end with a view of mathematics as being useful.

Student interviews

Students were similarly interviewed throughout the field testing. At the end of the school year, they were asked: "Would you recommend this course to students (juniors) considering a math course to take for next year?"

- Yes, because it seems like we learn things that are much more applicable to daily life and careers than other math classes.
- This course takes difficult skills and ideas that I may have misunderstood in previous courses, reviews and extends them.
- I think this class will prepare you for college with the layout of the class you get to learn many different types of math.

FURTHER WORK

Analysis of student placement and performance in their first college mathematics or statistics course is in process.

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SYMPOSIUM 5

THE NEW CENTURY PRIMARY MATHEMATICS TEXTBOOK SERIES

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International Conference on Mathematics Textbook Research and Development 2014 (ICMT-2014)
29-31 July 2014, University of Southampton, UK

THE NEW CENTURY PRIMARY MATHEMATICS TEXTBOOK SERIES: TEXTBOOKS IN CHINA WITH SPECIFIC CONSIDERATION TO CHARACTERISTICS OF CHILDREN'S THINKING

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This symposium provides an overview of the New Century Primary Mathematics Textbook Series developed in China. In developing this book series, characteristics of children's thinking were fully respected and considered, and lists of challenging and inviting tasks were designed.

Keywords: mathematical thinking, mathematical problem solving, New Century Textbook Series, China

SUMMARY

The process of children's mathematics learning is supposed to be lively, active and full of individual characteristics. An ideal series of textbooks should offer learning recourses with which students and teachers may work together. Therefore, while developing the New Century Textbook Series, the characteristics of children's thinking were fully respected and considered, and lists of challenging and inviting tasks were designed. An illustration of one of the textbooks is provided in Figure 1.

Figure 1. A textbook from the New Century Textbook Series

When taking in account the characteristics of children's thinking, a diverse and cumulative learning process was ensured. To promote understanding of core concepts in mathematics, it is designed in such a way that students are given sufficient time and space for exploration and discussion, thus contributing to all-rounded development of students in the domains of knowledge and skills, mathematics thinking, problem solving, and affection and attitudes.

In this symposium, examples, experiences and reflections in developing such a primary mathematics textbook series were shared.

SYMPOSIUM 6

REFORM OF CHINESE SCHOOL MATHEMATICS CURRICULUM AND TEXTBOOKS 1999-2014

Reform of Chinese school mathematics curriculum and textbooks (1999-2014): Experiences and reflections
Jian Liu

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REFORM OF CHINESE SCHOOL MATHEMATICS CURRICULUM AND TEXTBOOKS（1999-2014): EXPERIENCE AND REFLECTION

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This symposium provides an overview of mathematics curriculum reform in mainland China and the underlying social, political, economic, technological and cultural contexts over the 15 years from 1999 to 2014. In addition, the views and reflections of a sample of Chinese textbook developers' are discussed and analyzed with a focus on the issue of mathematics textbook diversification and further development in mainland China.

Keywords: curriculum reform, textbook development, China

Having worked in the Ministry of Education of China and been mainly responsible for the mathematics curriculum and textbook reform at the national level since 1999, in this symposium the presenter shared his personal experience and reflection about the mathematics curriculum reform in mainland China and the underlying social, political, economic, technological and cultural contexts over the 15 years from 1999 to 2014. Tracing back to the *21st Century China mathematics education prospective project*, which was founded in 1989, the presenter discussed the possible impacts of the research carried out by the project team on the New Century textbook reform in China. The timeline from 1992 to 2011 is shown in Figure 1.

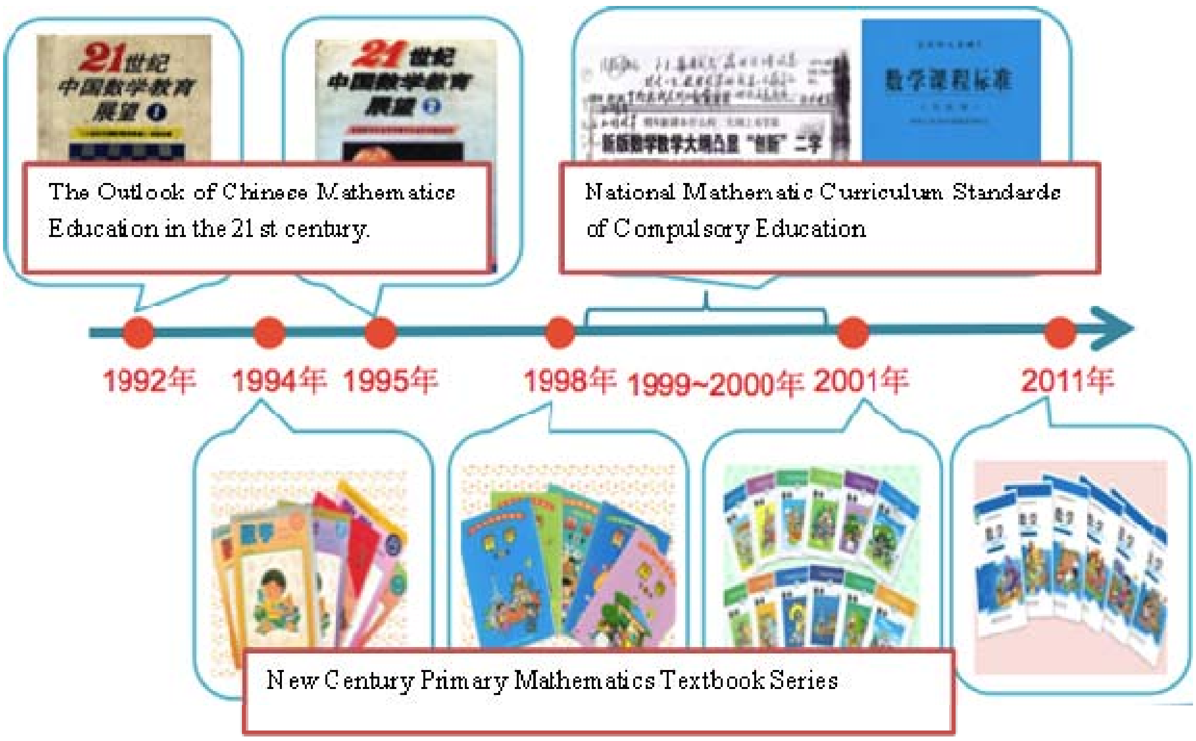

Figure 1. The timeline of curriculum reform in mainland China from 1992 to 2011

As well as mentioning the influence of the theories and ideas from international research (e.g., the work of Goodlad and associates (1979) on curriculum development), the symposium provided an overview of the basic structure of the mathematics textbooks of China and its diversifications, the general features of different mathematical textbooks, and the important role that these diversified textbooks have played in pushing forward the reform of mathematics education in China.

In addition, using the data collected from interviews with a number of chief editors of the reformed textbooks especially for this paper, the presenter discussed and analyzed Chinese textbook developers' views and reflections with a focus on the issue of mathematics textbook diversification and further development in mainland China.

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SYMPOSIUM 7
REFLECTIONS FROM THE PAST

Reflections from the past: A contemporary Dutch primary school mathematics textbook in a historical perspective
Marc van Zanten & Marja van den Heuvel-Panhuizenu

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REFLECTIONS FROM THE PAST: A CONTEMPORARY DUTCH PRIMARY SCHOOL MATHEMATICS TEXTBOOK IN A HISTORICAL PERSPECTIVE

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In the Netherlands, most currently used mathematics textbook series for primary school are influenced by the so-called 'Realistic Mathematics Education' reform. In this study, we investigated the historical roots of RME by carrying out a textbook analysis in which we compared a contemporary textbook series with two pre-RME textbook series. As the mathematical focus of our analysis we chose decimal numbers. Our study revealed that with respect to this topic onsets of the RME reform, which started in the 1970s, were already present in textbooks of the 1950s and 1960s.

Keywords: historical analysis, primary school, decimal numbers, mathematical content, performance expectations, learning facilitators, The Netherlands

INTRODUCTION

Approaches to mathematics education change and evolve over time (e.g., Walmsley, 2007). Knowledge of mathematics education from the past can lead to a better understanding of approaches of the present. Obviously, it is not possible to observe the enacted mathematics curriculum of the past. However, mathematics textbooks, as representatives of the potentially implemented curriculum (Valverde et al., 2002), can provide a view of approaches to mathematics education of years gone by.

Most Dutch textbooks are influenced by the so-called 'Realistic Mathematics Education' (RME) approach, which has been developed in the Netherlands. This reform dates from the 1970s. Until then, Dutch' mathematics textbooks generally had a mechanistic approach to the learning of mathematics, with much emphasis on practising formal procedures and the learning of algorithms by solving many similar bare number problems (De Jong, 1986). As a reaction, RME aimed, and still aims, to give children a better basis for understanding mathematics (e.g., Van den Heuvel-Panhuizen, 2001). Some characteristics of RME that can be found in textbooks from the 1980s onward are: the use of context problems as a source for learning mathematics; the use of models; the use of different calculation methods; and the opportunity offered to children to come up with 'own productions', such as self-constructed problems (De Jong, 1986; Van den Heuvel-Panhuizen, 2001).

In the present study, we investigated the historic roots of RME by carrying out a comparative textbook analysis of a representative contemporary textbook series and two mathematics textbook series dating from before the RME reform. Our aim was to get a view of the development of RME-related characteristics in textbooks over the years and to get a better understanding of the historical roots of RME. Our research question was: *Were RME characteristics present in a contemporary textbook series, already present in pre-RME textbook series, and if so in what way?*

As the mathematical focus of our analysis we chose decimal numbers, since this topic incorporates both procedural and conceptual challenges in its teaching and learning. Perhaps the greatest challenge is that prior knowledge of whole numbers can interfere with the learning of decimals (e.g., Ni & Zhou, 2005), for instance as students assume that a number with more digits always represents a greater value than a number with fewer digits (e.g., 5.068 is considered larger than 5.8) or that they interpret the digits behind the decimal point as a number in its own (leading to mistakes as '2.3 + 2.14 = 4.17'). In the Netherlands, these and other difficulties were addressed in RME by interpreting decimal numbers as measurement numbers, by making use of models, especially the number line, and by putting emphasis on estimation with decimal numbers (De Jong, 1986; Treffers, Streefland & De Moor, 1996).

METHOD

Textbook materials included in the analysis

We included three textbook series in our analysis: 'De Wereld in Getallen' [The World in Numbers, WiN], 'Nieuw Rekenen' [New Arithmetic, NA], and 'Functioneel Rekenen' [Functional Arithmetic, FA]. WiN dates from 2009 and was included because it is one of the most widely used contemporary textbook series. It has a market share of approximately 40% and has a history of three previous editions, dating back to the early days of RME in the 1980s. NA and FA, which date from before the RME reform, were included because these textbook series, although in the 1980s classified as belonging to the mechanistic approach to mathematics education, contain several innovative elements (De Jong, 1986). NA dates from 1969 and was in use in more than 35% of Dutch schools in the 1970s. It was still being used in the 1990s. FA dates from 1958 and was in use in approximately 5 to 10% of schools until the 1970s. Thus, our selection of textbooks covers over half a century.

In all three textbook series, decimal numbers are dealt with in grade 4, 5 and 6, so we limited our research to the materials for these grades. We analysed all textbook materials that are meant for all students, and included the teacher guidelines. Optional exercises meant for evaluation, repetition or enrichment were left out of our analysis.

Although the domain of decimal numbers has an overlap with calculation with money and with measurement, we only included the subject matter that primarily belongs to the topic of decimal numbers in our analysis. For example, exercises in which certain measures have to be converted in another measurement unit (e.g., 2.5 m = … cm) were considered to belong predominantly to the domain of measurement and were left out of our analysis, but exercises that focus on place value (e.g., 3.42 m = … m + … dm + … cm) were included, because here the students have to interpret the decimal digits.

Analysis Framework

In our analysis, we took the perspective of the addressed mathematical content and performance expectations into account (see Valverde et al., 2002) by looking at the task format and the type of calculation. Further, we looked at how the textbooks support learning and understanding (see Howson, 1995), which we refer to as 'learning facilitators'. Table 1 shows the framework we used for our analysis.

Table 1: Analysis framework

Perspective	Specification of the perspective
Format of tasks	Tasks with bare numbers Tasks with measurement numbers Tasks with money[1]
Type of calculation	Tasks on mental calculation Tasks on estimation Tasks on written calculation Tasks on using a calculator
Learning facilitators	Use of contexts as a source for learning Use of the number line as a model Use of other models Offering different calculation methods Offering the opportunity for 'own productions'

We used the task as our unit of analysis. With the term 'task' we mean the smallest unit that requires an answer from a student. We considered the corresponding directions given in the teacher guidelines as belonging to the task.

Analysis procedure

First, all tasks concerning decimal numbers that are meant for all students were identified. Then, the first author coded each task according to the framework. All codings were inspected on whether the code was in agreement with the teacher guidelines. To be sure that the coding was done correctly, the first author twice repeated the coding. When differences occurred in the coding results, all tasks with the code in question were checked again, and if necessary revised until all coding results were consistent.

RESULTS

Due to the space available in this paper, we only report part of our results. First, we give a brief overview of the exposure of tasks on decimal numbers in the three textbook series. Then, we report about the findings for the three perspectives in which we zoom in on one particular task format (i.e. tasks with measurement numbers), one type of calculation (i.e. tasks on estimation), and one learning facilitator (i.e. the use of the number line).

[1] Although tasks on money can also be regarded as belonging to the domain of measurement, we considered them as a separate category from tasks with measurement numbers.

Exposure of tasks on decimal numbers in general

In all three textbook series, decimal numbers are introduced in grade 4 and most attention is paid to this topic from the second half of grade 4 until the first half of grade 6 (see Table 2). All three textbook series offer tasks regarding place value, ordering and comparing decimal numbers with other decimal numbers, and with whole numbers and fractions, writing fractions as decimal numbers and vice versa, and carrying out operations with decimal numbers. Bare number tasks with decimal numbers are first offered in the second half of grade 4 (in all three textbook series). In FA and NA bare decimal numbers are introduced as another way for writing down fractions. In NA, this is illustrated with money. In WiN bare decimal numbers are introduced using measurement numbers.

Table 2: Number of tasks on decimal numbers in the three textbook series

	FA	NA	WiN
Grade 4, 1st half	35	131	146
Grade 4, 2nd half	1026	242	221
Grade 5, 1st half	396	534	386
Grade 5, 2nd half	486	919	581
Grade 6, 1st half	367	931	663
Grade 6, 2nd half	23	639	-

Exposure of tasks with decimal numbers as measurement numbers

Figure 1 shows the distribution of the three task formats. In WiN 23% of all decimal number tasks concern measurement numbers. This percentage contrasts with that in the pre-RME textbook series.

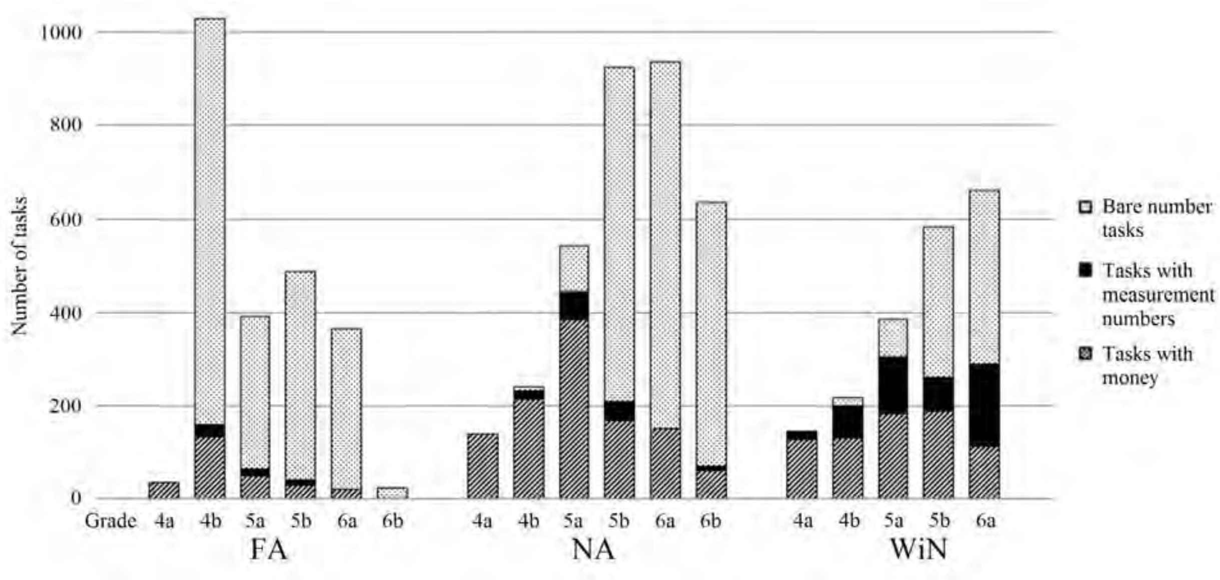

Figure 1. Exposure of different task formats for tasks with decimal numbers

In FA about 2% of all decimal number tasks belong to the category tasks with measurement numbers and in NA this is about 4%. Regarding the ways in which measurement numbers as decimal numbers are present, we found several similarities. For example, in all three textbook series measurement numbers are used to understand place value of the digits behind the decimal point. Furthermore, they are employed for mental arithmetic with decimal numbers in context problems. In both NA and WiN measurement numbers are also used for comparison and ordering of decimals numbers with different amounts of digits behind the decimal point. In FA the latter type of task is only present in tasks with bare decimal numbers.

Exposure of estimation tasks with decimal numbers

The degree to which estimation problems are offered is as follows: in FA 14% of all tasks on computing operations with decimal numbers concern estimation, in NA this is 32% and in WiN 36%. In all three textbook series students are expected to estimate the outcome of an operation (mostly a multiplication) before calculating it precisely. In FA this concerns 92% of all estimation tasks with decimals, in NA 66% and in WiN 31%. The relative low percentage in WiN is caused by the fact that in WiN most estimation tasks are not linked to precise calculation. All three textbook series offer tasks in which estimation takes place by calculating with rounded numbers, and by determining between what numbers an answer must lie (e.g., 2.4 × 7.6 is bigger than … and smaller than …). WiN also offers estimation tasks in which students have to choose what answer can be correct (e.g., 30 × 0.15 = 450 or 4.5 or 0.45). WiN and FA both also offer tasks that concern the recognition and correction of incorrectly placed decimal points (e.g., 3.58 × 52.3 = 1872.34; correct the mistake.).

Use of the number line as a model for dealing with decimal numbers

In WiN, the number line as a model for dealing with decimal numbers is used to illustrate the partitioning of the units into smaller and smaller units, to relate decimals and fractions; and to position and order decimal numbers (see Figure 2). Some tasks with bare decimal numbers and with decimal measurement numbers are accompanied with a number line. Once, the number line is used to illustrate an addition with decimal numbers. Furthermore, the number line does not only appear in the student books, but also in the teacher guidelines with an explanation of how a number line can be used in instruction.

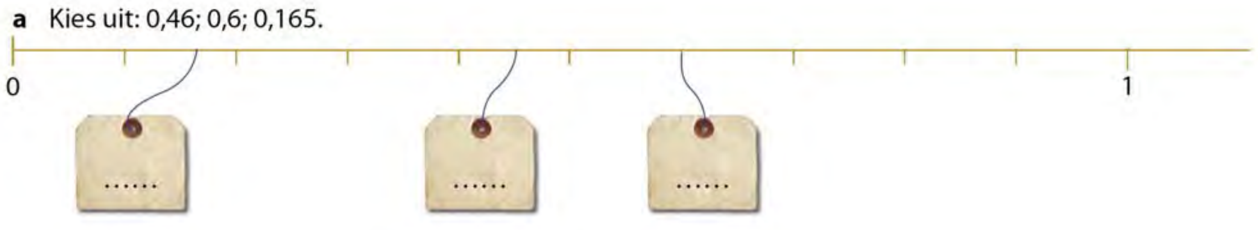

Figure 2. Positioning decimal numbers on the number line in WiN

In NA, we rarely found use of the number line as a model for decimal numbers. In this textbook series the number line is used to illustrate operations (see Figure 3), to fill in decimal numbers, and to illustrate that multiplying a multiplicand with a multiplier smaller than 1 makes the multiplicand smaller. NA also offers examples in the teacher guidelines of how to use the number line during instruction. In FA, the number line is not present (at all).

Figure 3. Visualisation of a multiplication with decimals on the number line in NA

CONCLUSION

Our study revealed that onsets of RME characteristics were already present in the researched pre-RME textbook series. The degree of RME characteristics is greater in the contemporary textbook series than in the pre-RME textbook series. In some cases, the onsets of RME characteristics in pre-RME textbook series differ from the RME characteristics in the contemporary textbook series, as came to the fore in the way the number line is used. However, sometimes the onsets in pre-RME textbooks are quite similar to contemporary RME characteristics, for example, the use of measurement numbers to support the understanding of place value.

Of course, this paper only reports on a few characteristics, on only one mathematical topic, and on a limited number of textbooks. Yet on the whole, our findings clearly indicate that the RME reform was built at least in part on approaches that were in potency already present in textbooks of the 1960s and 1970s.

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SYMPOSIUM 8

RESEARCH ON TEXTBOOKS USED IN CHINA FOR TEACHING GEOMETRIC TRANSFORMATIONS IN SECONDARY SCHOOL

Research on textbooks used in China for teaching geometric transformations in secondary school: From the perspective of the teachers' role
Chunxia Qi, Xinyan Zhang & Danting Huang

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RESEARCH ON TEXTBOOKS USED IN CHINA FOR TEACHING GEOMETRIC TRANSFORMATIONS IN SECONDARY SCHOOL: FROM THE PERSPECTIVE OF THE TEACHERS' ROLE

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Great challenges and impacts have been taken on teachers' ideas and practice by the new philosophy raised in mathematics curriculum reform in China. This new philosophy basically and radically shakes the authority of textbooks as well. Using Nicol's views about the levels of usage of textbooks, this symposium reports a qualitative analysis of the teaching of geometric transformations by six secondary school teachers. It was found that the usage of textbook reaches the level of elaborating and creating but most teachers mainly focused on elaborating the textbook. Meanwhile, great differences exist between novice teachers and senior teachers. Teachers transform their roles in mathematical communication, interaction with students, validation of knowledge, source of knowledge and students' autonomy. These transformations can improve the usage of textbooks.

Keywords: geometry, geometric transformations, textbook use, secondary school, China

INTRODUCTION

Dating from 1658, the book called *Orbis Pictus* written by Czech educator Comenius became the first textbook in the world. After five centuries' development, great changes have been taken on textbooks' connotation, functions, characteristics and structures. However, textbooks could be viewed as a materialized form of school curriculum, the most basic and popular teaching medium at schools and the basis of instructions. In spite of these functions, according to Chambliss and Calfee (1998), textbooks can still determine 75-90 percent of instructional content and activities in schools.

In the early 1930s, Bagley (1931) recorded the usage of textbooks. In 1955, Cronbach (1955) called for research on the usage of textbooks. However, so far, people have known little about how and to what extent the textbooks are used both in the class and after class, especially in school mathematics (Huang & Huo, 2005). Apart from that, the new ideas proposed by the recent mathematics curriculum reform in China basically and radically shake the authority of textbooks as well.

Literally, the position of textbooks changes from 'Bible' to 'material'. Yet, actually, this replacement of position means the textbooks' function turns from 'controlling teaching' to 'serving for teaching' (Guo, 2001). Meanwhile, this transformation is challenging and impacting mathematics teachers' ideas and practice so that teachers have to readjust their behaviours and choose proper roles for facing the changes. Besides, they also have to re-examine the functions and use of textbooks. As such, many questions arise. What kinds of roles do teachers play in reality, especially in teaching specific knowledge? What should we regard the textbooks as? How do we use the textbooks? To what extent can textbooks be used if teachers play different roles? What kinds of cultures are created by teachers in the class based on the effective behaviours in terms of teachers' role and the use of textbooks? What are the differences in using textbooks between mathematical teachers in different levels? The start of logic on researching the usage of textbooks is composed by the answers of the questions which have been listed.

FRAMEWORK OF RESEARCH

Mathematics teachers play various roles in different teaching links and generate diverse impacts so that their ways applying to textbooks are obviously distinct. This research tries to reveal the condition of the usage of textbooks and analyze the disadvantages in using textbooks more comprehensively in order to assist mathematics teachers to use textbooks more innovatively and effectively.

The role of teachers

Based on some analysis, mathematics teachers can be thought of as performing five different roles in teaching.

The first one is the role in mathematical communicating, which means the role to help student to get the ways to acquire some knowledge about math, such as conception, regulation and proposition. Except for textbooks and teachers, technology becomes another source for students to get information and communication in math. Another level of communication is the interaction with part of the factors from students, teachers and technology. Or teachers could be regarded as the only source for students to acquire mathematics information and communicate with.

The second role is in interacting with students, which mainly refers to the interaction between teachers and students during teaching and class management, which contains three different situations. One is that teachers are able to adjust their original teaching plan according to students' participation. Another one is that teachers are able to listen to students' answers attentively and respond them immediately. But what their aim is back to their teaching plan. The third one is that teachers follow predetermined plan strictly and make few responses to students' questions and answers.

The role in verifying mathematics knowledge is mainly related to the validation of the source of mathematics knowledge, which concludes verifying multiple sources such as teachers, technology etc. or teachers are the only source of validation.

The role in selecting mathematics problems refers to the source of problems. These valid problems could derive from digital programs and students themselves. Another way to get problems could come from teachers' and textbooks' questions. Or teachers are the unique sources who determine the problems for students to solve, debate and practice.

The role in students' behaviour and autonomy means that students have the autonomy to decide what to do and how to do, or students just obey, repeat and answer the questions provided by teachers.

Usage of textbooks

In order to learn about the usage of textbooks in teaching, this research would like to divide the usage of textbooks by teachers in mathematics classes into three levels according to the framework of Nicol and Crespo (2006). They are “adhering”, “elaborating” and “creating” Literally, “adhering” means that considering textbooks as “an authority” to decide what to teach and how to teach. During teaching, teachers always make no adjustment or modification on textbooks, or only make some superficial changes. “Elaborating” aims at regarding textbooks as “a guide” to tell teachers what to teach and how teach. Moreover, teachers will take advantage of other sources to amplify the questions, tasks and exercises in textbooks. “Creating” means that teachers can have a critical eye on textbooks. Therefore, teachers can find out the intention and limits of the books in order to optimize teaching structures, pedagogical sequence and teaching system through setting up appropriate problems.

RESEARCH DESIGN

This research intends to take video analysis to reveal and analyze the role of teachers and the usage of textbooks in teaching geometric transformation in junior high school. Samples used in the research are 6 mathematics teachers who come from different areas and stay in various developing stage. Among them, three teachers are called novice teachers whose teaching time is less than 5 years. The other three teachers are defined as senior teachers whose teaching time is more than 10 years. Table 1 shows the basic information of those 6 teachers.

Table 1 Basic information of 6 mathematics teachers

Code	Sex	Years of teaching	Diploma	Relative information	Subject
NT1	Female	2	Master	Have taught in a tutorial school in pre-service time	Translation
NT2	Male	2	Bachelor	No teaching experience	Axial Symmetry
NT3	Female	4	Bachelor	Have taught the whole junior high class	Rotation
ST1	Female	11	Bachelor	Responsible for the research on mathematics in their grade	Translation
ST2	Female	13	Bachelor	Have been teaching Grade 3 for many years	Axial Symmetry
ST3	Male	12	Bachelor	Have experience to teach senior high school	Rotation

Before taking videos of these teachers’ class, we explained the purposes of the observation and ask them to take it as a normal class. Researcher adopted participatory observation and non-participatory observation so as to guarantee the accuracy and integrity of the material. After taking videos, we had interviews with every teacher about 40 minutes in order to know their teaching links and design ideas profoundly.

Additionally, researcher selected the teaching videos about geometric transformations from each teacher's videos. Then six teachers are divided into three groups, one novice teacher and one senior teacher being grouped together. Based on the need of analyzing the distinction between novice teacher and senior teacher, two teachers in one group should choose the same part of knowledge to teach. Therefore, 3 different teaching subjects are picked up for each group. And after taking videos, their words were transcribed to provide the basic data to analyze the roles of teachers and the usage of textbooks in teaching geometric transformation.

RESEARCH RESULTS

Based on the framework of Nicol and Crespo (2006), with the combination of teaching activities and the usage of textbooks, the six videos have been spilt into teaching segments. Then it is convenient and accurate to analyze the role of teacher and the usage of textbooks in each segment. Take the analysis on ST1 as an example.

In the Teacher ST1 teaching geometric translation in that class, and the textbook she used was written by Beijing Normal University. The whole class could be divided into 5 parts which include setting the scene, abstracting the concepts, exploring the nature, enhancing and practicing and concluding the class respectively. Teacher used textbook in every part except in conclusion, so the last part will stay outside the whole analysis.

Segment 1:

Teacher ST1 introduced translation by asking students to watch a video about two spaceships.

> ST1: After observing the live video about the docking, what kind of movements did the two aircrafts make?
>
> S: (in chorus) Translation.
>
> ST1: Yes, translation. You have learnt it in primary school. Can you find translation in your life?
>
> S: (point to the curtain in the classroom) The Curtains.
>
> ST1: Excellent! You found the translation in our life. How did the curtain do the translation?
>
> S: From left to right, or from right to left.
>
> (Teacher performed by gestures)
>
> S: Sash window.
>
> ST1: How did sash window do the translation?
>
> S: When you open the window, it comes up to down. When you close the window, it comes from the bottom to the top.
>
> S: The blackboard in our classroom makes translation from right to left.
>
> ST1: Right, our blackboards do the translation every class.

(After the examples provided by another two students, the teacher revealed the theme of this class)

In this segment, students made judgments about translation first and search the movements in their memory. Actually, students had to perform translation in mind and built the connections with the main ideas in that class spontaneously. It seemed that students had the autonomy and freedom to select the objects which could do the translation, but actually it is the teacher who made use of questions and students' answers to control the core of the class. In other words, the communication between the teacher and her students had been confined to the simple questions. Teacher ST1 wasted the opportunities to deepen the concept. In this part, teacher chose the videos according to students' hobbies and current news which compose the contexts of following questions. Though the video and questions did not stem from textbook, they elaborated the usage of textbooks due to the intention of this part and the purpose of textbook reached consensus.

Segment 2: students abstracted the concept of translation and deepened their understanding about the two core factors through feeling this movement by using pens and triangles to imitate translation.

> ST1: What kind of docking could be regarded as a success? You can have group discussion.
>
> S: The two aircrafts' docking must be coincidence.
>
> ST1: If the two aircrafts become like this (teacher used pens to simulate aircrafts with a certain angle), what kind of problems did it happen?
>
> S: Directions, something wrong with the angle of the orbits.
>
> ST1: The docking failed because of the direction. Now adjust the directions to keep the two aircrafts in the same horizontal direction (place the two pens in parallel), does this mean a successful docking?
>
> S: (in chorus) No
>
> ST1: The two aircrafts fail to dock even after the direction adjustment, so what should be adjusted to realize a successful docking?
>
> S: Distance.
>
> ST1: So specifically, what is translation?

(Then students can feel the two core factors of translation by moving triangles and abstract the concepts of translation)

In this segment, teacher ST1 rearranged the teaching content after realizing students' problems in understanding translation. The teacher used pens to demonstrate docking, which showed communication based on questions and guided students to reasonable judgments. Questions raised in this process generated in teaching. Objects used in this class become the scaffold for students to understand direction and distance of translation. Therefore, the usage of textbook reached to the level of creating.

Segment 3: after concluding the concept of translation, the teacher let the students realize the direction and distance of every point identical as the whole triangle's movement by observing the corresponding points before and after operations. Students now could acquire the two characteristics of translation.

> ST1: Every point moved in the same distance, so the length of line segments is equal. You just summarized that the connections between each corresponding point are parallel and the number of connections is same. Whether this conclusion is correct or not should be verified. How to use Sketchpad to validate whether this hypothesis is right? Have you remembered the method to verify two identical line segments?
>
> S: Measure the length of the two segments respectively. (The teacher operated as students' description)
>
> ST1: How to prove two parallel line segments?
>
> S: We can make parallel lines ourselves.
>
> ST1: We have drawn parallelogram by sketchpad, do you remember how to make parallel lines? I'd like to ask someone to demonstrate here and others observe his steps.
>
> S: Firstly, choose a line segment AA´, then choose a point B or C outside AA´. For example, draw a line parallel with AA' across point B, if the new parallel line is identical with BB', so we can prove that the lines between two corresponding points are parallel. (Students was operating on Sketchpad and describing the construction.)
>
> ST1: Terrific! You can construct parallel lines to study the relationship of position, now we can use the same method to verify the lines between other corresponding points.

In this segment, the interaction among teacher and students is built on problems but lacks the communication of knowledge and ideas in mathematics. Students followed the questions raised by their teacher and verified the conclusion through Sketchpad, which means they participated class activities involuntarily. Additionally, Sketchpad enriched the means to validate knowledge and become another effective source for students to verdict. Generally, in this part, usage of textbook can reach the "elaborating" level. Not only did teacher arrange the exploration of translation's nature according to the textbook, but also she allowed students to experience the convenience which was brought by mathematical software.

Segment 4: in order to help students recognize the random direction of translation, the teacher broke the limits on horizontal or vertical direction and taught students to realize the translation by marking vectors. Moreover, she asked students to design pictures with the help of resources which is provided by teachers and present their accomplishment via local area network.

> ST1: Now I'll introduce a new button to you. In Geometry Sketchpad, there is a button controlling the direction and distance of translation, it is called vector. First of all, let's draw the line segment AB, then choose a point E outside AB. The order to select is from A to E. Then press the button mark vector and choose the line segment. The button translation will remind you whether to translate from A to E. When you press OK, connect AE and BB'. Now let's draw a parallelogram with marked vector first and construct a Christmas tree with translation.

(Students operated Sketchpad and design the pictures independently.)

ST1: You guys have designed all kinds of Christmas trees, let's appreciate others' work.

(Students presented their work to others and the teacher picked some students' pictures to show all)

Thanks to the teacher's generosity on passing the right of assessment to students, the activity for them to design the pictures embodied their autonomy completely. Besides, the plan of this segment stayed outside the textbook. Making better use of computers, the teacher promoted students' feeling on translation and realized the interaction among students and their teacher. This kind of usage could belong to creating. On the basis of the analysis above, researcher used arrows to describe the usage of textbook in Table 2

Table 2 Teacher ST1's usage of technology under different roles

Roles	Technology	**Adhering**	**Elaborating**	**Creating**
Communication of mathematics	Students, textbooks, technology and teacher			
	Sonic other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students. follows predetermined plan			
Validation of Mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it.			
	Teacher assumes active role, while students mainly listen, copy or answer questions.			

The head-to-tail ligation arrows in Table 2 indicate the changes both in teacher's role and the usage of textbook. The vertical components of the arrows express the transformation on the role of teacher, while the horizontal components indicate the alteration on the usage of textbook. The start and the end of the arrows represent specific behaviour in teaching segment. If the arrow closes to the border, it signifies that the teacher has undertaken two different roles or the teacher used two different technologies. The position where the arrows stay stands for the original meaning primarily. Take interaction with students for example, in segment 1(introduction), the usage of textbook get to the elaborating level, and the teacher mainly communicate with students by questions and answers. So the arrow lies under the second role and elaborating. In segment 2 (abstracting concepts), the usage of textbook reaches the creating level. Teacher ST1 used pens to imitate the movements of the two aircrafts innovatively according to students' participation. This kind of adjustment renders the arrow lie under the first role and creating status. In segment 3 (exploring the nature), the teacher elaborated the textbook again. She asked students to answer her questions, so the arrow lies under the second role and the elaborating status. In the last segment (enhancing and practicing), the teacher utilized the textbook creatively for she emphasized students' autonomy but still followed the original teaching plan. Consequently, the arrow in this segment stays the same as the arrow in segment 3. Table 3 to Table 7 indicate the usage of textbooks under other five teachers

Table 3 Teacher NT1's usage of technology under different roles

Roles \ Technology		Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks, technology and teacher			
	Sonic other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students. follows predetermined plan			
Validation of Mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it.			
	Teacher assumes active role, while students mainly listen, copy or answer questions.			

Table 4 Teacher NT2's usage of technology under different roles

Roles	Technology	Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks, technology and teacher			
	Sonic other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students. follows predetermined plan			
Validation of Mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it.			
	Teacher assumes active role, while students mainly listen, copy or answer questions.			

Table 5 Teacher ST2's usage of technology under different roles

Roles	Technology	Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks, technology and teacher			
	Sonic other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students. follows predetermined plan			
Validation of Mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it.			
	Teacher assumes active role, while students mainly listen, copy or answer questions.			

Table 6 Teacher NT3's usage of technology under different roles

Roles \ Technology		Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks, technology and teacher			
	Sonic other elements of communication	1 2	4	
	Teacher communicates exclusively		3	
Interaction with students	Modifies plan according to students participation			
	Listens to students and answers questions but goes back to plan		4 3	
	Little interaction with students, follows predetermined plan	2 1		
Validation of Mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation	4 1 2	3	
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves	1		
	Problems from teachers and textbook	2		
	Unique source of mathematical problems	4	3	
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it.			
	Teacher assumes active role, while students mainly listen, copy or answer questions.	4 1 2	3	

Table 7 Teacher ST3's usage of technology under different roles

Roles \ Technology		Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks, technology and teacher		4	1
	Sonic other elements of communication		2	3
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation		4	
	Listens to students and answers questions but goes back to plan		2 3	1
	Little interaction with students, follows predetermined plan			
Validation of Mathematical knowledge	Multiple sources of validation		4	
	Teacher as only source of validation		3 2	1
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves		4	
	Problems from teachers and textbook		3 2	1
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it.		4	
	Teacher assumes active role, while students mainly listen, copy or answer questions.		3 2	1

CONCLUSION

The change in the role of math textbooks in teaching

From Table 2 to Table 7, it is clear to see that less than 1/3 teachers adhered to textbook while teaching geometric transformation. The usage of textbook reaches the level of elaborating and creating but most teachers would like to elaborate the textbook. Specifically, the application of technology is the main character of using textbooks creatively. Such as teacher ST1 and ST3 validate their hypothesis by Geometry Sketchpad, and teacher NT1 and ST1 asked students to design patterns on Sketchpad. Technology helps students strengthen their understanding and facilitate the advancement of their innovation. Furthermore, the elaboration of the textbook is eminently embedded in the supplement to the exercise in the textbook and the complement in the introduction scene, which manifests the contradiction of the uniqueness educational scene in math and the universality of textbook pursuit as well. In addition, the exploration on transformation concepts always follows the procedures listed in the textbook.

Generally speaking, with the advancement of mathematics curriculum reform, teachers have decreased dependence on the textbook and they turn their attitude from “teaching textbooks” to “making better use of textbooks” but still show the materialized. In the six classes, although superficially, math teachers all presented their content by PowerPoint, actually lack of the guide of mathematics thoughts or methods. The six teachers demonstrated contents from the textbook or the information collected from internet and other books in accordance with static sequence. Although textbooks concern students’ cognitive characters and they are prepared by mathematics educators and mathematicians, textbooks are often arranged in a linear organization based on the findings of attribution, which is inconsistent with the characteristics of students’ non-linear learning. Therefore, the course’s content cannot completely duplicate textbook step by step. Teachers should "use teaching materials" and make the appropriate adjustments in the basic research materials, otherwise it will affect the effectiveness of teaching. Just as what Zhong (2010) says “Although textbooks lie everywhere, teachers must experience and grasp them on their own. Current textbooks may not be useful, we must continue to develop new materials and make them become ‘my textbook’”.

In addition, differences exist among teachers at different stages of development. Novice teachers’ reliance on textbooks is significantly higher than senior teacher’s. For example, teacher NT3 only compared the similarities and differences of rotation and translation after exploring the nature of rotation. In other parts, she carried out his teaching in accordance with the sequence and contents in textbook. In comparison, teacher ST3 used the textbook creatively. He added that rotation transformation is reversible transformation. Then he changed the exploration of rotation’s nature into validating the hypothesis by Sketchpad. Here comes to a conclusion, senior teachers are better at using textbooks and grasping the materials, which is consistent with the results from other studies (Borko & Livingston, 1989; Westerman, 1991).

Changes of teachers' role improve the usage of textbooks

In mathematics teaching, teachers transform their roles in math communication, interaction with students, validation of knowledge, source of knowledge and students' autonomy, which improves the usage of textbooks.

Firstly, the communication of mathematics diversified. Teachers and textbooks are not the only way to acquire mathematical knowledge; technology becomes an external source for students and teachers to get mathematical information. The technology expands the direct interactions between students and mathematical knowledge and provokes teachers to acquire, represent, broadcast and communicate knowledge through technology.

Secondly, the validation of mathematics knowledge produces multiple ways to verify mathematics instead of treating math teachers as the only source. When information technology begins to support the validation, it affects the validation that depends on deductive reasoning in the textbook. Current textbooks seldom concern about using information technology, especially professional mathematics software, which disobey the basic philosophy of the integration of technology and curriculum. During the compilation and revise of textbook, the application of technology needs attention. Experts who prepared the textbook should think about how to present basic philosophy of technology in textbooks and how to develop students' literacy in information via textbooks. At last, students' autonomy improved and they broaden the source to study instead of only from textbooks. When teacher ST2 asked students to explore the concepts of rotation, she allowed them to collect source from internet. Hence, technology provides source for students to study mathematics, which urges teachers to break through textbooks and expand their horizons.

SUMMARY

According to the analysis above, there exists a close relationship between teachers' roles and textbooks' usage in teaching mathematics. The transformation of teachers' role, especially the use of technology insist teachers in creating favourable mathematics learning environment, facilitating students' communication and activating students' effective learning behaviour. Meanwhile, the transformation also urges teachers to consider the applicability of textbooks and allows them to avoid the problem of "de-skilling" (Apple & Jungck, 1990; Shannon, 1987) through appropriate adjustment in content and orders.

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SYMPOSIUM 9

MATHEMATICS IN THE SCIENCE CURRICULUM

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MATHEMATICS WITHIN BIOSCIENCE UNDERGRADUATE AND POSTGRADUATE HIGHER EDUCATION IN THE UK

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As Bioscience is becoming more and more quantitative, undergraduate bioscience courses are facing problems of new students being under-prepared for the mathematical demands. This paper reviews how Higher Education courses are seeking to overcome these problems.

Keywords: bioscience, mathematics in biology, Higher Education, UK

SUMMARY

At a time when bioscience is becoming more and more quantitative, undergraduate bioscience courses are facing problems of new students being under-prepared. There are gaps in knowledge and understanding and issues surrounding attitudes to maths. This paper looked at the transition issues and identified where many of the gaps are and explained how Higher Education courses are seeking to overcome these problems through the use of context, authenticity and closer integration of mathematics in biology. The implications of recent UK government initiatives for the school curriculum were also discussed, including GCSE reform and the proposed post-16 “Core Maths”.

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MATHEMATICS: THE LANGUAGE OF PHYSICS AND ENGINEERING

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While physics is generally considered as the most mathematical of the natural sciences, the curriculum and assessment of physics at GCSE and A-level in England have become progressively less mathematical over time. This has led to a mismatch between what physicists and engineers in Higher Education in England require of entrants in terms of their mathematical capabilities and the actuality. Here I offer some ideas on how the linkage between physics and mathematics might be improved.

Keywords: physics, engineering, mathematics in physics, mathematics in engineering, Higher Education, England

SUMMARY

Physics is the most mathematical of the natural sciences. However, over time and for a number of reasons, the curriculum and assessment of physics at GCSE and A-level in England have become progressively less mathematical over time. Using data from published reports, I demonstrated the mismatch between what physicists and engineers in Higher Education require of entrants in terms of their mathematical capabilities and the actuality. In addition, I discussed how physics often becomes much harder to understand when the mathematical content is minimised. Finally, I offered some ideas on how the linkage between physics and mathematics might be improved and also provide some early indications from the Institute of Physics’ new Curriculum Committee which has started its work by considering A-level Physics.

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CHEMISTRY AND MATHEMATICS: A SYMBIOTIC RELATIONSHIP?

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This paper explored the role of mathematics in chemistry, and the importance of mathematical skills in ensuring that students can fulfil their potential in studying the discipline. The support provided to chemistry undergraduate students at the University of Southampton was outlined.

Keywords: chemistry, mathematics in chemistry, Higher Education, England

SUMMARY

This paper explored the role of mathematics in chemistry, and the importance of mathematical skills in ensuring that students can fulfil their potential in studying the discipline. This included discussion of the impact of prior attainment in mathematics on performance at degree level, and the support provided to chemistry undergraduate students at the University of Southampton in the form of ‘Mathematics for Chemists’ workshops and accompanying resources.

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FULL PAPERS

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MATHEMATICAL KNOWLEDGE AND SKILLS EXPECTED BY HIGHER EDUCATION: IMPLICATIONS FOR CURRICULUM DESIGN AND TEXTBOOK CONTENT

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One important function of school mathematics curriculum is to prepare high school graduates with the knowledge and skills needed for university education. Mathematics textbook writers operationalize curricular goals by choosing which content to cover. Because mathematical knowledge and skills needed for university education may differ from field to field, identifying the knowledge and skills scientifically that higher education programs expect in first year students may help making sound decisions about the contents of mathematics textbooks. In this study, we surveyed university faculty who teach first year university students about the mathematical knowledge and skills that they would like to see in incoming high school graduates. Data were collected from 122 faculty from social science (history, law, psychology) and engineering departments (electrical/electronics and computer engineering). Participants rated how important they thought certain mathematical topics and skills were to be successful in university and become a well-prepared professional. Results were compared across social science and engineering departments and among mathematical knowledge and skills. Implications were drawn for curriculum design and mathematics textbook content.

Keywords: mathematics curriculum, mathematical knowledge, mathematical skills, Higher Education, Turkey

INTRODUCTION

Mathematics curriculum is usually considered to serve three general goals; i) to meet needs of adult life and pre-professional employment (e.g., general financial literacy, manufacturing, clerical, agriculture, construction work), ii) to prepare students for further university education and iii) to develop students' general problem solving and analytical thinking skills (Cockroft, 1982; NCTM, 2000). For the second goal above, because there are many disciplines in universities that a student could follow, it is important that schools prepare students for higher education by differentiating mathematics curriculum.

Mathematical textbooks are important embodiments of mathematics curricula as they operationalize curricular goals about what to teach and how much weight is given to mathematical topics (Schmidt et al., 1997; Erbaş, Alacaci & Bulut, 2012).

In 2012, Turkey expanded compulsory education from 8 to 12 years. This means all students will need to stay in school until they attain a high school diploma. The change necessitates a reconsideration of goals and contents of high school mathematics curriculum to better meet the needs of the new and expanded body of high school students with a wider range of abilities, interests and needs.

In an effort to balance the mathematical needs of adult life and university education, curriculum developers in Turkey have reworked the structure of mathematics curriculum at high schools to allow adjustments according to a students' life after high school. One part of this effort is about allowing variation in mathematics curriculum according to the field of study a student wishes to follow at university.

Most of restructuring is based on guess-work about the mathematical needs of these disciplines. This study however is an effort to investigate empirically the mathematical needs of social science and engineering disciplines to be attained in high school as perceived by university faculty teaching students in these fields. If curricular and textbook content decisions are supported by correlational scientific and empirical data as called for by Fan (2013), it is hoped that students can save time and energy and be more successful and motivated as they see higher relevancy in what they study in mathematics in high school.

METHODS

A combined list of 43 mathematical topics from Turkish national mathematics curriculum for grades 9-12 was prepared for the survey. The list was supplemented by 6 more topics from International Baccalaureate Diploma Program (IBDP) that were not part of Turkish mathematics curriculum, as university faculty might have thought there were topics students needed to learn but were not covered by Turkish national curriculum. IBDP curriculum and textbooks are used in an increasing number of private schools in Turkey. The list consisted of 49 mathematical topics in all. In a separate section, participants were asked about the following mathematical skills; mathematical problem solving, mathematical modelling, mathematical reasoning, mathematical communication, mathematical representations, mathematical connections, analytical thinking and critical thinking. These topics and skills were arranged in a written survey for participants rating.

Participants were teaching staff from two leading universities in five departments in Ankara Turkey at various ranks; instructors, assistant, associate and full professors. There were 72 faculty from engineering (computer engineering 42, electrical/electronic engineering 30) and 50 from social science departments (psychology 17, law 25 and history 8).

In the survey, university faculty were asked to rate the mathematical topics and skills from 1 to 5 for how important they thought it was (1: not important at all, 5: very important) for students to attain in high school for being successful in university education in their field. They were also given space to add any other topic or skill that was not given in the list, but they might consider important.

Data were analysed by computing simple arithmetic means and standard deviations across social science and engineering departments and across individual departments for each of the 49 mathematical topics and the 6 mathematical skills.

RESULTS

Table 1 presents the mean values and standard deviations, representing the perceived importance for the 49 mathematical topics. To more easily compare topics, summary information is given in the last four columns of Table 1. Participants rated the topics from 1 to 5 (least important to very important), a mean value of 3,5 and above is considered "important." Hence values at or above 3,5 is shown by a plus sign and values below 3,5 is shown by a minus sign in the last four columns of the table.

Table 1: Perceived levels of importance of mathematical topics across departments

Topics	**Mean (SD)**				**Perceived Importance**			
	Law & History	Psychology	EE Engineering	Computer Engineering.	L&H	Psy	EE	CE
logic	4,455 (0,905)	3,765 (1,252)	4,258 (0,930)	4,805 (0,401)	+	+	+	+
proof	4,212 (0,857)	3,529 (0,943)	4,290 (0,902)	4,634 (0,488)	+	+	+	+
sets	3,212 (1,219)	3,176 (0,883)	4,226 (0,669)	4,415 (0,865)	-	-	+	+
relations	3,303 (1,186)	3,235 (0,831)	4,226 (0,497)	4,366 (0,767)	-	-	+	+
function	3,000 (1,090)	3,059 (1,249)	4,742 (0,445)	4,732 (0,501)	-	-	+	+
modular mathematics	2,333 (1,216)	2,647 (0,996)	4,290 (0,824)	4,537 (0,778)	-	-	+	+
exponential/radical numbers	2,273 (1,232)	3,353 (0,996)	4,516 (0,724)	4,244 (0,943)	-	-	+	+
divisibility	2,970 (1,358)	2,824 (1,286)	4,161 (0,779)	4,220 (0,791)	-	-	+	+
rate/proportion	3,939 (1,088)	4,294 (0,772)	4,484 (0,890)	4,268 (0,949)	+	+	+	+
vectors	2,394 (1,321)	2,588 (1,326)	4,452 (0,768)	4,439 (0,594)	-	-	+	+
line/circle in plane	2,333 (1,291)	2,588 (1,372)	3,903 (1,012)	3,951 (0,835)	-	-	+	+
distance in plane	2,424 (1,300)	2,647 (1,320)	4,097 (0,944)	4,122 (0,781)	-	-	+	+
point/line/angle	2,091 (1,042)	2,647 (1,115)	3,452 (1,028)	3,634 (1,178)	-	-	-	+
triangle/polygons	2,364 (1,194)	2,353 (1,222)	3,613 (1,054)	3,561 (1,119)	-	-	+	+
3D objects	2,212 (1,269)	2,294 (1,359)	3,516 (0,811)	3,293 (1,188)	-	-	+	-
tessellations	2,000 (1,061)	2,353 (1,170)	2,710 (0,739)	2,756 (0,916)	-	-	-	-
polynomials	2,485 (1,064)	2,706 (1,312)	4,484 (0,570)	4,195 (0,843)	-	-	+	+
quadratics	3,091 (1,234)	3,059 (1,345)	4,677 (0,541)	4,244 (0,799)	-	-	+	+
trigonometric ratios	2,242 (1,062)	2,353 (1,272)	4,645 (0,551)	4,000 (0,949)	-	-	+	+
trigonometry	2,182 (1,045)	2,294 (1,213)	4,452 (0,675)	3,927 (1,010)	-	-	+	+
similarity/triangles	2,606 (1,249)	2,588 (1,326)	3,419 (1,089)	3,415 (1,072)	-	-	-	-

transformations	2,121 (1,054)	2,647 (1,367)	3,645 (1,018)	3,537 (1,185)	-	-	+	+
proofs in geometry	2,515 (1,302)	2,412 (1,278)	3,419 (1,177)	3,488 (0,978)	-	-	-	-
complex numbers	2,303 (1,132)	2,412 (1,228)	4,387 (0,761)	3,390 (1,093)	-	-	+	-
exponential equations	2,152 (1,121)	2,824 (1,185)	4,387 (0,667)	4,098 (0,735)	-	-	+	+
logarithmic equations	2,121 (0,992)	2,882 (1,269)	4,387 (0,667)	4,439 (0,594)	-	-	+	+
induction / proof	4,030 (1,075)	3,588 (1,176)	4,226 (1,023)	4,488 (0,637)	+	+	+	+
sequences	2,788 (1,269)	3,176 (0,883)	4,065 (0,772)	4,146 (0,793)	-	-	+	+
matrices	2,242 (1,251)	2,765 (1,201)	4,323 (0,748)	4,659 (0,530)	-	-	+	+
linear eqn.	2,788 (1,244)	3,118 (1,219)	4,516 (0,570)	4,463 (0,711)	-	-	+	+
counting methods	3,030 (1,186)	3,353 (1,272)	4,000 (0,856)	4,659 (0,617)	-	-	+	+
Pascal/binomial expn	2,333 (1,164)	2,529 (1,281)	3,871 (0,846)	4,171 (0,863)	-	-	+	+
conics	2,152 (1,176)	2,235 (1,201)	3,355 (0,950)	3,341 (1,087)	-	-	-	-
circular region	2,424 (1,347)	2,529 (1,328)	4,000 (0,894)	3,439 (1,141)	-	-	+	-
probability	3,788 (0,992)	4,529 (0,515)	4,290 (0,938)	4,732 (0,593)	+	+	+	+
stat/data representation	3,455 (1,277)	4,882 (0,332)	4,032 (1,080)	4,634 (0,662)	-	+	+	+
stat/tendency-dispersion	3,091 (1,422)	4,824 (0,529)	3,548 (1,207)	4,512 (0,746)	-	+	+	+
limits/continuity	2,636 (1,109)	3,471 (1,213)	3,387 (0,956)	3,951 (0,975)	-	-	-	+
function graphs	2,727 (1,278)	3,290 (0,551)	4,765 (1,272)	4,098 (0,799)	-	-	+	+
derivatives	2,818 (1,176)	3,412 (1,393)	3,516 (0,851)	4,049 (0,662)	-	-	+	+
integration	3,030 (1,083)	3,387 (0,811)	4,824 (1,539)	3,927 (0,813)	-	-	+	+
vectors in 3D	3,000 (1,139)	3,516 (1,046)	4,882 (1,372)	3,683 (0,716)	-	+	+	+
plane transformations	3,485 (1,023)	3,118 (1,328)	2,613 (0,795)	3,122 (0,866)	-	-	-	-
finite random var.	2,636 (1,220)	3,471 (1,231)	3,387 (1,116)	3,951 (0,805)	-	-	-	+
stat/distributions	2,727 (1,232)	4,765 (0,437)	3,290 (1,189)	4,098 (1,020)	-	+	-	+
Bayes theorem	2,818 (0,917)	3,412 (1,121)	3,516 (1,313)	4,049 (0,893)	-	-	+	+
hypothesis testing	3,030 (1,186)	4,824 (0,393)	3,387 (1,383)	3,927 (0,959)	-	+	-	+
correlation/regression	3,000 (1,199)	4,882 (0,332)	3,516 (1,313)	3,683 (1,150)	-	+	+	+
interest	3,485 (1,278)	3,118 (1,364)	2,613 (1,334)	3,122 (1,029)	-	-	-	-

* H&L: History and Law, Psy: Psychology Departments, EE: Electrical/Electronic Eng Department, CE: Computer Engineering

For the departments of Law and History, only 5 topics (rates and proportions, logic, general methods of proof, and basic probability) out of the 49 topics were considered important to be attained at high school by the university faculty. For the department of psychology, 11 topics were considered important. Most of these topics were from statistics and probability strand; rates and proportions, logic and proof, basic probability, graphing data, measures of central tendency, statistical distributions, correlation and regression, and hypothesis testing. For the two engineering departments, most of the topics were considered important. Topics that were rated less than important were tessellations, geometric proofs, triangle similarities, conic sections, geometric transformations, and interest computations.

Table 2 shows the means and standard deviations for perceived importance of mathematical skills. In contrast to mathematical topics, there seems to be a stronger agreement among university faculty regardless of the discipline. They rated these skills as important to develop at high school for the education of students at university.

Table 2: Perceived levels of importance of mathematical skills across departments

Skills	H&L*	Psy	EE	CE	H&L	Psy	EE	CE
Math. problem solving	4,156 (0,884)	4,125 (0,719)	4,700 (0,466)	4,878 (0,331)	+	+	+	+
Math. modelling	3,781 (0,941)	3,875 (0,885)	4,367 (0,556)	4,634 (0,581)	+	+	+	+
Math. thinking	4,406 (0,837)	4,125 (0,719)	4,700 (0,466)	4,732 (0,501)	+	+	+	+
Math. communication	3,656 (1,208)	4,188 (0,834)	4,833 (0,379)	4,878 (0,510)	+	+	+	+
Math. relations	4,063 (1,076)	4,188 (0,981)	4,300 (0,596)	4,683 (0,471)	+	+	+	+
Math. representations	3,219 (1,263)	4,250 (0,856)	4,167 (0,699)	4,512 (0,637)	-	+	+	+
Analytical reasoning	4,563 (0,716)	4,688 (0,602)	4,667 (0,606)	4,951 (0,218)	+	+	+	+
Critical thinking	4,781 (0,491)	4,750 (0,577)	4,733 (0,521)	4,683 (0,650)	+	+	+	+

* H&L: History and Law, Psy: Psychology Departments, EE: Electrical/Electronic Eng Department, CE: Computer Engineering

DISCUSSION AND CONCLUSIONS

There is a clear difference in the perceived mathematical needs of engineering departments and departments of social sciences, as one would expect. Turkish national curriculum in mathematics and hence mathematics textbooks seems to be demanding too much in terms of scope of the curriculum for university-bound students. But even for engineering departments, it seems as if there a number of topics that might even be skipped from the national curriculum without hurting the mathematical preparation of high school students, such as conic sections and geometric proofs.

For social sciences, the findings draw a more dramatic picture: Only a few of the topics that are required in the national curriculum seems to be needed for these students in their university education. Further, there are some topics that they would need but are not present in the existing curriculum, as indicated by the ratings of participants. Most of these missing topics are from statistics and probability strand such as statistical distributions, theory of hypothesis testing, and correlation and regression. It seems as if students could benefit from a mathematics curriculum (and textbooks) that has a more robust treatment of statistical topics in Turkey for students intending to study social sciences at university.

While interpreting the findings of this study, it should be remembered that university faculty rated mathematical topics that they perceived to be important for the study of their disciplines only. Their ratings may not necessarily presume the attainment of topics that would be needed in an average adult life. It would be defensible to argue that specialized mathematical topics in high school for a line of university education should be considered in addition to the topics that would be needed for all high school graduates, university-bound or otherwise.

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PROBLEM SOLVING HEURISTICS IN MIDDLE SCHOOL MATHEMATICS TEXTBOOKS IN SAUDI ARABIA

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In Saudi Arabia, like many other countries, teaching solving problems is a main goal of teaching mathematics and a central concern of mathematics educators and mathematics curriculum developers. As textbooks are fundamental resources in Saudi classrooms, this study investigates how the national middle school mathematics textbooks in Saudi Arabia represent problem solving heuristics. We established a framework for coding heuristics into different categories such as "guess and check", "look for pattern", "draw a diagram", etc. The data were collected from three textbooks used in Grades 7, 8 and 9 through analyzing all examples problems in the textbooks. The findings show that all the textbooks represent a good number of problems and in total there are fourteen heuristics presented in the textbooks, though most of which are found in Grades 7 and 8. It is hoped that the findings of the study will shed light on the potential challenges and opportunities for textbooks developers to further improve textbook quality in terms of mathematical problem solving.

Keywords: textbook analysis, mathematical problem solving, Saudi Arabia

INTRODUCTION

Over the last three decades, educational researchers and policy makers have paid more attention to school textbooks content and their use by teachers and students, as they play an important part in learning and teaching process (e.g., Harding, 1995).

In Saudi Arabia, there is a considerable attention toward improving mathematics and science textbooks and that attention becomes more reliable after the Saudi Arabia first participation in the international assessments. The disappointing test results of students' performance in the Trends in International Mathematics and Science Study (TIMSS) were a high alert to raise the quality of education particularly in mathematics. To face these deficiencies in education, policy makers in the Ministry of Education have emphasized reforming textbooks alongside many improvements in mathematics education since textbooks contents mostly reflect what happened in the classrooms (Floden, 2000). After many stages of reform and improvement, the first edition of the new mathematics textbooks series was released in 2007 for primary schools (6years) and middle schools (3years), and in 2008 for secondary schools (3years). The whole series was developed by the Ministry of Education. In 2010 the new series has been adopted in all Saudi schools. The new textbooks were written to be more coherent than the previous textbooks, and the textbooks topics were built based on reasoning, problem solving and communication skills (Ministry of Education, 2007).

The education system in Saudi Arabia aims to support students' learning and develop their skills in all aspects and there is no doubt that developing students' ability in mathematics problem solving become a very high concern of mathematics educators policy makers and a main goal of teaching mathematics (Ministry of Education, 2007). As all teachers in Saudi Arabia are required to adhere to the textbook as a main resource for teaching, this study aims to examine to what extent the new textbook in Saudi Arabia reflect the mathematics curriculum goals in problem solving. More specifically, given the importance of problem solving heuristics, the study aims to address the following question: how the new national middle school mathematics textbooks in Saudi Arabia represent problem solving heuristics.

LITERATURE REVIEW AND FRAMEWORK

Problems solving has occupied a main part in mathematics education as Polya (1973) stated that to know mathematics is to solve problems. Moreover, students' ability to solve problems is a considerable indicator about students' general performance in mathematics (NCTM, 2000). Many researchers have defined the term "problem" in different ways for different purposes. According to Kantowsky (1980, P.195) "a problem is a situation for which the individual who confronts it has no algorithm that will guarantee a solution. That persons' relevant knowledge must be put together in a new way to solve the problem". That means a problem is something difficult to address and the students need to understand how to assign various sources of information to solve the problem.

The *Principles and Standards for School Mathematics* (NCTM, 2000) pointed that "Students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns, or trying special values or cases. These strategies need instructional attention if students are to learn them." (p. 182). It is essential to provide students with a general guidance for solving mathematics problems due to problem solving process is a mental operations that involve a set of procedures and heuristics in order to find solutions (Mayer, 1985). In fact, heuristics are another concept Polya used to analyse the problem solving process together with the four problem solving steps understanding the problem, devising a plan, Carrying out the plan and looking back. Polya described heuristic reasoning as reasoning not defined as final and strict but, as provisional and plausible only, whose purpose is to discover the solution of the present problem. Such reasoning is often based on induction or on analogy (Polya, 1973). According to Schoenfeld (1985) a heuristic is "a general suggestion or strategy, independent of any particular topic or subject matter, that helps problem solvers approach and understand a problem and efficiently marshall their resources to solve it" (p. 23). Schoenfeld (1979) noted that using heuristics in actual problem would be more practical to solve this problem than the four stages in Polya's model. He suggested that the main agenda of mathematics education research was to work out how problem solving strategies can be taught successfully, because under appropriate circumstances, many students can learn to use heuristics, with the result being a demonstrable improvement in their problem solving performance. It is important to realise that heuristics are tools that may make solving problems easer as Martinez (1998) stated, heuristics "is a strategy that is powerful and general, but not absolutely guaranteed to work. Heuristics are crucial because they are the tools by which problems are solved" (p. 606). In this study, we referred the concept of heuristics to Schoenfeld's definition, which is widely used in the literature (e.g., Schoenfeld, 1985; NCTM, 2000; Koichu, Berman & Moore, 2004; Fan & Zhu, 2007). Table 1 describes heuristics that represented in Saudi national textbooks.

Table 1: Problem solving heuristics descriptions

Guess and check	Make a reasonable guess of the answer, and check the result against the conditions of the problem to see if it is the answer.
Look for patterns	Observe common characteristics, variations, or differences about numbers, shapes, etc in the problems to find a solution.
Work backwards	Attack the problem from the outcomes or conclusions backwards to find what conditions the problem solver eventually need.
Draw a diagram	Draw a graph based on the information to visualize the problem.
Make a list	Construct on organized list containing all the possibilities for a given situation and find the answer.
Simplify the problem	Change the complex numbers or situations in the problems into simpler ones without altering the problem mathematically.
Act in out	Use people, objects to physically show what is exactly described in the problem.
Restate the problem	Approach the problem from a different angle.
Use equations	Use the letters as variables to represent unknown quantities.
Use a model	Translate the problem in different way by Using lines, points.
Use Venn diagram	Represent the problem by using Venn diagram.
Logical reasoning	Reasoning from one or more true statements.
Draw a picture	Drawing figures or graphs.
Think of a related problem	Recall related problem solved before to solve the recent problem.

METHODOLOGY

To investigate how middle school mathematics textbooks in Saudi Arabia represent problem solving heuristics, we selected three textbooks for Grades 7, 8 and 9 that are currently used in Saudi middle schools in order to collect, code and analyse data.

Textbook analysis

In this study, we analysed the student books as each textbook consists of student book and work book. Analysis procedure was based on problem solving heuristics introduced in the main text, e.g., in examples, due to teachers using them in class discourses or groups work.

Coding procedure

According to the conceptual framework of problem solving heuristics we assigned the units that contain mathematical problems then we categorised the heuristics we had found. In order to measure the reliability of coding, an external coder was invited during the study. The Inter-rater Agreement according to the Intra-class Correlation Coefficient (ICC) was 0.98 on heuristics in general and 0.97 on heuristics existence in each book thus, this result indicates a high agreement in coding.

RESULTS AND DISCUSSIONS

The results from the coding revealed that the Saudi middle school textbooks represented a variety of problem solving heuristics and the results can be seen in Figure 1.

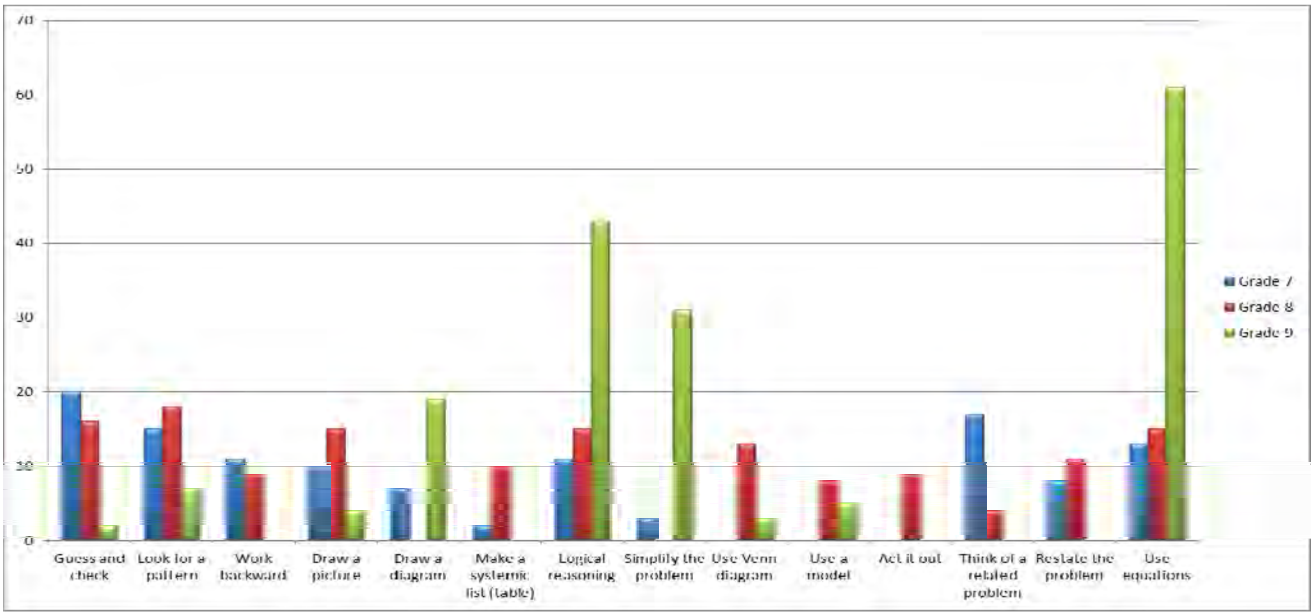

Figure 1. Problem solving heuristics in Saudi middle schools textbooks

As we can see from Figure 1, fourteen different heuristics are introduced in the three books. The distribution of these heuristics showed that the three textbooks represent about the same number of these heuristics. In other words, the data revealed that grade 8 textbook represents the highest number (12) of the (14) heuristics, followed by grade7 (11) heuristics and the lowest number was coded in grade 9 (9) heuristics. Furthermore, five of the heuristics were common across the three textbook. It can be noted that the majority of problems in the three textbooks were solved based on Polya's four stages.

It can be also seen that “guess and check” and “look for a pattern” are nearly the most frequently used heuristics represented in the 7th and 8th grade textbooks, whereas these heuristics seem to be less present in grade 9 textbook. The result is interesting in comparison with what Fan and Zhu found in their study, namely “guess and check” and “look for a pattern” were the least frequently used heuristics in mathematics textbooks in China, Singapore and USA at the lower secondary grade level (Fan & Zhu, 2007). Furthermore, if we look at all the heuristics presented in the grade 7 textbook, we can find that the most frequent heuristic is “think of a related problem”, whereas in the grade 9 textbook, there is no “think of a related problem” heuristic at all.

It should be also pointed out that the numbers of “logical reasoning” and “use equations” heuristics in the three books are increased gradually from the grade 7 to 9 textbooks.

Moreover, the data revealed that "use equations" is the major problem solving heuristic represented in the grade 9 textbook. This result reflects the importance and usefulness of these two heuristics. In fact, Polya (1973) once explained that using equations is like "translation from one language into another and the difficulties which we may have in setting up equations are difficulties of translation" (p. 174). The result also showed that there are 11 and 9 examples using the "work backward" heuristic in the grade 7 and 9 textbooks respectively. Charles and Lester (1984)'s study found that using a verity of heuristics like logical reasoning, work back word and make a list for teaching Grade 5 and 7 students improved students' ability to understand the problem and choose the right solution to solve the problem. The question here is whether the "work backward" heuristic is more suitable for students to learn at lower levels that higher levels.

Figure 1 also shows that "act it out", "use a model", "use Venn diagram" and "make a systemic list" are the least frequently heuristics used in both grade 7 and 9 textbooks, whereas these heuristics are represented frequently in the grade 8 textbook. In comparison, Fan and Zhu (2007)'s study reported that "act it out" and "use a model" were among the four least heuristics presented in China, Singapore and USA textbooks. It seems worth further investigation on whether using "act it out" and "use a model" are more appropriate for students at certain levels with certain backgrounds. Finally, it was found that most heuristics in the three textbooks are introduced and labelled as special topics, which reflects the textbooks authors' vision of focusing on heuristics as a tool to solve problems.

SUMMARY AND CONCLUSION

This study examined Saudi mathematics textbooks at the middle schools level in order to reveal how they represent problem solving heuristics. The results show that there are 435 problems examples in the three textbooks that have been solved by using various problem solving heuristics, and in total, there are 14 different heuristics in the these textbooks, which appears to be fairly reasonable in terms of variety (e.g. see Fan & Zhu 2000; 2007). In consideration of the fact that textbooks are potentially implemented curriculum and Saudi Arabia's educational context, we think the results to a large degree indicate how problems solving is being taught in Saudi classrooms and it is clear that problem solving heuristics get a very high attention in middle schools textbooks. As the majority of examples were solved by one heuristic, we suggest it will be helpful when such problems are solved by two or more heuristics, when it is appropriate, so the students can learn from different solutions for the same problem.

As the data show, the frequency of using some types of heuristics was low in some textbooks whereas, the frequency of these types was high in other textbooks. It seems that the distribution of various heuristics among the three textbooks was not sufficiently organised. From this point, it is necessary for textbooks developers and mathematics educators to consider heuristics' distribution in the three textbooks and reconstruct these heuristic regarding the usefulness of these heuristics to the students at this level. It was also found that the three textbooks represent and label problem solving heuristics as specific topics and that might lead teachers and students to treat these heuristics separately. It would be beneficial to consolidate the heuristics with other topics to help students to tackle the problem better.

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SOCIAL AND MATHEMATICAL PRACTICES ASSOCIATED WITH THE DEVELOPMENT OF MATHEMATICAL MODELS OF POPULATION GROWTH APPROACHED IN TEXTBOOKS

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In this article we investigate which practices (mathematical and social) can be identified with regard to population growth. We start with the analysis of the historical and cultural context in which they were developed two classical models for the study of population growth: the model of Malthus and the Verhulst model. Whereas the study of the dynamics of population growth conquers space in the school environment in disciplines of ordinary differential equations, we direct our focus for the analysis of textbooks of this discipline. This analysis allows us to conclude that, from the epistemological point of view, the authors present the models already structured and with mathematical language and notation as currently agreed. Mathematical practices are reduced to applications that differential equations can generate in relation to growth of populations of different species.

Keywords: social practices, population growth, mathematical modelling, differential equations

INTRODUCTION

The present article is intended to identify the mathematical and social practices in relation to the population growth, focusing the construction of classical mathematical models – Malthus' and Verhulst's models - to describe and estimate populations.

According to Arrieta (2003), social practices are guided by the production and reproduction of the humanity's practices throughout time, with intentionality and for specific situations. In this article, we are also interested in practices which require or make use of mathematical knowledge, and are undertaken by not-necessarily-scientific communities but, in a certain way, are also influenced by those. Addressing us, in this way, to the mathematical practices

As stated by Vilela (2009), these are conditioned by the own structure of language, which limits and regulates the development possibilities of mathematics in the specific practices. The author ponders that the mathematical practices also constitute intentional social practices in each situation and context, being determined by the normative force of formulation from some groups. School mathematics therefore seems to be constituted by means of appropriation and transformation of socially elaborate knowledge, not depending solely on the individual, but also on the median relation to the other and to the culture.

In the text, we look first at the historical and cultural context in which the population growth models were developed. Then, considering that the study of population growth gains relevance within the school environment, in subjects such as Ordinary Differential Equations (ODE), we drive our focus to the analysis of textbooks on this matter.

SOCIAL PRACTICES AND MATHEMATICAL PRACTICES IN THE DEVELOPMENT OF POPULATION GROWTH MODELS

In order to deal with the social and mathematical practices associated to the development or use of population growth models, we initially described the origin of the two classical models – Malthus' and Verhulst's - addressing these authors' practices with the objective to deduct their models.

Malthus's Model

One of the earliest essays regarding the analysis of the dynamics of the world's population was the work *An essay on the principle of population*, written by Malthus in 1798. In this year, industrial agglomerations began to grow, the factory's proletariat came to be, and the aggravation of misery was strongly related to the growth of taxation on the poor. Malthus wrote his Essay in the midst of a rapid population growth and its corresponding penury, influenced much more by the distributional of wealth than by the great number of inhabitants (Mantoux, sd). He characterized two postulates: 1. "That food is necessary for the existence of man"; 2. "That the passion between the sexes is necessary, and will remain nearly in its present state" (Malthus, 1798, p.4). In keeping with Malthus (1798), such postulates seem to have been fixed by the laws of nature, once it had knowledge of humankind whatsoever.

The analysis of the original works written by Malthus allows us to affirm that his model was not written in terms of a mathematical language. This can be justified by the social practices that may have influenced Malthus in the elaboration of his publications. Nowadays, the so conventionally called 'Malthus's model' considers that the variation of a population growth is proportional to the population in each instant, which means to say that the population grows in geometrical progression or in exponential growth. One associates to Malthus's hypothesis a mathematical language in terms of the solution of an ordinary differential equation separable of first order together with an initial condition, customarily written $\begin{cases} \frac{dP}{dt} = kP \\ P(0) = P_0 \end{cases}$, where *t* is the time; *P(t)* is the population in the time *t*, P_0 is the initial population and *k* is the constant of proportionality.

Considering the ideas proposed by Malthus and the model associated to them for more prolonged periods of time, the population would become infinitely great, which does not constitute an acceptable estimate. It would be necessary to consider the existence of some factor which must reduce the growth rate and inhibit the exponential growth. The estimates and search of this factor would become the main objective of Verhulst's complementations to the enunciates presented by Malthus.

Verhulst's Model

The results of Verhulst's investigations on the demographic growth came to light by means of several publications in the period of 1838-47. Verhulst's descriptions reveal the influence of his professor Quetelet's ideas on his methods and publishing of his works. Verhulst's interest in studying the population growth prediction, due to his relations to Quetelet and/or the interest in overcoming the limitations to the exponential modeling proposed by Malthus, characterize social practices that, in a historical and social context, grant some structure and meaning to the theoretical formulations on the growth of the world population.

In general terms, Verhulst exposed the essence of his growth theory in the publications Notice sur la loi que la population suit dans son accroissement (1838) and Recherches mathématiques sur la loi d'accroissement de la population (1845), defending that the population growth has necessarily a limit and that it does not grow indefinitely, as Malthus exposed in his formulation. Establishing P as population, t as time, s and n undetermined constants, b the normal population having denoted M as the value (in module) by which it is necessary to multiply the natural logarithm to convert it in decimal logarithm, Verhulst (1845, p.8) wrote:

> We can substitute the differential equation $\frac{Mdp}{pdt} = s$, related to the geometrical progression by $\frac{Mdp}{pdt} = s - n(p-b)$.

In the publication of 1845, Verhulst introduced the term logistic to refer to this population growth equation, presented more details on its properties and estimated its parameters. In his publication Note sur la loi d'accroissement de la population (1846) Verhulst announced that the condition that establishes a limit to the population growth is not proportional to the superfluous population, as he had assumed in his earlier publications, which led Verhulst to a revision of the original model. In 1847, Verhulst exposed with more details to the notes of 1846, dealing with aspects differing greatly from the earlier texts and assumed that:

> The obstacles to population growth increase in proportion to the overabundant population by the total population, in such ways that $\frac{Mdp}{pdt} = s - \frac{n(p-b)}{p}$.

It is possible to notice a change of hypothesis in Verhulst's publications and, consequently, in his mathematical model. Conforming to Verhulst (1846, p.226), "these results shall not be considered definite" This demonstrates that the problem of either the acceptation or not of a model depends on factors that affect the modeler, such as the historical context, his objectives and available resources.

Last century's literature conventionally called Verhulst's model a logistic model and the solution to the problem $\begin{cases} \frac{dP}{dt} = kP(L-P) \\ P(0) = P_0 \end{cases}$, where t is the time, P (t) is the population in time t, P0 is the initial problem, k is the proportionality constant and L is the population limit.

POPULATION GROWTH IN TEXTBOOKS

The study of the population growth dynamics gains relevance in the school environment, in higher education courses, in the ODE subject. Aiming at investigating how the population growth problematic is approached in such environment, we analyzed textbooks for the ODE subject. This subject is part of the higher education curriculum structure at several areas in Brazilian universities, and two books are more frequently mentioned in the bibliography to this subject: 1) *Equações Diferenciais Ordinárias Elementares e Problemas de Valores de*

Contorno (Elementary Ordinary Differential Equations and Boundary Value Problems) by authors Boyce &DiPrima, published in 2002. 2) *Equações Diferenciais Ordinárias com aplicações em Modelagem* (Ordinary Differential Equations with Applications in Modeling) by Zill, translated by Patarra, in 2003.

The thematic of population growth is approached in Boyce and DiPrima's book in the chapter on the Ordinary Differential Equations of First Order, named Autonomous Equations and Population Dynamics. The section begins with a definition to these and, in the sequence, the author discusses these equations in the context of a specie's population growth and decline, considering Malthus's exponential growth, as shown in Figure 1.

Exponential Growth. Let $y = \phi(t)$ be the population of a species in a given instant t. The simplest hypothesis on the variation of the population in that the variation rate of y is proportional to the current value of y, that is,* $\frac{dy}{dt} = ry$ (1)

Where the constant of proportionality r is named rate of growth or decline, depending on whether it is positive or negative. We shall suppose here that $r > 0$, in such way that the population is increasing.

Solving the Equation (.1) subject to the initial condition $y(0) = y_0$, *(2)*

we obtain $y(t) = y_0 e^{rt}$ *(3)*

** Apparently, British economist Thomas Malthus (1766-1834) was the first to observe that many biological populations grow at a rate proportional to the population. His first article about populations appeared in 1798.*

Figure 1: Exponential growth as proposed by Malthus and presented by Boyce and DiPrima (2002, pp 40-41)

The authors emphasize the methods to solve the differential equation, although in the book makes mention of a historic aspect in the origin of the equation as a model to analyze the population growth. Furthermore, the authors also call attention to the fact that Malthus's model is reasonably precise for many populations during a certain period of time. Still, sometime the limitations (space, food and other resources) eventually reduce the growth rate, inhibiting the population growth. The authors then present, as shown in Figure 2, the model proposed by Verhulst which takes such considerations into account.

Logistic Growth. In order to consider that the growth rate in fact depends of the population, we substitute the constant r in the Equation (1) by a function $h(y)$, obtaining thus the modified equation $\frac{dy}{dt} = h(y)y$ (4).

Now, we will choose $h(y)$ in such way that $h(y) \approx r > 0$ when y is small, $h(y)$ decreases when y increases and $h(y) < 0$ when y is sufficiently great. The simplest function to have these properties is $h(y) = r - ay$, where a a positive constant is also. Making use of this function in Equation (4), we obtain

$$\frac{dy}{dt} = (r - ay)y \qquad (5)$$

The Equation (5) is known as Verhulst's equation or logistic equation. It is for many times convenient to write the logistic equation in the equivalent way* $\frac{dy}{dt} = r\left(1 - \frac{y}{k}\right)y$ *(6)*

** P. F. Verhulst (1804-1849) was a Belgian mathematician who introduced the Equation (5) as a model for the human population growth in 1838. He referred to this growth as logistic growth; therefore, the Equation*

(5) is, for many times, called logistic equation. He was not able to test the precision of his model due to the inadequate census data and he was not given much attention until many later years. The reasonable concordance of the model to the experimental data was demonstrated by R. Pearl (1930) for populations of Drosophila melanogaster (common fruit fly or vinegar fly) and by G. F. Gause (1935) for populatuions of Paramecium and Tribolium (flour beetle).

Figure 2: Population growth according to Verhulst and presented by Boyce and DiPrima (2002, p 41)

According to what we presented in the previous section, it is possible to observe a variation-type approach in Verhulst's publications, whose intention is predicting the future state of the population growth. On the other hand, Boyce e DiPrima (2002) introduce a logistic model idea, discussing the formulation of a mathematical expression that would establish a limit to the population. The authors do not present any discussion on the origin of such formulation but, starting from Malthus's model, they include a term that would confer the model an asymptotic characteristic. The historical context is pointed in a footnote, where Verhulst and the date to one of his publications is mentioned. Alongside that, also in a footnote, the authors briefly indicate the context to the development and the validation of the model. Specific mentions to the growth or decrease of the population appear in the exercises throughout the book and, so it seems, it is revealed an intention of the authors to associate Verhulst's model to the analysis of the population growth in different species.

In the other book we analyzed, Zill (2003), the population growth modelation is worked in different moments and circumstances. In the Introduction to Differential Equations chapter, more particularly in the section named Differential Equations as Mathematical Models, Zill addresses Malthus's model as presented in Figure 3.

Population Dynamics One of the first attempts to modeling the human population growth by means of mathematics was made by English economist Thomas Malthus in 1798. Basically, the idea behind the Malthusian model is the hypothesis that the rate to which the population of a country grows in a determined instant is proportional to the total population in that instant. In other words, the more people there are in an instant t, the more people there will be in the future. In mathematical terms, if P(t) is the total population in the instant t, then this hypothesis may be expressed by*

$$\frac{dP}{dt} \propto P \quad \text{or} \quad \frac{dP}{dt} = kP \qquad (7)$$

where k is a constant of proportionality. Although this simple model does not take into account several factors that may influence the human population, regarding both its growth and decline (immigration and emigration, for instance), it results in being reasonably precise in the prediction of the United States population between 1790 and 1860. The populations which grow to a rate described by (7) is still used to model the growth of small populations in a short interval (growth of bacteria in a Petri plate, for example).

Figure 3: Exponential growth as proposed by Malthus and presented by Zill (2003, p. 23-24)

Zill (2003) presents the Malthusian model idea the way it became known in the scientific community; yet, without mentioning the group of evolutions this idea has until it configures the form it is today known.

The chapter Differential Equations of First Order presents some methods to solve the ordinary differential equations and, in the following chapter, Modeling with Differential Equations of

First Order, Zill proposes some applications with this type of equations. That indicates the division of the content in delimited parts, in such a way to program the teaching of content. In this same chapter, at the Linear Equations section, the author presents models to the type of growth and decay, but does not approach specifically the problematic of the population growth as it was treated by Malthus. In other words, social and mathematical practices associated to the population growth do not integrate the book author's repertoire.

In the section Non-Linear Equations, Zill addresses Verhulst's model, arguing that the growth rate of a population depends on the number of present individuals, and presents a logistic model as shown in Figure 4.

Population Dynamics If $P(t)$ denotes the size of a population at an instant t, the exponential growth model begins with the hypothesis that $\frac{dP}{dt} = kP$ to some $k > 0$. In such model, it is assumed that the relative growth rate or specific, defined by $\frac{dP/dt}{P}$ ....(8)

is a constant k. Real cases of exponential growth for a long period are difficult to be found, because these limited resources of the environment will, at any instant, restrict the growth of the population. Thus, it is expected that (8) can decrease to the measure P increases.

The hypothesis that the growth rate (or decrease) of a population depends solely on the number of present individuals instead of on a time-dependable mechanism can be written as $\frac{dP/dt}{P} = f(P)$ or $\frac{dP}{dt} = Pf(P)$. (9)

The differential equation in (9), widely used in models of animal populations, is called Density dependence hypotheses.

Logistic Equation Suppose a determined environment is able to sustain no more than a fixed K number of individuals in its population The quantity K is called support capacity of the environment. Thus, for the function f in (7.9), we have $f(K) = 0$ and simply establish $f(0) = r$. [...]~The simplest possible hypothesis is that $f(P)$ is linear - that is, $f(P) = c_1P + c_2$. If we use conditions $f(0) = r$ and $f(K) = 0$, it results in $c_2 = r$ and $c_1 = -\frac{r}{K}$ and, therefore, f assumes the form $f(P) = r - \frac{r}{K}P$. Equation (7.9) then becomes

$$\frac{dP}{dt} = P\left(r - \frac{r}{K}P\right). \qquad (10)$$

Renaming the constants, the non-linear equation (10) is the same as

$$\frac{dP}{dt} = P(a - bP). \qquad (11)$$

Around 1840, Belgian mathematician and biologist P. F. Verhulst interested in mathematical models to predict the human population of several countries. One of the equations studied by him was equation (7.11), where $a > 0$ and $b > 0$. Equation (11) was known as logistic equation. Its solution is called logistic function. The graph to such function is called logistic curve.

Figure 4: Population growth proposed by Verhulst and presented by Zill (2003)

We can observe that the author makes some investment to mention Verhulst's ideas and one observes that the contents in this book are also presented without referring to the historical context and to the problems which originated them. Problems related to the growth or decrease of populations appear in Zill (2003) being that in one of these one must find a logistic model for the United States population by means of data in a table.

Thus, in a general way, the analyzed textbook authors begin the discussion on population growth models considering Malthus's exponential growth. Afterward, they call attention to the fact that Malthus's model is reasonably precise for many populations during a certain period of time, but at some moment, the imitations (space, food and other resources) will eventually reduce the growth rate, inhibiting the exponential growth. Because of that, they present the logistic model developed by Verhulst in the sequence.

CONCLUSION

From the historic viewpoint the books refer to the authors of the population growth models presented (those of Malthus and of Verhulst). Yet the books do not mention the social practices related to the development of such models. Besides, they address the idea of Malthus's model that populations grow at a rate proportional to the population, and they make reference to this author's work of 1798. Concerning the logistic model, they only use Verhulst's work of 1838 in order to present their considerations.

From the epistemological point of view, the authors present the models already structured, with a notion of mathematical language as it is conventional today. In this sense, the mathematical practices aim to indicate the applications that ordinary differential equations can generate regarding the growth (or decrease) of populations in different species.

Thereby, the books seem to summarize the approach to the population dynamics in the school environment as an application to the ordinary differential equations, paying little attention to the social and mathematical practices associated to such dynamics.

Following a mathematical model structured by population dynamics, methods for solving such ordinary differential equations are discussed in order to obtain the solution to the equation. What appears to be configured is that the analysis of the population variation is overshadowed by the problematic of the solution to ordinary differential equations. That is, the focus of mathematical practice is more on the ODE solution method than on its formulation and its relation the growth (or decrease) phenomena of a population.

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MATHEMATICS TEXTBOOKS, IN PORTUGAL: THE UNIQUE TEXTBOOK

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Textbooks are some of the most relevant elements for the study of a school disciplines' history. Textbooks are sensitive to national contexts and can be seen as probes of the state and structure of mathematical education, its goals and its organization. In Portugal, by 1947, an educational reform established that a disciplines' textbook for each grade of a cycle would be the same for all the Liceus (secondary schools). This paper will discuss the tenders for the approval of the unique mathematics textbooks, in the period 1947 and 1974. We aim at deepening our understanding of the system created in order to adopt a unique book and of its pros and cons. The main historical sources were composed from the Education Ministry's General-Secretariat Historical Archives, the Lisbon´s Newspaper Library and the National Library.[1]

Keywords: history of mathematics education, historical analysis, secondary school, Portugal

INTRODUCTION

The time span in this study begins in 1947, soon after the end of Second World War. In these times, a new political, social and economic order was established in Europe, which sets education as a priority. Political leadership was authoritarian and Portugal did not accompany the economic pace of the most developed nations. However, we observe some development and a drive towards strengthening the industry that needed skilled

[1] This work is financed by national funds through FCT - Foundation for Science and Technology under Project Promoting Success in Mathematics -PTDC/CPE-CED/121774/2010 contract.

manpower to succeed. By the reform carried out, in 1947, by the Minister of National Education Pires de Lima, the educational system consisted of a mandatory primary cycle (6-9 years old), followed by parallel branches for secondary education: the Liceus and the Technical Schools. The Liceus course encompassed three cycles: 1st (10-11 years old), 2nd (12-14 years old) and 3rd (15-16 years old), this course, especially the last cycle, was oriented to studies at the universities. The Technical Schools aimed the preparation of workers at several skill levels and of students to pursue studies at the polytechnic institutes. Regarding textbooks, the 1947 Pires de Lima reform established that a disciplines' textbook for each grade of a cycle would be the same for all the Liceus. Also, this textbook was the only support to teaching authorized, apart from logarithm tables. The emergence of a unique textbook was not peaceful; some teachers published their arguments against this system in teachers' bulletins, as well as, in the press (Almeida, 2013).

In the following, for matters of fluency in writing, we´ll use unique book instead of unique textbook. This text presents part of a study regarding the approval of unique books, with particular focus on mathematics books. Here we will look at the process of approval and the organized tenders' for approving the 2nd cycle's unique books.

THE MATHEMATICS UNIQUE BOOKS

Before the scholastic legislation that forced the adoption of a unique book, the teachers' council of a school could choose the book to be used, in each discipline, for the following five school years. The books were chosen amongst the ones previously approved by the Ministry of Education.

In 1947, the choosing of books was taken away from schools. The process for selecting a unique book begins with the opening of a tender issued by the Ministry of Education, to which the authors submit their books. After the tendering, all the books were reviewed by a pair of mathematics teachers, called the jury. Finally, from the analysed set of books, the jury should select the one to be the unique book, which would be used by teachers and students the following five years. During this five year time, the authors of a unique book could propose, in new editions, amendments they deem important. The words "*Officially Approved as unique book*", an official stamp and a number, guaranteed a book's authenticity (Figure 1).

Figure 1. The reference to being a unique book and both the official stamp and number of a unique book (Algebra Book, 2nd cycle)

Some of the mathematics books approved referred to the topic studied – Arithmetic, Algebra, Geometry and Trigonometry. So, some covers of approved books did not refer the discipline, but only named the topic.

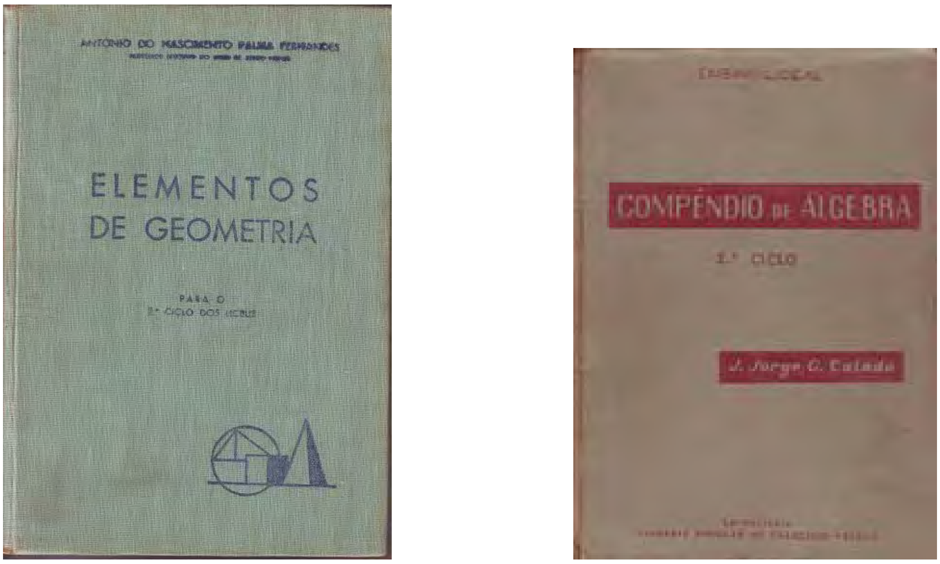

Figure 2. The covers of the first unique Geometry's book and Algebra's book (2nd cycle).

By observing Table 1, that shows the organized tenders for Algebra´s book (2nd cycle), we can verify that the first unique book was approved in 1954. In Table 2, are displayed the organized tenders for Geometry's book (2nd cycle), we can see that that the first unique book was approved in 1952. It also draws our attention that in some of the tenders there was not an approved unique book. This comes from the difficulties that arouse when the new approval system was put in practice.

Table 1: Organized tenders for Algebra´s book (2nd cycle)

Year	Tender	Number of books	Unique book
1948	No	----	----
1949	No	----	----
1950	Yes	2	No
1951	Yes	6	No
1954	Yes	7	Yes
1959	No	--	--
1960	Yes	4	Yes
1965	Yes	1	Yes

The decision to establish a unique book had warning voices. A mathematics teacher, Laureano Barros (1950) expressed his opinion using a national newspaper. In his point of view, this system could relegate to oblivion some good books for the work of students and teachers. In the same article, Barros (1950) points out the huge responsibility of both authors and jury, the former in writing the books and the latter in evaluating and approving them, since the book occupied a central place in the teaching process and they were endorsed for five years.

Table 2: Organized tenders for Geometry´s book (2nd cycle)

Year	Tender	Number of books	Unique book
1948	No	----	----
1949	No	----	----
1950	Yes	2	No
1952	Yes	4	Yes
1957	Yes	2	Yes
1962	Yes	1	Yes
1967	Yes	1	Yes

Next, we address some of the difficulties in this system. A book approval depended on its consonance to the syllabus, scientific rigor and suitability to support teaching. Gathering a jury with enough competence for the job was not easy, as the authors were not allowed to take part in one. In each tender, authors had a given time to submit their book. In 1948, it was introduced a new syllabus so the authors needed time to write a good book, this forced the delay of the opening tenders. Such as authors, jury members were given a rate of time to analyse and write a report about the book they were required to review. This amount of time was often considered not enough for the task and led to a poor quality of the performed evaluation. The members of the jury, alone, analysed the books and made a report on each one of them. Sometimes, the positive aspects pointed by one of the jury members were reason for disapproval by the other jury element. Some of the articles published in teachers' bulletins, concerning the unique book, use these problems to argue against this system. The articles agree that the low price of the books, which was officially enforced, was one of the few advantages of this system (Almeida, 2007).

CONCLUSIONS

The analysis shows that the unique books' system was established in 1947, but the approval of the first unique book for the 2nd cycle occurred only in 1952, further the endorsement of a unique book five years, but a delay on the opening of a tender could maintain the same book for some more years. Also, there were years that had none approved unique book.

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THE CREATION OF MATHEMATICS IN SCHOOL TEXTBOOKS: PALESTINE AND ENGLAND AS EXAMPLE

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The language of mathematics textbooks, including symbols and diagrams, constructs particular views of the nature of mathematics and expectations about students' participation in mathematical activity. In a collaborative project between the Institute of Education and Birzeit University, we developed an analytic framework for examining the nature of mathematics and mathematical activity in textbooks. This framework, based on those developed by Tang, Morgan, & Sfard (2012) for the verbal mode and by Alshwaikh (2011) for the visual, enabled us to take account of the multimodal nature of mathematical texts. We applied the framework to analyse a sample of topics from the textbooks used in Palestinian schools and to a smaller sample of topics from textbooks commonly used in England. The research showed that, for younger students in both countries, mathematics is construed as involving practical activities. For students in Palestine, however, abstract mathematical reasoning is also prioritised from a much earlier age. This raises questions about how textbooks in the two countries may support students to move towards abstract mathematical reasoning.

Keywords: semiotics, discourse analysis, nature of mathematics, learner agency, Palestine, England

INTRODUCTION

In this paper we present and illustrate an approach to analysis of textbooks that addresses the ways that mathematics and learners are construed. Our study was motivated originally by the many complaints regarding Palestinian mathematics textbooks. In particular, studies have identified that they are densely packed with abstract concepts which makes mathematics a hard topic to learn and teach (Al-Ramahi, 2006; Alshwaikh, 2005). Some have suggested that the nature of textbooks is linked to the very modest achievement of Palestinian students in international and local studies (Rewadi, 2005).

In a project funded by The British Academy, *Analysing the Palestinian school mathematics textbooks* (Alshwaikh & Morgan, 2013), we examined mathematics textbooks used in grades 4 to 10 (9 to 15 years old) in Palestine using a multimodal analysis that considers language and diagrams in mathematical meaning making. A comparison was made with textbooks used in the UK. We take the theoretical view that language, diagrams and other systems of communication function to construe the nature of our experience of the world and of the identities and relationships of participants. This view draws on multimodal social semiotics (Halliday, 1985; Kress & Van Leeuwen, 2006; Morgan, 2006). We also draw on the work of Sfard (2008) to consider the characteristics of mathematical discourse.

In order to analyse the textbooks we posed two research questions: What image of mathematics is presented in Palestinian and English textbooks? How is the learner of mathematics presented in these two contexts? We will give a short description of the contexts of the study, followed by the methodology and the analytic framework used in this study. Then we present an illustrative example of our analysis and, finally, introduce some concluding remarks.

THE CONTEXTS: PALESTINE AND ENGLAND

In both Palestine and England, the curriculum is mandated by the state. However, the degree of centralisation and control varies. In Palestine, the Ministry of Education is responsible for producing textbooks and distributes them to governmental schools. The other two types of schools (UNRWA and private) are also obliged to use the same textbooks from grade 1 (age 6 years) to grade 12 (age 18).[1] In contrast, schools in England are free to choose textbooks from a wide range produced by commercial publishers. In this study we chose to analyse textbooks from a series very widely used in schools in London at Key Stage 3 (Years 7-9, age 11-14 years) and a textbook for Key Stage 4 (Years 10-11, age 14-16) published by one of the commercial organisations responsible for setting the national examinations at age 16.

ANALYTIC FRAMEWORK AND METHOD

The first stage of the project was to construct a framework for analysis that would enable us to consider the multimodal nature of mathematical texts. This framework was constructed drawing on two main sources: the analytic framework developed by Tang, Morgan and Sfard (2012) for application to examination papers and that developed by Alshwaikh (2011) for application to geometric diagrams. The major components of the framework addressed in this paper are derived from Halliday’s ideational and interpersonal meta-functions of language. An initial version of the framework was produced and iteratively refined through application to sample chapters from both Palestinian and English textbooks. The framework was applied to analyse texts on different topics and at different grade levels in order to check the general applicability of the framework across the school mathematics curriculum. An extract of the developed version of the analytical framework is in Table 1: each component is elaborated by questions that guide our analysis and indicators that allow us to identify relevant characteristics of the verbal and visual text. The structure of this framework is based on that proposed by Tang et al. (2012).

While some textual characteristics were counted during the analytic process, we have only used relative numbers of occurrence of specific indicators to support the construction of qualitative descriptions of the texts. Independent analyses of sample chapters were conducted by each of the two authors as part of the process of refining the framework. These give us confidence in the reliability of the analytic method.

AN EXAMPLE ANALYSIS

To illustrate the application of the framework in Table 1we will use a geometry unit from the Palestinian textbook for Grade 7 (an extract from this unit, translated from Arabic, is shown in Figure 1) and a section from an English textbook for the same age group (Year 8) (an

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[1] There are however, some private schools that follow international curricula, such as International Baccalaureate –IB, IGCSE and SAT, with different textbooks.

extract is shown in Figure 2). Both deal with the topic of congruence as we wish to compare the treatment of the same topic in both contexts. Given the limited space available in this paper, we will give details of the analysis using only that part of our framework shown in Table 1.

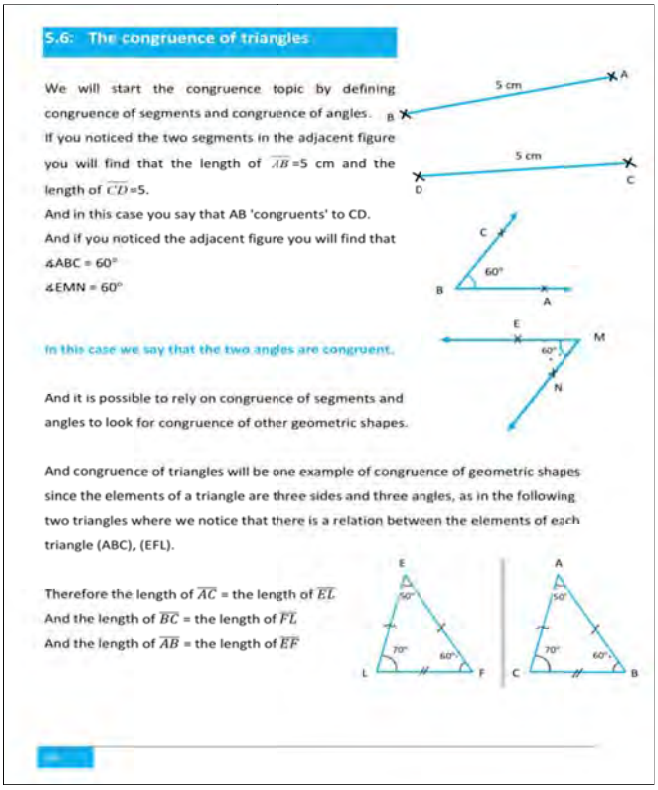
Table 1: An extract of the analytic framework, showing its structure

	property of the discourse	specific questions guiding analysis	indicators in verbal text	indicators in visual text
How is the nature of mathematics and mathematical activity construed?	**specialisation**	To what extent is specialised mathematical language used?	- vocabulary used in accordance with mathematical definitions - ‘conventional’ expressions - mathematical symbols	- ‘conventional’ mathematical diagrams, charts, tables, graphs and labelling systems - Conceptual diagrams (showing properties and relationships, *not actions*)
	further properties include: *objectification, alienation, logical structure, status of mathematical knowledge*			
How are the learners and their relationship to mathematics construed?	**agency**	What kind of activity is the learner expected to engage in? What possibilities are there for learners to make decisions?	- ‘thinker’ or ‘scribbler’ processes ascribed to the learner - imperative instructions or open questions?	- presence or absence of labelling (suggesting form of engagement with diagrams)
	further properties include: *authority; formality*			

The nature of mathematics and mathematical activity

Our framework identifies a number of properties that contribute to the image of mathematics and mathematical activity. Here we focus only on the issues arising from analysis of the **specialisation** of the two texts. The Palestinian text has a high density of specialised mathematical words (*congruence*, *segments*). Mathematical symbols ($\overline{AB}$, $\measuredangle$ABC) are used both within the verbal parts of the text and in independent symbolic statements. The diagrams consist of representations of named mathematical objects such as triangles and segments. These objects are identified conventionally by letters labelling vertices and their properties are communicated by conventional marks on the sides and angles. Most of the diagrams are conceptual (Alshwaikh, 2011), displaying the properties of objects and relationships between them rather than representing a process; the dominance of conceptual visual elements is a common characteristic of specialised scientific and mathematical text (Kress & Van Leeuwen, 2006).

In the English text, the density of specialised vocabulary is lower. There is more “everyday” language, referring, for example, to *shapes* rather than naming specific mathematical objects. There is also reference to the use of practical equipment such as *tracing paper*, *piece of card*, etc., making it clear that the text is about concrete school activity rather than abstract mathematical reasoning. There is almost no use of mathematical symbolism. The diagrams include a wide variety of irregular objects, contrasting strongly with the overwhelming focus on triangles in the Palestinian text. Although, as in the Palestinian text, most of the diagrams are conceptual, the diagrams in the English text lack conventional features; in most cases the vertices of the polygons are not labelled. This lack of labelling is related to the lack of mathematical symbolism elsewhere in the text.



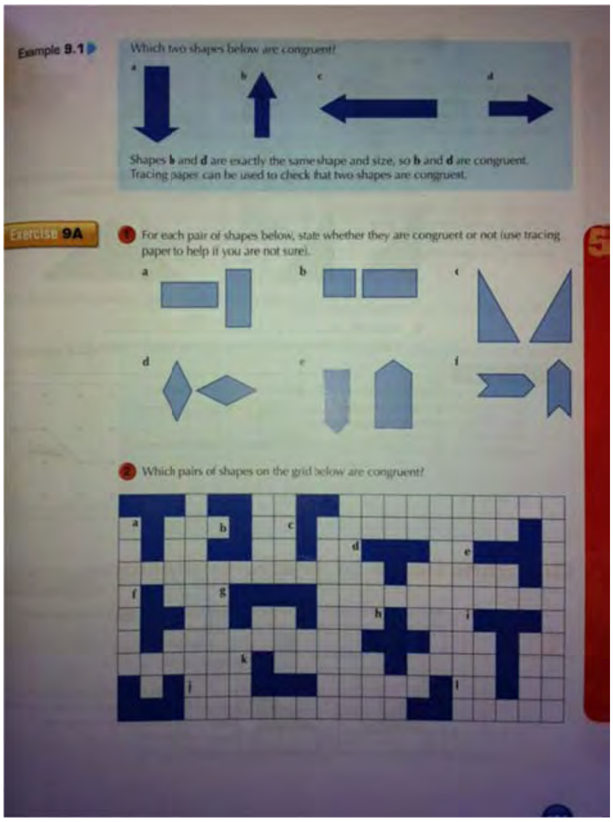


Figure 1: extract of (translated) Palestinian text

Figure 2: extract of English text

In summary the Palestinian text maintains the mainstream conception about mathematics as timeless, impersonal and dealing with a specialised domain that is separate from everyday experience (Davis & Hersh, 1981; Morgan, 1996). While this specialised image of mathematics is also present in the English text, this text is more mixed, containing features that link to more everyday and concrete objects and activities and lacking use of conventional notations.

The nature and role of the learner

In order to address the question of learner *agency*, we distinguish between engagement in material processes (e.g. *measure*, *calculate*) that construe a role as a 'scribbler' and in mental processes (e.g. *consider*, *prove*), construing a 'thinker'. According to Rotman (1988), doing mathematics involves undertaking both of these roles: performing operations and reflecting on them. The Palestinian text engages learners (using an inclusive *we* or *you*) in mental processes (e.g. *define*, *notice*) as well as in material processes (e.g. *find*) that construe learners as 'scribblers'. In the section shown in Figure 1, these roles are combined (*if you noticed the adjacent figure you will find*). There is thus expectation that learners will be 'thinkers' (e.g. *show*, *prove*, *notice*, *consider*), engaged in observation, reflection and reasoning as well as operating on mathematical objects. In the visual component of the text, figures are labelled with specific measurements or marks indicating equality. Learners are thus construed as observing and reasoning about the properties of the shapes. The learners' activity is elicited not only through imperatives but also through use of questions allowing choice in the mode of response, (e.g. *If you try to measure AB and RP, what do you notice?).*

The English text construes learners as active mainly in 'scribbler' activity, including manipulation of concrete objects. Moreover, where there are mental processes, such as *notice*, these tend to refer to observation of facts rather than to engagement in reasoning. In the visual component of the text, the lack of labelling with measures or marks indicating properties suggests that the task of determining congruency is one of visualisation rather than analysis. In some cases, shapes are positioned on grids, suggesting that learners might be expected to check equality of lengths by counting. The use of questions such as: *Which pairs of shapes are congruent?* allows a choice of method. However, the absence of any formal definition, method or example of reasoning and the marking of use of concrete manipulation as less desirable (*use tracing paper to help if you are not sure*) mean that learners seem to be expected to rely on everyday notions of shape and size and informal methods of visualisation and measurement rather than focussing on specific properties.

In summary, the Palestinian learners appear to be expected to engage in both the scribbler and thinker activities that make up the work of a mathematician. They are construed as attending to specific mathematical properties of shapes and reasoning about these. In contrast, the English learners are construed as using their own informal methods to compare shapes and the thinker role is largely absent or implicit.

CONCLUDING REMARKS

The two texts deal with the same topic of congruency in different ways: While the Palestinian text uses a specialized discourse, emphasising formally defined objects and reasoning about properties, the English text uses a mixed discourse, emphasising processes of practical manipulation and visualisation. Both sets of learners are expected to engage in material activity; the Palestinian text, however, also construes the learner as a 'thinker'. The combination of scribbler and thinker activity suggests that the Palestinian text as a whole seeks to apprentice learners into specialised mathematical discourse (cf. Dowling, 1998), while the English text does not offer this opportunity for apprenticing.

However, while Palestinian textbooks appear to construe learners as engaged in reasoning, the actual teaching and learning of mathematics in Palestinian schools reflects a different reality. Observation of a number of geometry lessons in Palestine suggests that, in practice, teaching focuses mainly on the sections of the textbook identified as 'Exercises and Problems', which construe a predominantly scribbler role. Solving problems usually come after an example, and the student is expected to mimic what has been done in the example, using similar wording and a similar problem. Students are thus expected to act as scribblers: to follow what the authors have done or simply to repeat the activity already illustrated. Our analytical framework allows us to identify what opportunities are offered by the texts but the extent to which these are manifested in classrooms depends on how teachers make use of the texts provided. Knowledge of the context is thus essential to interpretation of the text.

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TEXTBOOK AND TECHNOLOGY: AN ANALYSIS OF MULTIMEDIA LEARNING IN BRAZIL

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This is part of a broader research project aimed at obtaining a deeper understanding on how geometry and technology intertwine in the textbooks and what kind (nature and goals) of proposals are made. Amongst ten sets of textbooks that have been evaluated and approved by the Brazilian Government, to be distributed in schools (freely), only three of them have "digital learning objects" (DLO). Mayer (2009) presents a cognitive theory of multimedia learning considering how people learn from words and pictures. The technology evolution prompted new efforts to understand the potential of multimedia as a means of promoting human understanding – a potential that Mayer (2009) called the promise of multimedia learning. The need for intertwine research on textbooks and the DLO included on them is also justified by the fact that such DLO expand Valverde et al. (2002) comments that textbooks are the print resources most consistently used by teachers and students in the course of their joint work. The project considers a qualitative research with an interpretative approach and here we will discuss some aspects of the DLO. Preliminary results show that the required level of interaction with pupils is very low and it can be considered only has a detractor for students, and a domestication of media (Gadanidis & Borba, 2008). Deeper analysis is needed in order to understand what is really the role of DLO in textbooks and possibly, to steer some possibilities for improving its conceptualization in and for facilitate the multimedia learning.

Keywords: technology, multimedia learning, digital learning objects (DLO)

INTRODUCTION

There is a large investment in Brazil by the federal government in the evaluation and free distribution of textbooks to public schools, and this material, according to Valverde et al. (2002), is the main resource used by teachers in preparing their classes. In 2014, elementary schools (11-14 years old) received some collections of textbooks along with "digital learning objects" (DLO) previously approved by a governmental evaluation. In 2015, high schools (15-17 years old) will also have these resources integrated into textbooks. The multimedia contents that go along with the books are…

> curricular themes dealt with through a set of digital educational objects intended to teaching and learning processes. These objects must be presented in the audiovisual, educational electronic game, simulator, and animated infographic categories; or associate all or some of these categories in the hypermedia style, where every object must be individually identified, stored in media, and be open to be made available in virtual environment (Brasil, 2011, p.2, our translation).

It is important to highlight that it is foreseen that the "digital educational objects included in each multimedia content are complementary and must be articulated with the content of the printed volumes, in both the students' books as in the teachers' manuals" (BRASIL, 2011, p.2). Furthermore, it is recommended that the multimedia content promote the diversity of interactive objects, with different usage possibilities by teachers and students.

Each DVD that goes along with the Type 2 collection volume is considered an integral part of the collection, and contains between 10 to 20 multimedia contents, each of which has a one to five DLO of the already mentioned categories: audiovisual, educational electronic game, simulator or animated infographic (BRASIL, 2011).

Since it is a recent initiative, the study of these new media is pertinent. In this paper, we highlight the partial results of a larger research program that analyses the DLO, and the Mathematics textbooks, in the extent of Geometry[1]. The purpose is to discuss a set of DLO from some Brazilian textbook collections, focusing on interactivity and multimedia learning. For such debate, here are the theoretical aspects that support the analysis.

INTERACTIVITY

Machado et al. (2012) state that interactivity happens through a process that simulates the sensation of a user interacting with another person, with a video or any other resource. Regarding the case of video, used as an example, the authors note that "the receptor communicates with the narrative, understands what it wants to transmit, and comprehends that the outcome of this story depends on the choices he makes" (p.4). That is, the user acts vividly in front of a media. "Due to a technology that permits the creation of a communicational system through computers, the production of an interactive environment where the user can explore and interact with the content is possible" (p.4).

As the Cambridge dictionary has it, to interact is to communicate with or react to another person or thing. According to Cannito (2009), we can classify interactivity in three levels of controlling carried out by the user: 1) Reactive: the spectator reacts by making choices pre-defined by the producer, where there is low interaction; 2) Coactive: a certain control over choices and sequential steps is possible; 3) Proactive: when there is a high level of interactivity, so that the user can change both the structure, and the media content, also becoming a producer when feeding the environment with his creations. Before this classification, Machado et al. (2012) highlight that even with the different interactive degrees that each level has, they are still a kind of interactivity, and in media of the Proactive type there is the highest feeling of participation, but the Reactive type also creates this same reaction, although with a different force.

In Mathematical Education, the different digital resources developed to be used in classrooms seek to create an opportunity for the student/user to interact with the computer (or other portable resource) to explore mathematical questions from simulations, manipulations etc. However, this interaction does not always happen in a way that enables a high level of student

[1] "A Geometria nos livros didáticos e a integração das tecnologias digitais", financed by FAPESP.

participation in the development of the given activity (Proactive level). This is the main challenge for the developers of these materials.

This debate leads to the concept discussed by Gadanidis and Borba (2008) about “media domestication”. Some media or activities are created to be understood and used in a manner analogous to what the student would do using pencil and paper, without a differential that would be specific to the media’s characteristics. In this case, it can be said that it is just another “attire”, often more modern etc., for an already existing form. An example of such a case is the use of a digital object to show a function chart: the charts are probably more colorful, and more attractive than those the teacher would draw on the board, but there is no qualitative difference in the purpose of the use of each of the resources (blackboard or projector), which is the student’s chart visualization.

This kind of usage has potential – in, for instance, introducing the content in a more attractive, beautiful way – but it does not make good use of the contents the media could provide, such as the exploration of a function’s coefficients from an analysis of its charts with such software, for instance. In this way, the peculiarities of the computer media would be explored, and it is understood that its use is not yet domesticated.

COGNITIVE THEORY OF MULTIMEDIA LEARNING

The Cognitive Theory of Multimedia Learning (CTML) is a theory based on three suppositions suggested by cognitive research on how people learn through words and images: ***double channel***, which holds that human beings have separated channels (visual and verbal) to process images and audible representations; ***limited capability of supposition***, which holds that only part of the information can be processed in a channel; and ***active process***, which holds that multimedia learning happens when the student engages himself in the cognitive processes to select, organize, represent, and integrate the information with the previous knowledge (MAYER, 2009).

Mayer (2009) argues that multimedia learning happens when the student selects words and images relevant to the processing of the verbal and visual memories, respectively. Finally, when the student integrates the verbal and the visual representations. In this respect, the conditions in which the multimedia messages are presented in a material and how they are manipulated by the teacher are important (BARROS, 2013), stimulating the creation of environments that facilitate experiences that lead students towards learning. Aiming to examine powerful resources of multimedia design to promote expressive learning, Mayer (2009) presented 12 elements that may be used to base the creation of environments with multimedia messages, seeking multimedia learning. These elements were elaborated from experimental studies, and are validated by the theory in which people learn from words and images.

DLO AND PERSPECTIVES TO THE CLASSROOM

Analysing DLO through Mayer’s perspective (2009) does not mean verifying how many of the principles described by the author are “contemplated” by the media. This exercise was done specifically to elaborate a panel, and observe, for instance, which element is more present in the DLO.

However, it is believed that a relevant analysis of this material brings qualitative aspects to the debate/reflection, aiming to contribute both to the teachers' practice and to improve new media that can be created from this initiative supported by the government.

The "statistics charts" DLO is an example of a "simulator" in which, from a problem, as the participation in a company's profits, three charts are presented. Following that, the user must choose the chart that best represents the given problem. If he clicks the wrong chart, a message will tell him that his choice is wrong, and a new chart will be selected, until the correct answer is chosen.

Figure 1: Statistics charts (wrong and right answer)

In this DLO, the principles of "coherence" (people learn better when information that is not relevant to the topic is omitted), and "signalizing" (people learn better when the characteristics that are of importance to organizing the material are included), for example, can be highlighted. It includes no voice, disregarding principles such as "personalization" (people learn better from a multimedia presentation when words are displayed in conversational style, instead of a formal style), and "voice" (people learn better when the narrative is in human voice, instead of a machine's voice), and in an isolated form, it is believed that the DLO can be considered very simplistic by the student. However, if it were to be used in the classroom, where the teacher could raise questions about the theme, debate the suggested problems, and also correct possible wrong solutions the students might suggest when answering the question, more of Mayer's (2009) principles would be incorporated, hence encouraging students' learning.

There are others DLO that can be explored in classroom, but their focus is on the usage by the student, studying at home. The DLO "Solving second degree equations" is an example of this. It is also classified as a simulator. There is a second degree equation in it in which the student can add values to its coefficients.

Figure 2: Solving second degree equations (option of equation and coefficients)

From the determined equation, its solution is displayed on the screen, while the student clicks along.

Figure 3: Solving second degree equations (steps of the solution)

Besides incorporating few of Mayer's principles (2009), this type of DLO is not very different from exercises found on books or available on the internet. The only difference is the student's ability to choose the coefficients.

In general terms, it is noticed in the many of the DLO that accompany the textbooks there are few possibilities of interactivity. It is hence possible to say that they are limited to the "Reactive" level, in which the student can only choose from a certain limited set of options, many of which are limited to the act of reading concepts and formulas. There is no active participation from the student in carrying out a mathematical investigation. The differentiated potential of the digital media is explored to some extent, which renders the DLO a non-domesticated media. That is, as Gadanidis and Borba (2008) emphasize, this use of technology is what can be called "media domestication", not exploring the different ways technology can contribute to print media.

FINAL COMMENTS

Other DLO could be cited, showing other content examples. In general terms, it is important to emphasize – and it is common to most DLO (regardless of the mathematical approached content) – that the analysed set has few resources that effectively enable interactivity (Coactive and Proactive levels), leading to student's investigation, to coming up with hypotheses, to exploring suppositions, in a way that differs from pencil and paper, not being a "domesticated" resource (Gadanidis & Borba, 2008). It is also possible to notice that it is shown in the principles of Mayer (2009), which are barely contemplated.

As is known, some of the resources certainly can be used by teachers with this perspective, integrating the media in their classes so as to lead the student to more expansive contemplation of the problem. However, if a DLO enables little interactivity, it will involve greater effort on the part of the teacher to use it in a non-domesticated way, often discouraging the integration technology into teaching practice.

Public policies to stimulate technology integration into classrooms are being developed by the Brazilian government, with one possibility being to incorporate DLO to the textbooks collections. This is an initial study of this kind of DLO, and new results and researches are necessary so that requirements such as greater interactivity and less media domestication can be considered in the elaboration of new digital resources, so that they can effectively reach the classroom.

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CHOOSING TEXTBOOKS WITHOUT LOOKING AT THE TEXTBOOKS: THE ROLE OF THE OTHER'S INTERPRETATIONS

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Textbooks are one essential resource for teaching. They reflect the author's interpretations of the official documents jointly with their conceptualizations and visions on the teaching and learning process. Thus, as a mediator between official curricula and teachers' practices, and hence a way of shaping students' visions of mathematics and the learning of it, the choice of textbooks to work with seems of fundamental importance – and it is also linked with teachers' specialized knowledge for teaching. As such, teachers should play an important role in analyzing and discussing all the possibilities in order to make informed choices. In Brazil, textbooks are evaluated by the government and a general guide which teachers use to make their choices of the textbooks is elaborated. Thus, teachers choose the textbook they will use in their classes through the "eyes and visions of others." This paper is part of a broader research project, one of the foci of which is on obtaining a deeper understanding of the different aspects focused on the textbooks and addressed on the guide in order to start an active discussion on ways to make informed choices with "one's own eyes". Preliminary results reveal the existence of some mismatch between what is expressed in the guide and what mathematical correctness and adequacy expected in textbooks evaluated and approved by the government (that is, by the same group who elaborate the guide).

Keywords: national curriculum, textbook evaluation, choosing textbooks, mathematical correctness

INTRODUCTION

Textbooks are one essential resource for teaching (Valverde, Bianchi, Wolfe, Schimdt & Houang, 2002). They reflect the author's interpretations of the official documents jointly with their conceptualizations and visions of the teaching and learning process. Thus, being a mediator between official curricula and teachers' practices (Rezat, 2012; Valverde et al., 2002), and, thus, a way of shaping students' visions of mathematics and its learning, the choice of textbooks to work with seems of fundamental importance. Such choice is also linked with teachers' specialized knowledge for teaching (Carrillo, Climent, Contreras & Muñoz-Catalán, 2013), how practice is developed (Ribeiro & Carrillo, 2011) and the learning opportunities provided (Hiebert & Grouws, 2007). As such, the choice of the textbooks to be used at schools should ideally be informed by and reflect a consideration of the content of the textbook – both in terms of the nature of the proposed tasks and the methodological approaches considered.

In Brazil, textbooks are given freely to all students at public schools. Recognizing the role of textbooks in and for teachers' practices (Valverde et al., 2002), and trying to ensure the quality of such textbooks, the Brazilian Government established a National Program of Textbooks (PNLD)[1] involving six phases, from the conceptualization of textbooks itself until the teachers' chosen textbooks arrive to schools and students. Two of those phases concern the elaboration by a set of "experts" of a Textbook Guide[2], and the evaluation of the proposed collections (from year 6 till year 9 – Ensino Fundamental II – and from 1st till 3rd year of the next school stage – Ensino Médio) of textbooks by teachers, choosing the collection they consider the best to develop their students. Those two stages are intertwined, as the Guide is elaborated in order to inform teachers about the content of textbooks, both in terms of the mathematical content (a quantitative and qualitative description) as well as concerning the methodological approaches proposed. Grounded on such recommendations, teachers choose the textbook they expect[3] to use in their practices. Thus, teachers choose the textbook they will use in their classes through the "eyes and visions of others" which, by the importance they assume in practice and in students' learning, seem, at the least, to sustain a non-reflexive practice that is easily justified by the absence of choice of the effectively used mediators.

In that sense, and as the textbook guide is assumed to be major mediator to support teachers' choices for textbooks, in order to get a deeper and broader understanding on the effective links between the content of such guides and the effective content of textbooks as a mediator for teachers' choices, we address the following research question: what are the Guide's expressed interpretations (potentialities and limitations) concerning the textbook's content and what are its implications for teachers' choices?

THE BRAZILIAN CONTEXT CONCERNING TEXTBOOKS CHOICES

In order to improve (at least theoretically) the quality of textbooks provided to students (freely at public schools), the Brazilian Government established the PNLD, composed of six phases: (1) Launching the official document from the government (on the basis of which textbook companies propose their collections); (2) textbook evaluation (textbooks are evaluated according to the criteria presented in the official document – in such evaluation are involved teachers from schools and professors/researchers from the Universities); (3) Guide publication (from the approved textbook collections, a textbook guide is elaborated, with the evaluators' views on each collection, including both quantitative (number of pages and of exercises) and qualitative (methodological approaches, language) perspectives; (4) Textbook selection by teachers (using the Guide, teachers must present a list of textbooks they would prefer to use in their practice. The rank of the choices is indicated by each school, thus implying a collective discussion); (5) Purchase of the textbooks by the governmental responsible entity[4] (after negotiating with the textbook companies, now it is always possible to distribute to schools the textbook of their choice); (6) Textbook distribution to schools (at this stage a huge logistic problem must be solved, taking into consideration the size of the country).

[1] From its original name in Portuguese: *Programa Nacional do Livro Didático (PNLD).*

[2] From now on, we just call it Guide.

[3] Teachers need to evaluate and grade the textbook collections they consider to be the more adequate (the best), indicating their order of preferences, for its use in practice. Thus, in some cases, due to negotiations between the government and the textbook companies, teachers have to use textbooks they have not selected (or that are even in the three main choices).

[4] Fundo Nacional do Desenvolvimento da Educação.

SOME THEORETICAL NOTES

Teachers' practice is influenced by a large set of factors, with teachers' cognitions being at the core of these (Ribeiro, 2013). On the other hand, the resources teachers have and how they use them influence such practice, and thus students learning opportunities (Hiebert & Grouws, 2007). As textbooks are the main resource teachers' use in their practice (e.g., Valverde et al., 2002), and that fact that, in Brazil, teachers actually have a say over the textbook they will work with in practice (at least in theory), is perceived as a very positive aspect in the path to improve practice and students' results. Thus, as teachers – and in particular their knowledge – are an essential aspect in promoting students' results (e.g., Nye, Konstantopoulos and Hedges, 2004), and the fact that practice is grounded mainly in the use of textbooks, the content of such textbooks (e.g., ideas expressed, methodological approaches, mathematical correctness, adequacy of the mathematical language used, the kind and nature of the connections promoted and allowed and representations used) play a crucial role in supporting students' learning and improving their results (also considering international tests, such as PISA or TEDS-M).

In that sense, the importance of textbooks' quality is unquestionable (something already known – e.g., Rezat, 2012; Valverde et al., 2002), but in the scope of our research, also unquestionable should be the quality of the Guide elaborated by a group of "specialists"[5] on the basis of which teachers will ground their choice of the textbook. Such quality, and how teachers perceive it, is linked with teachers' specialized knowledge (Carrillo et al., 2013), as the ways teachers use such textbooks are intertwined with the visions and knowledge they have both of the mathematical contents and of the didactical aspects to approach and link such contents. Thus, the content of the Guide is of fundamental importance for the teachers' choice of textbooks.[6] In parallel with Jaworski's (2008) ideas on the need for attention on teachers' trainers (in order to improve training), focusing on the Guide, there is the need for attention not only for its content and expressed ideas, but also (indeed, probably mainly) the authors of such documents, as this will deeply influence teachers' choices in the textbooks they will use and, thus, on the possible views, learning and students' results.

CONTEXT AND METHOD

This paper is part of a broader research project focusing on textbooks and on some aspects of their content, in particular those dealing with geometry topics. Here, due to space constraints, we present some results from a preliminary analysis of only two of the ten approved collections in Brazil (Brasil, 2012). Such analysis concerns different aspects focused on in the textbooks and addressed in the Guide in order to obtain a deeper understanding of how one can contribute to effectively allow teachers to make informed choices with "one's own eyes", and not "with others' eyes". Such Guide and textbook collections are considered our instrumental case study (Stake, 2005), and by its analysis we aim to contribute, ultimately, some recommendations for its improvement. We will not differentiate the analyzed collections, as our aim is not to compare but to obtain a deeper and broader understanding of the aspects under focus. In order to contribute to its improvement, there is the need for a focus

[5] Such a group is comprised of teachers, researchers, and mathematics educators, which are part of the evaluation board of the textbooks.

[6] Such specialized knowledge, in the context of what is defined by the Brazilian Government, will be used only in classrooms when using the textbooks, and thus, after these have been chosen.

on the most problematic identified aspects. Therefore, here we will focus on some of the critical aspects identified in the Guide, linking them with what was mentioned in the Edital (stage 1).

In order to get a broader overview of the textbook guide, its content and implications for teachers' choices, we choose here to focus our analysis, and discussion, on the geometry content and the methodological approaches/ideas mentioned on the guide as considered to be the ones each collection express. We opted for such two focuses, and not focusing only on how the different content topics are exposed in the guide, in order to also allow a deeper and intertwined reflection and discussion on the role of such a guide, complementing the broader overview.

In order to better understand the analysis, we have to mention that the textbook guide, besides having comments concerning the textbooks, also mention the process of the evaluation of textbooks, some general appointments concerning textbooks, some issues concerning what they call "today's mathematics," and some reflections (from the textbook guide authors) in the scope of mathematics education. For the analysis process we focus on what is mentioned in the guide, trying to give sense to its content, and only afterwards do we look at the textbooks in order to try to perceive the correspondence between the two documents. We have to stress also that, although at the moment we have only analyzed part of collections, we intend to analyze all of them, and textbook guides in order to obtain a broader understanding of the content of both of these mediators at their different levels.

ANALYSIS AND DISCUSSION

An analysis of the textbook guide on the two collections under study reveals that it provides a general overview of the content of the collections (in terms of the mathematical content, and its correctness), as well as some comments on the methodological approaches (concerning the teaching and learning process), which seems to sustain the textbooks' authors' visions, perceptions and interpretations of what "should be the ideal" methodological approaches. One aspect that calls our attention concerns the fact that, although all textbooks have been evaluated (on phase 2) on the textbook guide (only elaborated from the approved collections), there are references to some mathematical errors, repetitions and some excessive focus on some aspects of mathematics (number and operations, and algebra). The presence of such mathematical errors and excessive focus on algebra in textbooks moves, even more, the responsibility of classroom practice and students' learning to the teacher and their knowledge, in order to allow for exploring of the topics, with a mathematically correct and adequate vision, and on dealing with the proposed activities in order to allow for the eradicate from student's visions of the algebrization of mathematics, elaborating connections amongst and between different topics and representations. One other aspect that calls our attention concerns the mention to a too exhaustive list of some topics, leaving almost aside, obviously, some others that are supposed to be included (which the guide considers "relevant"), such as basic notions of statistics and probabilities.

Looking specifically to the comments on geometry, the Guide points to the inexistence (continuity) of tasks promoting the development of connections between 2D and 3D shapes, as some mismatches of articulations involving (and between) the notions of geometric transformations (symmetry, rotation and translation) are also identified – which corresponds to some of the already identified difficulties in students and teachers (e.g., Gomes, Ribeiro, Pinto and Martins, 2014; Jones, 2002). Thus, it seems that those textbooks have been elaborated, and evaluated positively, allowing for the perpetuation of some of the aspects that research has already identify as problematic.[7] Such a fact may lead to a questioning of the role of research in and for textbooks' elaboration, but also in and for analyzing and evaluating the textbooks and writing the guide – as it will be used by teachers to choose the textbooks, which reinforce the role of teachers' specialized knowledge. Although in some cases there are references to a good articulation (but not connections or different representations) between some contents of geometry and other topics, in the same vein one can also find the use of the same word (view) for different, non-equivalent, meanings, leading, once more, to the exploration of the contents in an inappropriate way on the textbook – reinforcing, thus, also, the need of a sustained teachers specialized knowledge at the time of exploring such concepts.

Concerning the methodological approaches, a general overview allows one to say that both collections assume that teaching is a delivery process, with it being mentioned in the Guide that "it follows the usual model". But even so, the Guide mentions that there is some space for the use of problem solving, motivating the students to use different strategies, besides comparing and discussing the results with colleagues. Such double approach (delivery process and problem solving approach) seems somehow strange, unless "problem solving" is considered by the Guide authors as a process for verification of the learned contents – which, cannot be the case for much of the work related to problem solving (e.g., Schoenfeld, 1985)

SOME FINAL COMMENTS

A preliminary analysis to the Edital published by the Brazilian Government (Brasil, 2011) brings to the fore the role attributed to and importance of the textbook guide, as it recommends, explicitly, for teachers to analyze the Guide, arguing that its contents are intended to be a contribution to "helping on the choice of the textbook that teachers may consider more adequate to work with their students, and the political-didactical project of their schools" (p. 7). Thus, teachers are incentivized not to read the textbooks directly but are supposed to work with the Guide elaborated by a set of "experts," and in that sense, to make their choices with a "second vision" of and from the textbooks. On the other hand, by analyzing the Guide one can find some inconsistencies on what is said, on what was supposed to be evaluated and also what was supposed, ultimately, to be explored with students in classrooms – the minimal requirements should be to explore the mathematical topics in their correct form (and we would say also in its adequate form, considering the school stage and the prior and future possible connections).

Our preliminary results reveal that further research is needed in order to identify "all" the problematic aspects concerning the triad of textbooks, Guides and teachers knowledge, allowing contributing for the effective improvement of these aspects, aimed at contributing for teachers to make better informed choices of the textbooks they will work with.

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MATHEMATICS TEXTBOOK USE IN ENGLAND: MINING OFSTED REPORTS FOR VIEWS ON TEXTBOOKS

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According to TIMSS data, the use of textbook in mathematics classrooms in England is relatively low in comparison to other countries. Although the reasons for this might be varied, the pronouncements of Ofsted, the official body for inspecting schools in England, might have an influence. This paper reports on a text analysis of almost 10,000 publicly-available Ofsted secondary school inspection reports and mathematics- specific commentaries from the year 2000 until now. The analysis focused on what Ofsted said over this period about textbook use in general and about the use of mathematics textbooks in particular. The analysis was conducted by first 'scraping' the reports from the Ofsted website and then utilising basic text mining and analysis methods to extract information on these documents. While the analyses showed that the occurrence of comments by Ofsted on textbooks appeared to be relatively minor, interpreting these findings from text mining alone was not straightforward. A further qualitative analysis of a sample of Ofsted publications found mention of 'over-reliance' on textbooks. Such allusion to 'over-reliance' on textbooks might have negative connotations and may have contributed to the relatively low use of textbook in mathematics classrooms in England.

Keywords: TIMSS, textbook use, Ofsted, England, text mining, text analysis

INTRODUCTION

Textbook use in mathematics classrooms in England is, according to TIMSS data, "lower than that in the highest-attaining countries [in TIMSS]" (see Askew et al, 2010, p.34). Askew et al, go on to say that given the paucity of relevant research, "the reasons for these differences in textbook use are not clear", although the lower proportion of teachers in England reporting use of textbooks may reflect "a greater use of internet resources", "initiatives by the National Strategies promoting the use of alternative resources alongside published textbooks", and "a view amongst the educational establishment that schools over-rely on textbooks rather than undertaking their own detailed planning [of lessons]".

Askew et al do not elaborate on who might comprise the "educational establishment" that might hold the view that "schools over-rely on textbooks", so this paper reports on research that set out to establish what Ofsted, the official body for inspecting schools in England, might have said about textbook use of the period from the year 2000 until now. The analysis was conducted by first 'scraping' almost 10,000 publicly-available Ofsted secondary school inspection reports and mathematics- specific commentaries reports from the Ofsted website and then utilising basic text mining and analysis methods to extract information on these documents. As interpreting the results of the text mining was not straightforward, a further small-scale qualitative analysis was conducted as a way of illuminating the text mining analysis.

OFSTED, THE OFFICE FOR STANDARDS IN EDUCATION

Ofsted, the Office for Standards in Education, Children's Services and Skills to give its current full title, says that it "regulates and inspects to achieve excellence in the care of children and young people, and in education and skills for learners of all ages in England" (Ofsted, 2014, preface). It does this by conducting inspections of schools, colleges, and providers of further education and skills (including providers of initial teacher training).

The process of inspection has been subject to some critical scrutiny, mostly focused on issues of validity, reliability, and consistency. For example, Campbell and Husbands (2000) found the methodology of inspection to be insufficiently reliable for the consequences which flow from it, a finding echoed by Jones and Sinkinson (2000) and Sinkinson and Jones (2001). Inconsistencies affecting inspections were reported by Penn (2002), while more recently Baxter & Clarke (2013) found considerable tensions in the inspection system.

While the role played by Ofsted in promoting school improvement is currently being examined (e.g. Jones & Tymms, 2014), what remains as yet relatively unexamined is the influence of Ofsted on the professional practice of teachers in England. Here it is necessary to note that Ofsted has recently had to state publicly that it "does not favour any particular teaching style and inspectors must not give the impression that it does" (Ofsted, 2015, para 178). Ofsted goes on to state that, for example, inspectors "should not criticise teacher talk for being overlong or bemoan a lack of opportunity for different activities in lessons unless there is unequivocal evidence that this is slowing learning over time" and that "when observing teaching, inspectors should be 'looking at' and reflecting on the effectiveness of what is being done to promote learning, not 'looking for' specific or particular things" (ibid).

METHODOLOGY

The procedure that was used for data mining was loosely based on the 'knowledge discovery in data' methodology using The Cross Industry standard Process for Data Mining (CRISP-DM, Bosnjak, Grljevic & Bosnjak, 2009). CRISP-DM distinguishes several phases. The first phase, Organizational Understanding, concerns an understanding of what data is actually on the web, what does it say, and how could it be useful for us. The second phase, Data Understanding, involves knowing the precise format of the data. In phase three, Data Preparation, the data is transformed into a format that is understandable for the tool that is to be used to perform the analyses. Phase four, Modelling, is the phase that is used for the actual analyses. Phase five, Evaluation, determines the truthfulness and usefulness of the analysis results. Finally, phase six, Deployment, involves the distribution and publication of the results of the analyses, as is done by this paper, and is therefore not given further consideration.

Organizational understanding

As noted above, the first phase of the analysis, Organizational Understanding, concerns an understanding of what data that is on the web, what it says, and how it could be useful for the analysis. Here we can note that the Ofsted website provides publicly-available inspection reports for every school. Every report has a judgment attached to it which is mentioned on the website and within the report itself. The current judgments are: grade 1 (outstanding), grade 2 (good), grade 3 (requires improvement) and grade 4 (inadequate). Before January 2012 grade 3 (requiring improvement) was called 'satisfactory'. The website also contains interim reports.

Data collection and data understanding

For the second phase, Data Understanding, a 'scraper' was set up using Scrapy (http://scrapy.org/) and this was used to 'scrape' the Ofsted website. The scraper collected the URLs of all historical inspection reports and interim reports since the year of first publication, 2000 (N=9559, 1.39 GB of data). This set of reports was then downloaded using a mass downloader. All documents were in PDF format. Table 1 provides an overview of all the Ofsted documents that were collected, and includes additional mined data, namely the average days between inspection date and report, as well as medians, minimum and maximum.

Table 1: overview of downloaded Ofsted documents

Year	Reports	Size *)	Avg days between inspection date and report	Med	Min	Max
2000	212	34.2 MB	171	158	91	1257
2001	278	38.7 MB	101	99	65	347
2002	178	30.5 MB	114	100	79	822
2003	190	33.7 MB	118	99	72	1409
2004	274	35.4 MB	137	94	51	1192
2005	302	51.1 MB	120	73	13	1178
2006	639	58.2 MB	106	26	10	2566
2007	884	122 MB	61	27	7	1975
2008	835	132 MB	42	29	6	1042
2009	896	128 MB	43	30	2	439
2010	1062	150 MB	36	26	10	286
2011	1139	193 MB	39	26	8	800
2012	1000	175 MB	29	22	4	212
2013	1481	239 MB	28	22	9	974
2014	189 **)	7.17 MB	35	31	-1 ***)	120
TOTAL	9559	1.39 GB				

Data preparation

For phase three, Data Preparation, the PDF files were first converted to plain text format. Two datasets were prepared from these 'raw' materials. One dataset consisted of three corpora of Ofsted files: 1) all files from 2000 to 2004, 2) all files from 2005 to 2009, and 3) all files from

2010 to March 2014. These files were all added as corpora to a software programme called TextSTAT (version 2.9c), "a simple programme for the analysis of texts. It reads plain text files (in different encodings) and HTML files (directly from the internet) and it produces word frequency lists and concordances from these files" (to quote from the TextSTAT website). Two mathematics-specific reports (Ofsted, 2008; 2012) were added separately making a total of five documents. A second dataset consisted of three year files for the years 2004, 2008 and 2013 respectively. These were imported into Rstudio web, a web-based frontend for the statistical package R. The tm (text mining) package (Feinerer & Hornik, 2014; Meyer, Hornik & Feinerer, 2008) was used to make a 'corpus' for every data set. The three corpora were then subjected to several transformations:

- Making all characters lower case
- Removing punctuation marks from a text document
- Removing any numbers from a text document
- Removing English stop-words
- Stripping extra whitespace from the documents

Finally, for this phase, what are called Document-Term matrices were constructed for every document, allowing us to perform several analyses of the documents. In the analysis reported in this paper we looked at the top 10 words and the relative frequency of words between the year-pairs 2004 and 2008, 2008 and 2013, 2004 and 2013.

Modelling

Both data sets were analysed using the respective software packages. The first set was explored quantitatively on frequencies of the following words: textbook, text book, over-reliance, resource (resources), math (maths, mathematics, mathematical), conceptual, procedure, procedural, algorithm, total number of unique words, and total number of words. Table 2 gives the frequencies for the five corpora. The last line shows the total number of documents in the data set/corpora.

Table 2: frequencies for the corpora

	2000-2004	2005-2009	maths2008	2010-2014	maths2012
textbook	2597	304	7	30	5
text book	346	64	0	20	1
over-reliance	185	61	0	37	0
resource	29472	9371	7	3611	35
math	41463	17100	387	20224	485
conceptual	201	67	8	81	17
procedure	16200	10343	2	6207	0
procedural	23	2	2	4	1
algorithm	27	3	1	3	1
unique words	88212	121112	3911	46360	4503
total words	28464140	12676440	28685	11149713	36404
total no reports	1132	3556	1	4871	1

FINDINGS AND DISCUSSION

Table 2 shows there are distinct differences in frequencies of certain key words, even when taking into account the total number of (unique) words in the texts. There also seems to be an increase in the number of reports, with these on average containing fewer words. This might be explained by the introduction of more interim reports and letters. A qualitative analysis also shows there certainly are examples of quotes within the reports that are very telling with regard to the vision on learning and teaching mathematics. One report stated:

> …They develop a methodical approach to finding solutions rather than relying on memorising a collection of algorithms…

Another:

> …Weaker factor: teaching procedural rules rather than developing understanding…

Observations like these, however, must be scrutinized carefully and take into account the context of the word use as well. As it stands, the word counts might be an indication of certain developments, but may not yet be reliable enough.

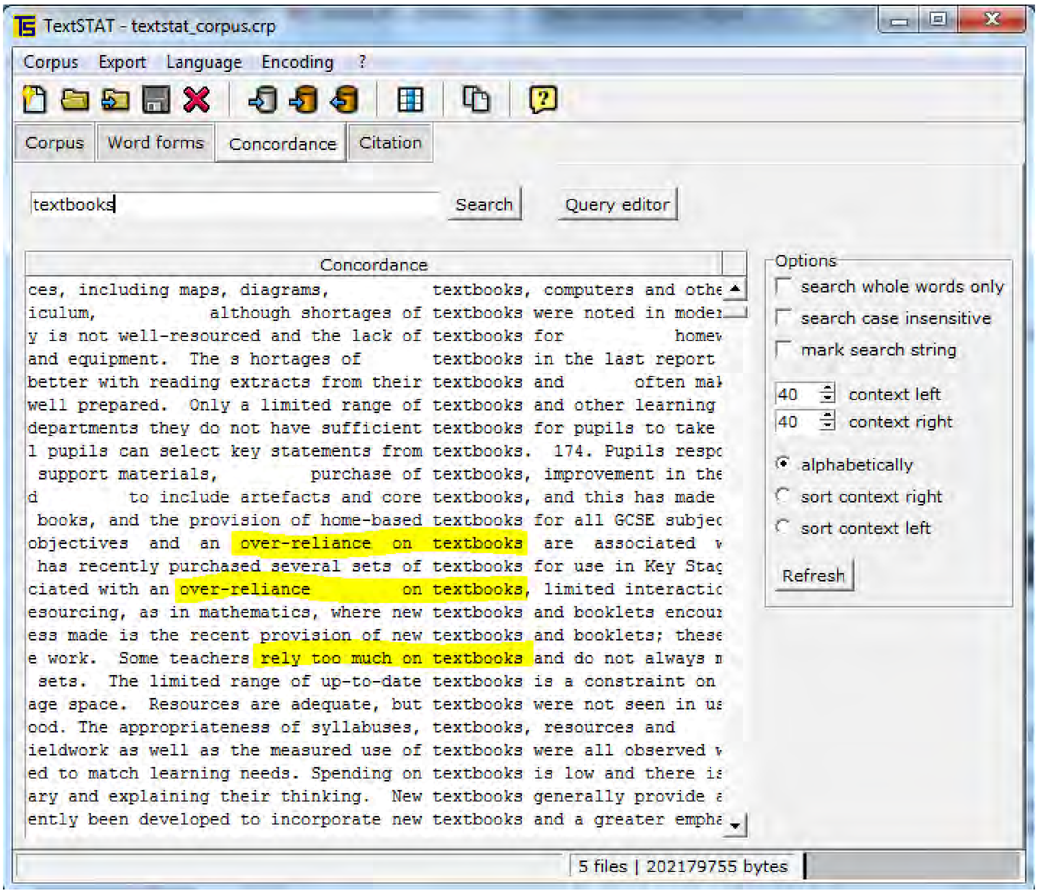

Figure 1: example of concordance use with key word ‘textbooks’, ‘over-reliance’ highlighted

The sharp decrease in the use of the words ‘textbook’ and ‘text books’ might indeed indicate that textbooks have been valued less and less over the years. As such, it is to be expected that comments on the ‘over-reliance’ of textbooks would decrease as well. For example, the 2005-2009 documents flagged up several uses of the word ‘over-reliance’, as also demonstrated in Figure 1. Examples included:

“…dominated by an over-reliance on textbooks…”
“…also sometimes an over-reliance on textbook activities…”

Even in these sections one could argue that textbooks and textbook activities are quite different. Further, the quantifier ‘sometimes’ seems less pronounced than ‘dominated’. ‘over-reliance’ is also used for different topics:

…is therefore an over-reliance on work sheets…
…over-reliance on web-based mathematics packages…

Work sheets can certainly be seen as different from textbooks, but arguably could be grouped under ‘resources’. Another example is:

…over-reliance on whole school initiatives….

This might not be the type of ‘over-reliance’ one is searching for, and it certainly is not connected to the topic of this paper, that of textbooks and resources. The same thing happened to ‘resources’ where URLs with ‘resources’ in them were included.

Nevertheless, a further qualitative analysis of a sample of Ofsted publications found mention of what Ofsted called ‘over-reliance’ on textbooks: “in over a third of classes there was an over-reliance upon a particular published scheme” and this “usually led to pupils spending prolonged periods of time in which they worked at a slow pace, often on repetitive, undemanding exercises, which did little to advance their skills or understanding of number, much less their interest and enthusiasm for mathematics” (Ofsted, 1993, p. 16).

Finally, there is the issue of ambiguity in terms. In mathematics education ‘procedural’ and ‘conceptual’ are often used to denote procedural skills and conceptual understanding. The words’ moderate occurrence and ratio might indicate a preference by Ofsted for ‘conceptual’ rather than ‘procedural’, an observation befitting criticisms along these lines. If one looks at the word ‘procedure’ then we are presented with a very high word count. This is mostly caused by the word ‘procedure’ being used for more purposes than learning and teaching, for example a ‘safety procedure’. Still, if only a small proportion of the word implies a ‘procedural’ aspect of learning, it could ‘normalize’ the ratio procedural/conceptual. It is exactly this *contextual* aspect that would need to be taken into account by future text analyses.

In addition to these challenges for interpretation, there also are numerous technical challenges. One issue concerns the processing phase. Even in this experiment the transformation of PDF files was not straightforward. Converting PDF documents to text format depends on whether the file is not password protected. Another challenge concerned special Unicode symbols, missing spaces and other formatting issues. This might have influenced some of the resulting data; it is however hypothesized that given the large amount of data and documents the influence was relatively small. One interesting aspect to explore would be whether the expressed views differ over judgments: are there differences and similarities between inadequate and outstanding schools, for example. In a next phase we aim to explore the usefulness of more advanced text analysis and mining methods like sentiment analysis, association mining, semantic analysis, and Latent Dirichlet Allocation (LDA) which is a form of so-called topic modelling (e.g. Blei, Ng, & Jordan, 2003). Worded in laymen terms these techniques these techniques might allow us to discover themes and patterns in the reports' choice of words, for example whether certain words and topics occur more frequently for certain (types of) schools. As Figure 2 further exemplifies, these might be visualized as well. In any case, like any model, the biggest challenge in all of this seems to be the *interpretation* of the findings.

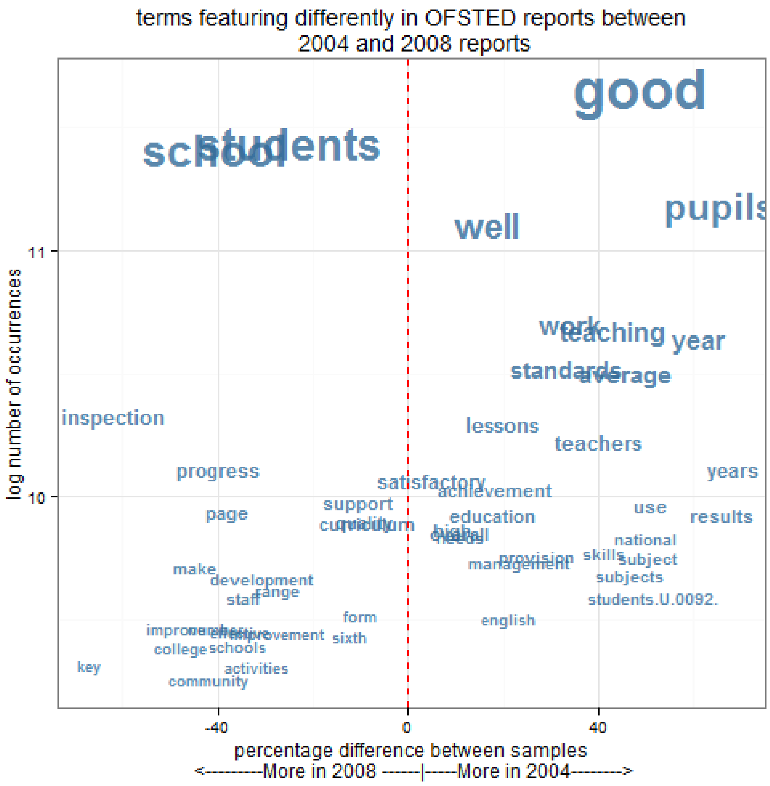

Figure 2: example visualization for Ofsted reports in 2004 and 2008

CONCLUSION

This paper reports on the first results of text mining analysis of the most recent publicly-available Ofsted inspection reports and interim reports from 2000 to March 2014. During this period it may be expected that policy changes have taken place, which might be concurrent with the relevant head inspector. This analysis focuses on what Ofsted has written about (mathematics) textbooks in documents over this period, and how this might relate to changes in Ofsted leadership. Although we conclude that interpretation of the current results proved to be difficult, this paper serves two purposes: first, to show that web-scraping and text analysis methods can be used to analyse policy changes (in this case on text-books and mathematics); second, to demonstrate an application of basic text analysis techniques. In relation to Ofsted's own policies, their re-issued Schools inspection policy FAQs (Ofsted, 2014) contains the following Q & A which is illustrative for this topic:

> Q16. Does Ofsted discourage the use of textbooks to support teaching?
>
> A16. No. Ofsted has no preference as to whether textbooks or other teaching materials are used. Inspectors are interested in the standards achieved by pupils and the progress made by them. Teachers are free to use whatever resources they see fit to prepare for, and teach a

lesson including textbooks. Inspection reports rarely criticise the use of textbooks unless they are out of date. Inspection reports are more likely to criticise an over-reliance on worksheets, which may be poorly reproduced, incomplete or insufficiently linked to earlier work.

In future research we aim to utilize more sophisticated techniques to see whether claims like these can be further substantiated.

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CO-DESIGNING ELECTRONIC BOOKS: BOUNDARY OBJECTS FOR SOCIAL CREATIVITY

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The European 'MC-squared' project has a number of 'Communities of Interest' (CoI) (Fischer, 2001) in European countries that work on digital, interactive, creative, mathematics textbooks, called cBooks. A community of interest consists of several stakeholders from various 'Communities of Practice' (Wenger, 1998). In this paper we outline the creation of an English CoI describing the development of a cBook on numbers and equivalence. We use a design-based research methodology approach for teachers, designers, researchers, teacher-educators jointly working on cBooks as 'boundary objects' (Akkerman & Bakker, 2011) to facilitate thinking about creative mathematical thinking and social creativity. We illustrate our design-based approach through the example artefacts created during the different stages of development of the cBooks. The details of our approach provide a blueprint for the formation of CoI's by working on digital, interactive, creative, mathematics textbooks.

Keywords: digital textbook, interactive textbook, e-textbook, creative mathematical thinking, creativity

INTRODUCTION

This paper describes how the European MC-squared project[1] is creating a new generation of digital, interactive, creative mathematics electronic textbooks, called *cBooks*. To analyse our findings we use literature from communities of practice and communities of interest. cBooks are potential boundary objects that are used to cross boundaries between different stakeholders. This paper describes the formation of the communities of interest (CoI) in England.

THEORETICAL FRAMEWORK

The theoretical framework for the project is provided through the lens of communities of practices, CoP's (Wenger, 1998) and communities of interest, CoI's (Fischer, 2001). Teachers who co-design and use resources for teaching, can contribute to their own professional development (e.g., Jaworski, 2006). As these designs eventually are used in classrooms, students are included as actors in the framework. Members of the CoI are seen as *boundary crossers*. In this view, a boundary is defined as "a socio-cultural difference leading to discontinuity in action of interaction" (Akkerman & Bakker, 2011, p. 133). Boundary

[1] See for more information http://www.mc2-project.eu/

crossing usually refers to a person’s transitions and interactions across different sites (Suchman, 1994). Boundary *objects* refer to artifacts doing the crossing by fulfilling a bridging function (Star, 1989). In the context of this project, it can be hypothesized that a cBook is as a boundary object that brings different CoP representatives together and hence functions as a catalyst for crossing boundaries. A cBook enables group members to work together and scaffolds the “collaboration” fostering the opportunity for creativity.

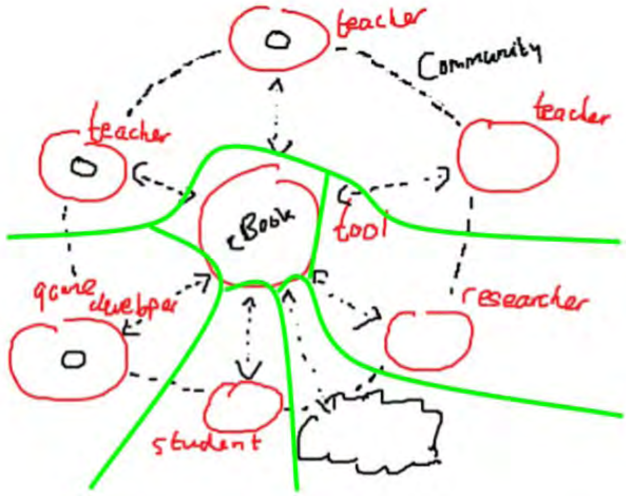

Figure 1: visualization of a community of interest consisting of several communities of practice, working on a boundary object cBook. The green lines denote the boundaries.

The MC-squared project aims to harness the structure of these CoI’s to stimulate social creativity (SC) and creative mathematical thinking (CMT): human creativity emerges from activities that take place in contexts in which there is interaction among people and artifacts that embody knowledge from various communities (e.g. Csikszentmihalyi, 1996; Engeström, 2001). With regard to CMT the project aims to not impose a fixed definition of what CMT means but to let the members of the CoI decide and define it for themselves. The remainder of the paper will describe the evolution of a cBook about numbers, expressions and equivalence, linking it to our research lens.

CREATION OF A C-BOOK

This section described how the CoI developed an idea from the first embryonic stage to a first prototypical version of a cBook.

Birth of the idea: a catalyst for creativity

During the first meeting we used a strategy that the English CoI set out to use throughout, namely to ask CoP representatives (CoI members) what challenges there are in their daily classroom or in English maths education that need addressing. The idea behind this is that CoI members not only think about 'low hanging fruit' but also about actual applications of cBooks that address challenging issues in the classroom. By taking a real example, creative solutions are required. In addition it enables CPD and deep learning for the teacher experience as well as considering other creative solutions. Taking the culture of English classrooms into account it is imperative for adoption that the cBooks that come from this project are rooted in genuine challenges rather than ‘nice activities at the side’. The process of thinking about these challenges involved using poster format papers to express ideas and opinions. One of the ideas concerned a number challenge. The idea was that to stimulate creative mathematical thinking (CMT) students should not be restricted to one 'good answer' but be made aware of the different ways you can obtain an answer. It was envisaged that by posing a number students could be asked to find as many ways as possible to write 36, with 36 in the center and possible solutions pointing outward.

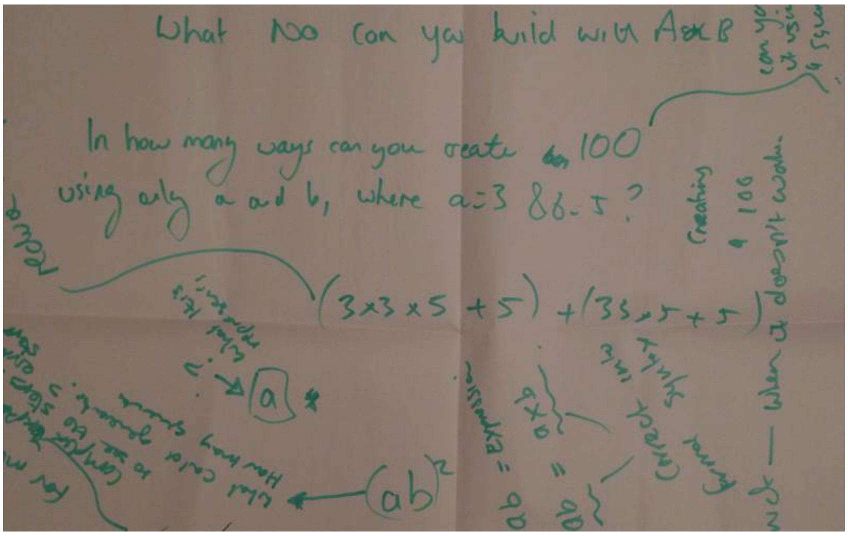

Figure 2: drawing during the first CoI meeting

Building on the initial idea

The initial idea was then extended on a larger paper poster. Several aspects were incorporated in the elaboration:

- The general topic was equivalence and involved several ‘layers’: target audiences could be both at primary school and secondary school;
- A progression (scaffolding) in task types was needed;
- The cBook should 'force' the use of certain operations;
- Software should give pupils feedback on the 'correctness' of inputs;
- The cBook should address understanding the order of operations;
- It was reiterated that an important feature should be that pupils would be free in the answer they could give

Another group developed numbers ideas that focused more on expressions. These ideas were merged with the initial idea. After a short introduction to the software, a first prototype of an activity was authored. This prototype was developed further after the meeting[2].

Further developing the prototype

A cBook of the prototype was developed using DME authoring environment. During these developments several suggestions were made by fellow CoI members in the communication workspace within the software. Amongst others, the social process in the CoI led to several additions and modifications:

[2] Development was done by the first author and not the CoI creator.

- Randomization. Every instance of the book is slightly different.
- Equivalence checking and feedback on whether the answer is correct.
- An early contribution to the discussion emphasised the need for a problem posing element: pupils could be asked to make up their own activity and appropriate text. An open textbox and drawing widgets were integrated in that version.
- Further ideas about fractions were added by two CoI members. This shows the 'network' nature of evolving ideas, as this idea was also mentioned by another participating teacher. Equivalence of fractions is a topic that fits both in 'fractions' as a topic as well as this activity about equivalence relations. It could very well be that these ideas will converge in the future.
- Another comment concerned the first page, which needs an attractive introduction that explains the importance of being able to find equivalent expressions and why this is useful.
- Subsequent comments seem to be converging to an agreement about the status of the book and starting raising detailed questions about terminology, usage in the classroom, and the care that one would need to take when introducing the topic to children due to implicit assumptions.
- At this stage there was a quick action-reaction cycle in the c-Book: one CoI member responded further, causing more reactions etcetera. The comments were evaluated and either led to revisions or a rejection of the idea(s). Some proposed ideas or changes were harder to implement than others and therefore we 'stored' those for a while for (i) further reflection on how we could implement them, or (ii) future implementation. The latter includes ideas about game-like elements, isomorphisms and more feedback.

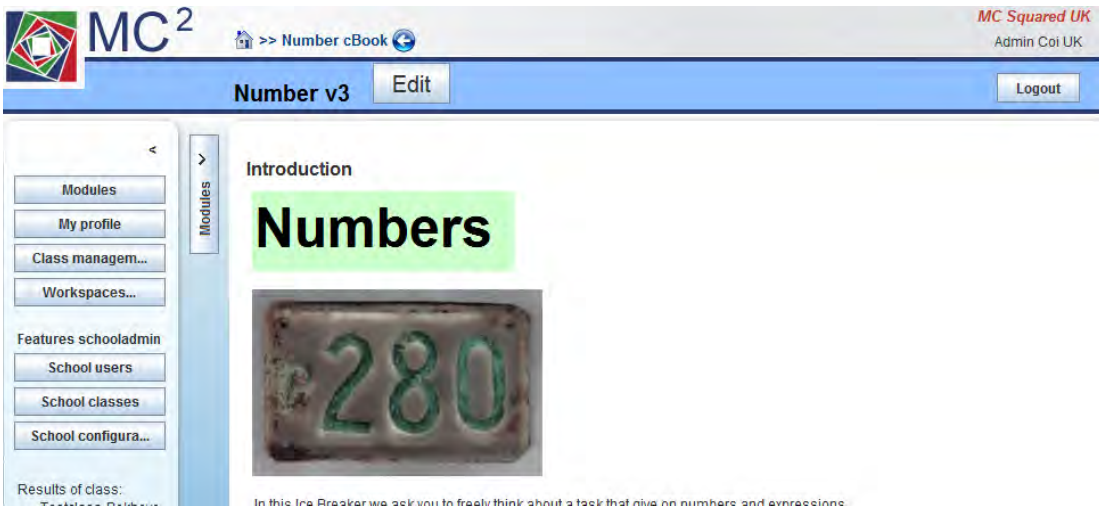

Figure 3: introduction c-book Numbers

The end result of the CoI process was a Numbers cBook with seven pages: an introduction, two tasks asking students to construct expressions that were equivalent to a randomized number, two tasks asking students to construct expressions that were equivalent with an algebraic expressions, an open problem posing task asking students to make up a task for a fellow student and share. The end ‘product’ is not finished yet and will be developed further, but the final version shows how the SC process has contributed to its evolution.

EVALUATION AND DISCUSSION

The CoI formation and authoring of cBooks is only in its initial phase but it is already possible to reflect and draw conclusions on the creative process and its outcomes. It is worth noting that the UK CoI is a heterogeneous group of professionals with different backgrounds, which is promising as this may lead to innovative and creative exchanges and developments. These different backgrounds can certainly be seen as different ‘Communities of Practice’ with every individual representing it. Together they make up a ‘Community of Interest’ that functions as a springboard for new resources to be used and developed to facilitate exchange, integration and thus creative thinking and working. The boundaries between the different CoI members are apparent as well: teachers are very much in a different environment than, for example, researchers. Working together on a cBook provides a useful boundary object to cross these boundaries. Not only do CoI members work on CMT, as described before, boundary crossing within the CoI also is an important source and trigger of social creativity (Fischer 2001). Obviously these are not the only boundaries, in principle every new CoP that joins the CoI provides a new potential boundary that can contribute to the evolution of the cBook. This certainly will be one of the foci in the course of the project. Central in the project are cBooks, which in this case example, can be viewed as boundary object: through their successive versions cBooks act as boundary objects allowing and supporting the CoI's boundary crossing efforts. This is demonstrated in figure 4, an excerpt of the communication within the authoring tool.

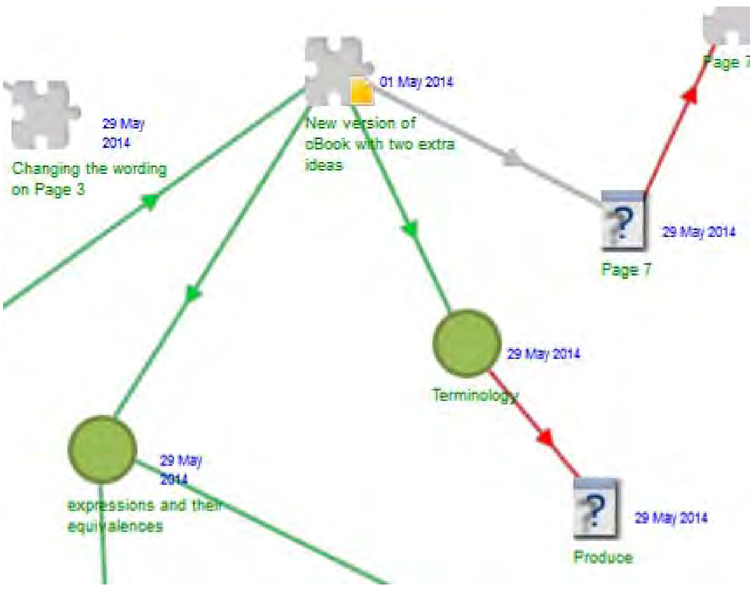

Figure 4: small anonymized excerpt of the communication about the cBook.

The example of the numbers cBook evolution in our view demonstrates how this process augmented SC, as the book evolved: (i) first one idea from one individual; (ii) this led to discussing this idea with one other person in the CoI; (iii) a CoI member took those ideas and made it into the first prototype in the tool (see Bokhove et al., 2014 for more on the authoring tool); (iv) this was disseminated to the rest of the CoI, critiqued and co-edited with improvements. One challenge in using the cBook as boundary object is that of an apparent process-product dichotomy: are we working on new books or are we working on processes?

How do the different stages of the cBook demonstrate the SC process? From figure 4 we can certainly see that the CoI gave rise to new, creative feature, but it will be interesting to see whether and how this process changes over time. The evolution of the cBook has confirmed and deepened some aspects of our CMT approach. Rather than giving a list of expressions and students having to identify whether they are equivalent or not the main aim is to let pupils themselves decide what numbers and/or expressions they want to enter. This philosophy of not providing a question with an answer but leaving the 'correct' answer open underpins these activities. In conclusion, this paper describes some of the first cautious steps of the MC-squared project in which new, creative, electronic maths book are created. Although we are still at the initial stages we are already observing that that these cBooks function well as boundary objects for a Community of Interest. Furthermore, they seem to be able to function as catalysts for augmenting social creativity and creative mathematical thinking.

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HOW DO TEXTBOOKS INCORPORATE GRAPHING CALCULATORS?

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Textbooks are still the main support and mediator in the educational process, but the use of technology such as computers, graphing calculators and, recently, other interactive devices have enabled a new approach to the study of mathematics. The purpose of this paper[1] *is to present an analysis of the complexity of activities proposed to Portuguese students in the trigonometry chapters of 11th grade textbooks as they incorporate graphing calculators in the intended students' actions. More specifically, in this paper we: a) characterize how textbooks in the 11th grade integrate graphing calculators in the study of trigonometry; b) analyze the use of the calculator in the textbooks and the use of the calculator as suggested by the syllabus and see if they convergent; c) analyze research activities and problem solving with the use of the graphing calculator in textbooks. An apparatus for data collection discriminating between two dimensions was created: the Level of Intended Use of the calculator in the presentation of mathematical topics, and the Level of Proposed Use of the calculator as specific tasks are proposed to students.*

Keywords: technology, graphing calculators, trigonometry, Portugal

INTRODUCTION

The use of technology such as computers, graphing calculators and other devices, has enabled a new approach regarding the study of mathematics, which opens up multiple didactical perspectives. In 1988 a curricular renewal in Portugal highlighted the importance of the use of technology in mathematics reflecting the guidelines proposed by the National Council of Teachers of Mathematics (1989)' standards and suggested the use of the graphing calculator. By 1995 its use was quite widespread and in 1998 it was officially required for the solution of specific items in the national exams.

Textbooks however showed no such fast evolution. They couldn't keep up with the syllabus as to include suggestions for tasks and activities regarding technology, and more specifically, the graphing calculator. Only by 2004, a noticeable effort was made by publishers to transfer students working materials from paper to other types of media such as CDs, interactive CDs, online resources, etc. Notwithstanding, the suggested activities do not seem to take full advantage of the calculator (Carvalho, 2006).

[1] This work is supported by public funds through FCT - Foundation for Science and Technology in the context of the project "Promoting Success in Mathematics" (PTDC/CPE-CED/121774/2010).

The purpose of this paper is to study the complexity of the activities proposed by textbooks authors as they incorporate technology, reflecting especially on the complexity of the intended students' actions. More specifically, this paper will:

a) characterize how textbooks in the 11th form integrate graphing calculators in the study of trigonometry;
b) analyze the use of the calculator in the textbooks and the use of the calculator as suggested by the syllabus and see if they convergent;
c) analyze research activities and problem solving with the use of the graphing calculator in textbooks.

METHODOLOGY AND THEORETICAL FRAMEWORK

Content analysis (Bardin, 2004) of the trigonometry chapters of all nine Portuguese mathematics textbooks for the 11th grade in 2004/5 was performed for the first two objectives. The 11th grade was chosen, as it is an intermediate year between the 10th grade, where students had already worked with the graphing calculator, and the 12th grade which ends with a national exam.

In the trigonometry chapter of the program, teachers should focus on the definition and range of functions, the study in a unit circle, the concepts of periodic function and trigonometric functions and use mathematical models appropriate to respond to real life problems. The use of the graphing calculator could therefore allow for a simultaneous use of the techniques and skills with arithmetic and algebra, to allow a balanced approach enhancing capabilities of students to solve problems and engage in modelling activities (Waits and Demana, 2000).

Different students can use the graphing calculator in different ways within the same activity (Graham et al, 2003). Moreover, as Christiansen and Walther pointed out (1986), the ways in which mathematical content is organized, the purpose of the action required, and teachers' role in the classroom help to differentiate degrees of complexity in students' activities. We found these distinctions useful in the analysis of textbook content.

Two dimensions emerged from the content analysis of the chapters: 1) the level of intended use of the calculator, and 2) the complexity of the description of the content presented to students or in the tasks students were asked to perform. The connection between these two dimensions was expressed in a double entry table filled for each of the four subchapters of the trigonometry chapter of each textbook.

We discriminated the *intended use* of the graphing calculator by the textbooks into three levels: level 1 in which the calculator is used as a *scientific calculator* and its graphical capabilities are not required; level 2, the *basic level*, where the calculator is used without requiring its multiple functions and operations; and level 3, which allows the *comprehensive* use of the calculator. According to program expectations, an adequate usage of the calculator in this school year should be mainly at level 3.

The usage of the calculator in the textbooks was classified into four categories of *task complexity*: 1) *explanation of mathematical processes*, very basic use of the calculator, usually in the introduction of a new item from the syllabus and not requiring action from the students; 2) *immediate computation*, when no more than two operations are required in finding results; 3) *closed answer* task when more than two operations are need; and 4) *open answer*, when the solution requires advanced knowledge as well as a critical analysis of the results (of the task). These categories were applied in two distinct types of occurrences in the textbooks, one for the explanation of mathematical processes and another for the tasks proposed to students, usually exercises. We expected textbooks to show a balanced distribution of task complexity among these four categories.

EXAMPLES OF TEXTBOOK USE OF THE GRAPHING CALCULATOR

In the following we give examples of levels of intended use found in the textbooks. The first describes the procedures for solving the equation cos x = -0,45 in the sexagesimal system (textbook A). The student is guided through several steps and graphing capabilities are not used (figure 1).

Figure 1. Solving a trigonometric equation using the inverse function of the calculator.

According to the second dimension the presentation of the mathematical process is performed by a closed answer task in this second example.

The second example (textbook B) is an exercise. It asks students to use the graphing calculator to draw the graphs of the functions sine and cosine in the same screen and to describe in way in which the two graphs are related. This is an open answer task and the calculator is intended to be used at level 2 as it requires basic calculator features.

The problem of the tides is an activity in the textbook C that seeks to study the height of the tide at a given port and uses a trigonometric expression as a function of time. Two questions are posed: what is the height of water at 5:30 am? When does high tide occur and what is the height of the water at that time? It is an open answer task that makes use of the full capacity of the calculator and the textbook proceeds at level 3 of the intended use. The official program suggests this activity and is also included in another book (book D).

The textbook suggests students to start by the observation of the graph on the calculator, and only then to answer questions, either analytically or using the graphing calculator. Students are counselled to take detailed notes of the process that led to the solution. The book gives detailed technical suggestions (that the calculator is put to work in radians, the function should be introduced and choose an appropriate viewing window, the machine as the X represents the time t, is set to vary from 0 to 24, etc.) (Figure 2). Although they suggest a preview window, or here in this textbook or in any other, no particular attention is devoted to display windows.

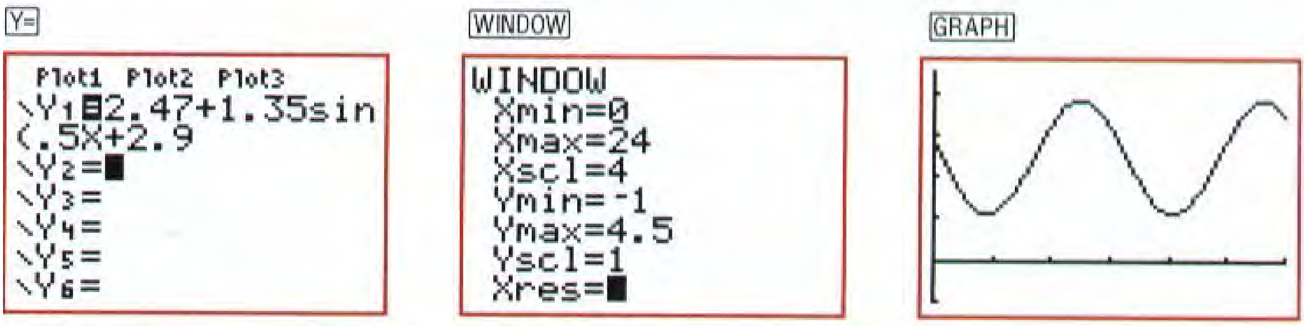

Figure 2. Procedures to obtain the graph of tide height as a function of time.

From the third window in figure 2 the student can begin to make assumptions and draw conclusions on the problem in general and the questions raised. The authors of the manual C highlight the solution of the activity by analytical processes, or using the graphing calculator (figure 3) and they state that any of the questions could be solved using only one of these processes. However they opted to present both, suggesting that they are complementary.

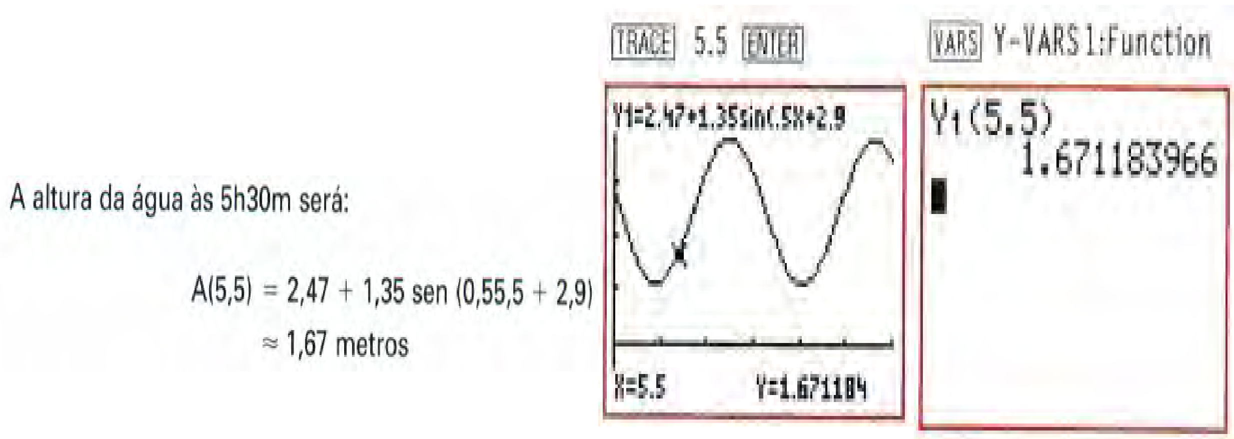

Figure 3. Analytical resolution and graphing resolution of the task, “Tides”

In the follow up of the activity “Tides”, it is suggested to the student the solution of another activity with cosines. This last activity, in addition to analytical processes and the use of calculator, also stimulates mathematical communication in writing to request that the students draw up a short essay on the reasoning in the resolution of an issue.

Textbook D also features an exercise related to the problem of "Tides" translated by a trigonometric function sine. This book presents two other problems: one represented the cost of electricity from a small business data by a cosine function, and the other an optimization problem given by a trig function tangent. An optimization problem similar to this can also be observed in the manual C. These activities also denote a level 3 intended use of the calculator.

In short, both books C and D followed a similar methodology, either in the application of trigonometric functions to real-life problems, whether the use of calculators in the proposed resolution, or even on the similarity between the problems treated. Both could have improved its approach. In the manual D able would have used the table values to determine the parameters of the functions do not leave this task to the calculator. Likewise in the textbook C, could have discussed the influence of parameters on the behaviour of functions.

CONCLUSIONS

Overviewing the nine books, we observe that there are substantive differences among them. A minority (two textbooks) made no use at all of a calculator. Most books (five) made a moderate use, i. e. at larger intervals the calculator is used but it is not logically integrated in a task sequence, and very few open answer questions are included. A minority of books (two) showed an integrated use displaying a well-balanced distribution of both the levels of usage and the tasks devised for the calculator.

The use of level 2 intended use, however, should not be seen as limitative as there are level 2 tasks that could lead to more meaningful learning than others that require a level 3 use.

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WHAT OFFICIAL DOCUMENTS TELL US ABOUT TEXTBOOK USE IN TIMES OF CURRICULAR CHANGE: THE CASE OF THE "NEW MATH MOVEMENT" IN PORTUGAL

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This paper discusses the influence of the use of textbooks in elementary teachers' practices when a new structure was created in the Portuguese educational system in 1968. Mathematics syllabus incorporated the ideas from the New Maths Movement and teachers were faced with a novel situation requiring teaching new contents and the adoption of new teaching methods. Official documents report teachers' difficulties and misunderstandings that may be traced to the use of textbooks.

Keywords: textbook use, history of mathematics education, historical analysis, modern mathematics, New maths movement

INTRODUCTION

Compulsory education in Portugal comprised four years of primary education to all students since 1960. Those who had means to continue their studies had to choose between the technical schools, oriented for a future job position, and the "*liceu*" for those who expected to pursue higher education studies. The intention to extend compulsory education to six years by unifying the first two years of the branches of secondary education was a project of the Ministry of Education since 1958. However, this involved a huge enterprise namely the creation of new schools and the preparation of teachers. An intermediate step was the creation in 1965 of *Telescola* (a low cost school system supported by televised lessons) for students of ages 10-11 in remote rural areas. Teachers of the several content areas broadcasted lessons by television and students attended these lessons in the presence of a monitor who assisted them in their work (Almeida, 2013).

THE CREATION OF CPES

In 1968 compulsory education was extended until the 6th grade, a new educational structure, *Ciclo Preparatório do Ensino Secundário* (CPES, *Preparatory Cycle of Secondary Teaching*), was created and Preparatory Schools were established. Teachers of mathematics might have a degree in Biology, Physics, Geology, Pharmacy and Mathematics. Few of them actually graduated in mathematics and in the first years of the reform most were primary teachers. The mathematics syllabus incorporated the ideas brought by the New Maths Movement (Almeida, 2013; Matos, 2009; Wielewski & Matos, 2009).

This paper is part of a historical study, whose sources are interviews, documents and a textbook, intending to contribute to the understanding of the culture that was developed during the late 1960s and early 1970s in the teaching of mathematics at the CPES. Vinão Frago (2007) argues for the need to consider the historical dimension of the school culture that comes to resist to reforms through the development of traditions and ways of doing that rule the whole process of teaching and learning. According to Chervel (1988), the history of a school discipline is an important complement of the school culture, since original elements are created within it that comes to influence later the culture of the society.

THE REFORM IN PRACTICE

A considerable effort was done in order to prepare teachers to the new modern math ideas through several updating courses. Along the indications to adopt new (heuristic) teaching methods, those courses essentially emphasized scientific knowledge and the importance for students to acquire basic mathematical structures, namely set theory (Bento, 2012; Wielewski & Matos, 2009). However, as Clandini and Connelly (1988) point out, short term courses fail to lead to a positive reform since teachers have narrative histories that rule their way to see their students and their classrooms stressing that “knowledge as an attribute can be given; knowledge as a narrative cannot” (p. 157).

Two memos issued by the Inspectorate in October and December 1969 (*Ofício circular nº 4116 and Ofício circular nº 5603*, respectively) report difficulties and misconceptions detected during the first year of the implementation of the mathematics syllabus (1968/9) and exemplify some of the misuses specially related to set theory, followed by a set of recommendations. The memos were written by inspector Redinha (Bento, 2012) with the collaboration of Sebastião e Silva, the author of the prescribed curriculum and the main responsible for the introduction of modern mathematics in Portugal.

One of the consequences of the introduction of the new math ideas in the mathematics syllabus of the CPES, according to the memos, was the abnormal time teachers devoted to set theory, probably because many teachers may have identified it with the mathematical structures instead of a unifying concept. Although Freeman and Porter (1989) argue that textbooks do not dictate the content taught in schools, its use vary under certain constraints, among which teachers’ knowledge of the subject, convictions about what should be taught and the recognition of the textbook author as an authority in the field. It is reasonable to suppose that, in face of a new situation they did not dominate, teachers turn to the textbook as the main source for their practice. Rezat (2006) offers a model for analysing textbook use based on activity theory. His model adopts the three dimensional shape of a tetrahedron as depicted in Figure 1.

In this model the teacher is seen as a mediator between the student and his mathematical knowledge but the knowledge of the teacher is mediated, among other elements, by the use of an artefact, the textbook. Rezat (2006) goes on to sustain that the artefact (textbook) is in this way placed in the centre of the activity system, which contradicts the notion that human activity is directed to gain control of their artefacts.

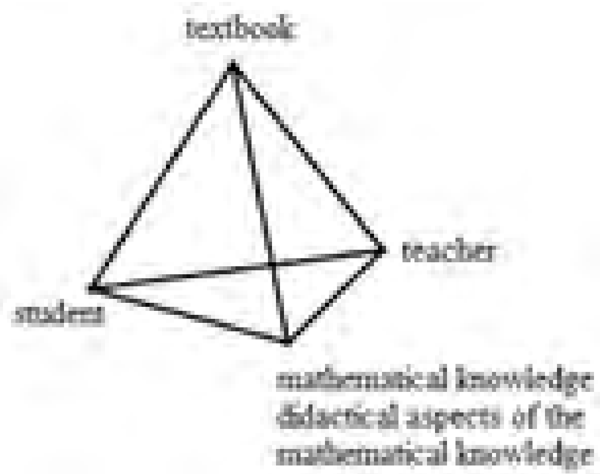


Figure 1. A model for analysing textbook use (Rezat, 2006, p. 413)

THE TEXTBOOK, THE PROGRAM, AND TEACHING PRACTICES

Teacher practices reported in the memos may be influenced by the use of a textbook of the time pervasively used in the first years of the reform (Sousa, 2013). Its author was a highly regarded mathematics teacher, António Augusto Lopes, a member of the Portuguese commission for the reform of modern mathematics, working directly with its chairman Sebastião e Silva (Almeida, 2013; Wielewski, G., Matos, & Wielewski, S., 2010). The analysis that follows compares recommendations from the memos with related textbook approaches and contrasts them to teacher practices (as referred in the memos).

The first issue concerns representation. The author of the textbook chooses Venn diagrams as the main way to introduce and represent sets and their operations whereas the prescribed curriculum and the orientations in the memos analysed stress that sets should be defined, preferably, by the listing of their elements, referring that diagrams can be used by didactical reasons but that students should not be forced to use them. The memos also propose a simplified version of the set-builder notation (e.g. {Portuguese rivers}), but it is not used in the textbook. Although the textbook uses exhaustively the listing notation, diagrams do occupy a great part of the treatment of set theory. One original representation uses drawings or pictures within braces (Figure 2).

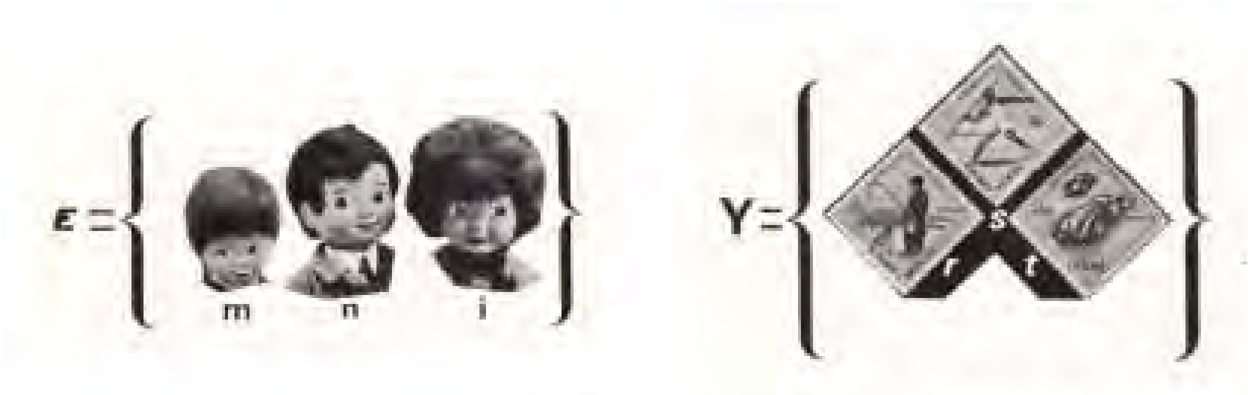


Figure 2. Set representations in the textbook (Lopes, 1968, p. 17 and p. 89).

The memos refer that a kind of mysticism was being created regarding the use of diagrams and strongly appeal for its use to be restricted and not used in written tests for students. One example condemned in the documents with sets of flowers in a garden parallel another example contained in page 36 of the textbook. Although incorrect uses do not appear in the textbook the same does not happen in teachers' productions.

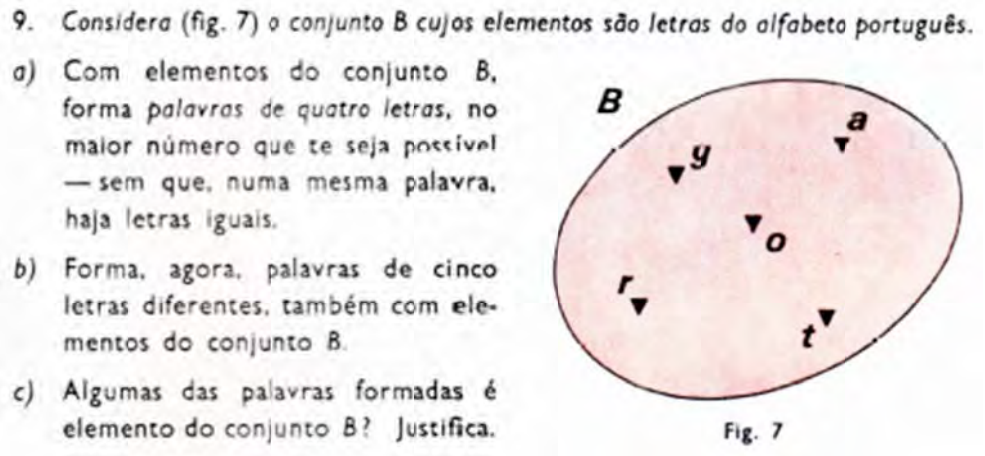
Another issue still related to representation concerns the use of examples with letters. The memos stress that they may be used to represent concrete elements previously defined but one should not make an unrestrained use of examples with letters. An example of a question proposed by a teacher that uses a set whose elements are letters is shown in the memos noting that one does not know whether those letters are to represent themselves or concrete elements, thus creating confusions in the mind of the students. In the textbook one can observe several of those examples, although a reference is always made to the universe of study that may consist of concrete elements or of the letters of the Portuguese alphabet. In this regard the memos demand for an exclusion of the notion of universe that is considered to be prone to create confusions in the students as well as in the teachers.

Treatment of topics not included in the programme is also reported in the memos. Numerations systems in bases different than the decimal were included in the syllabus of the 2nd year of the CPES but were eliminated in 1969 (*Ofício circular nº 3889*). The author of the textbook chooses to treat this topic in the 1st year and the memos note that this is not included in this grade and that it is not acceptable to introduce contents that are not referred. They also refer that the study of the cardinal of the reunion of two non-disjoint sets is to be eliminated however it is included in the textbook (Lopes, 1968, pp. 149-50). Other topics such as sets with binary relations (namely equivalence ones), sets ordered by a relation, the set of the subsets of a given set or the notion of equality of sets ($A=B \Leftrightarrow A\subset B$ and $B\subset A$) are treated in the textbook and go beyond the demands in the prescribed curriculum.

The textbook establishes connections with other disciplines. Examples include sets of historical personalities and of Portuguese monuments and provinces. In some cases the students are asked to look for books of other content areas to answer to the questions. The memos note that a problem of mathematics should never turn out to be a problem of another discipline.

However the connection between mathematics and language created most confusion among teachers and the documents report actual examples in which students are asked to form the greatest number of words as possible using the elements of a set of letters among others. Memos comment that this kind of questions is not a valid way to assess students' knowledge and that they reveal an approach that is neither adequate nor efficient. Again, some exercises of this kind are to be found in the textbook (Figure 3).

Finally, and commenting the excess of time devoted to set theory, the memos ask the teachers not to spend time making drawings in the board of the elements of a set. Some activities for students proposed in textbook include drawings, cuts and the construction of sets. For example, in page 91 of the textbook one asks students to "using coloured paper cut — after you draw them — a dog, a rabbit and a cat. Form the sets that are proper parts of the set {dog, rabbit, cat}". More examples of this kind are to be found along the text, sometimes, as it is the case of the example here given, in the treatment of topics not included in the curriculum.



9. *Considera (fig. 7) o conjunto B cujos elementos são letras do alfabeto português.*

a) Com elementos do conjunto B, forma *palavras de quatro letras*, no maior número que te seja possível — sem que, numa mesma palavra, haja letras iguais.

b) Forma, agora, palavras de cinco letras diferentes, também com elementos do conjunto B.

c) Algumas das palavras formadas é elemento do conjunto B? Justifica.

Figure 3. An exercise asking students to form words using the letters of a given set (Lopes, 1968, p. 14).

CONCLUSIONS

The confrontation between official documents and the prominent textbook leads to the conclusion that many teachers, not dominating the new orientations for mathematics education, and probably not having studied the prescribed curriculum, seem to have used the textbook as the main source to update their mathematical knowledge and to orient their practice. The memos emphasize the need for teachers to confront the mathematics syllabus and the accompanying orientations with the content of the textbook adopted. We know from the interviews conducted so far that most teachers only had contact with modern math ideas when entered the CPES and that they relied on the textbook to take acquaintance of the contents of the mathematics syllabus. Since it implied a completely new way to teach mathematics, whose rationale was not known to them and thus could not be assimilated or accepted, and not having pursued higher studies in this area, many teachers may have misunderstood the task they were expected to perform.

Some of the problems reported in the memos seem to have lasted for a long time. As mentioned above, the reputation of the author of the textbook is highly recognized in the Portuguese mathematics education community. Apparently he did not, however, anticipate the problems that could result from an enlargement of the compulsory education and the deficient preparation of teachers. When confronted with the opinion of the Inspectorate as exposed in the memos issued in 1969, the author explains to Almeida (2013) that he intended to write a textbook that could serve also as a source for teachers to deepen their own mathematical knowledge and that "the intention was a good one but probably it was an error, I don't know if it was an error, but we were all learning" (p. 385).

In the present time we are confronted in Portugal with new syllabuses for all grades of elementary and secondary mathematics education that neither are understood nor accepted by many teachers namely those who have developed a deepened consciousness of the demands of the mathematics teaching and learning. In what concerns elementary mathematics education a series of texts were developed with the intention to assist teachers to the teaching of the new programs of study. There is the possibility that many teachers are to experience difficulties in interpreting those texts and it may be the case that textbooks will come to influence the mathematical knowledge taught in schools.

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TELLING NEW STORIES: RECONCEPTUALIZING TEXTBOOK REFORM IN MATHEMATICS

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This paper examines the content of geometry textbooks to begin to address the questions: When the narrative forms of contemporary textbooks are compared with those that are historical, how have the mathematical stories changed? And what implications may those changes have for teachers and students? This exploratory analysis interprets written geometry textbooks as mathematical stories in order to understand the content design curriculum as a mathematical genre. Three distinct sequences of content found in recent U.S. geometry textbooks are highlighted and compared with that of Euclid's Elements due to its large influence on geometry curriculum (Sinclair, 2008). This study employs a framework theorized in Dietiker (2013), which metaphorically positions mathematical objects as characters, procedures as actions, and representations as settings. Thus, the mathematical content in textbooks is interpreted as an art form, defining the changes of mathematical content in a sequence as mathematical events and the rhythm of raising and answering questions as its mathematical plot. Curricular reform, then, is recast as changes to previous stories, which involve changes in emphasis, form, characters, and plot. This analysis raises new questions about the sequential structure of content within geometry curricula and identifies potential reasons reformed curricula are resisted or misunderstood by teachers.

Keywords: mathematical stories, curriculum reform, geometry, geometry curriculum, USA

INTRODUCTION

In addition to offering entertainment, stories are educative in nature (Egan, 1988; King, 2003). Egan (1988) argues that stories are a "powerful form," which sparks our imagination and helps us "make sense of the world" (p. 2). King (2003) uses competing creation stories to explain that stories are passed from one generation to create cultural knowledge. Schram (1994) nicely brings these ideas together, writing:

> A story is a beautiful means of teaching religion, values, history, traditions, and customs; a creative method of introducing characters and places; an imaginative way to instil hope and resourceful thinking. Stories help us understand who we are and show us what legacies to transmit to future generations. (p. 176)

However, if stories work to preserve and pass on some form of our past, then in what ways might they limit our future? And what happens when these stories change? These questions inform the focus of this paper into how mathematics curriculum has changed over time, as well as the implications for its revision. How might viewing mathematics curriculum as a story shed light on how curriculum has changed and what might be implications of these changes? This paper introduces an analytic that can help to compare different mathematics textbooks that tell what appears to be the "same story" differently.

In many ways, storytelling describes my work as a curriculum designer of textbooks. Like Schram, I try to help students understand the traditions and customs of mathematical practice. These lessons, or *mathematical stories*, introduce characters, such as rectangles or arithmetic expressions, which act in settings, such as graphs or symbolic representations. Rather than being limited to a "story problem," the phrase *mathematical story* in this paper refers to the unfolding of content connecting the beginning with ending, as described in Dietiker (2013). Curricular reform, then, is recast as changes to previous stories, which may involve changes in emphasis, form, characters, or plot. When contemporary textbooks are compared with those that are historical, how have the mathematical stories changed? And what implications may this have for mathematics teachers and students?

To start, I present an interpretation of the oldest of mathematical stories (Book 1 of Euclid's *Elements* (Fitzpatrick, 2008)) and then describe ways the mathematical stories contemporary geometry lessons are similar and different. Recognizing that interpretations of a text vary within and among individuals, I do not assert that my reading should be taken as "the reading" of text. Instead my goal is to show how the narrative forms in mathematics textbooks offer a way to recognize how curriculum has changed.

HISTORICAL GEOMETRY STORY

Euclid's accomplishment in producing a continuous logical narrative of geometry was foundational for what western mathematics has since become (Netz, 2005). Regardless of Euclid's original intent, which remains unknown, the Elements became a textbook and a primary source of geometry curriculum (Sinclair, 2008). The overarching sequence of the Elements story contains four discrete parts: definitions, postulates, "common notions" (often referred to as axioms), and propositions. Book 1 starts with 23 ordered definitions, "an equilateral triangle is that having three equal sides" (Fitzpatrick, 2008, p. 6), which introduce some mathematical characters by name and verbal description. At times, certain important relationships between characters are revealed, such as how a diameter "cuts the circle in half" (p. 6).

Following the definitions are postulates and "common notions," assumptions on which the rest of the story resides. The generation of these is not explained by Euclid; they are just given. In this sense, they are not stories themselves, but instead are the products of previous stories. Just as the outcome from an earlier episode of a soap opera may be referenced, axioms are information to be taken as true in the present mathematical story.

The final section of *Elements* consists of propositions, separate narratives with mathematical characters and action. Together, these propositions are sequenced to incrementally build a broader story, so each represent an episode of this mathematical story. Each episode starts with a claim, which is accompanied with step-by-step instructions of a construction for the readers to observe or perform. Euclid also provides a diagram of the final construction and cites axioms, postulates, and previous propositions to validate his claims. The first few episodes involve claims about possible compass and straightedge constructions. Later claims involve properties of a character or relationships between characters, such as "In isosceles triangles the angles at the base are equal to one another" (Fitzpatrick, 2008, p. 11).

Therefore, *Elements* starts with character introductions, then relates the endings of previous stories, and then offers episodes where claims are made and justified. What other story forms have this structure? Plays similarly start by introducing characters and often contain details of what happened prior to the start of the story. Just as the propositions build on previous propositions, plays contain acts that build upon prior acts. In this case, the reader is an actor, acting out a part. Taking into account the nature of proof, Euclid's play is similar to a court drama, where a claim is made and evidence is provided in a deductive sequence.

However, nothing about a play communicates the constructive nature of the propositions. Since the propositions are arguments that certain constructions are possible, another literary genre is suggested: an instruction manual. For example, assembly directions accompanying my furniture often start with a list of screws, tools, and other "characters" involved in the constructive action. It is also common to find a list of directives that are not explained or justified, such as, "Use two or more people to move and install the cabinet. Failure to do so can result in back or other injury." These claims, which readers are to accept as true, suggest the existence of prior stories that prompted their production. Following these, an ordered list of directives is provided in separate "steps" forming acts.

CONTEMPORARY GEOMETRY STORIES

How do the geometric stories in contemporary geometry textbooks differ? I selected three geometry textbooks to explore this question. One of these texts, written by Moise and Downs, was one of the dominant geometry texts used in the 1960s, 1970s, and 1980s. *Moise and Downs Geometry* (1982) interestingly represents a reform as part of the "New Math" movement of the United States. The other two texts were published more recently. One of these, *Holt Geometry* (2007), represents a geometry textbook designed by a large publishing house with a large market share. The other, *CME Geometry* (2009), is a textbook designed to develop mathematical habits of mind (Cuoco, Goldenberg, & Mark, 1996). Ten lessons in each textbook were analyzed for the mathematical stories at the start of the lesson, focusing on those that address geometry content related to that in Book 1 of the *Elements.*

Moise and Downs Jr. Geometry

In *Moise and Downs Geometry*, most lessons contain the *definitions – background – claims with justification* sequence. In fact, many have a series of claims, followed by step-by-step proofs similar to the *Elements*. However, a few lessons with subtle differences were found. Lesson 5-10, entitled "Quadrilaterals, Medians, and Bisectors," is such a case. Before introducing a formal definition, Moise and Downs provide examples and non-examples of quadrilaterals that invite readers to make judgments about the possible characteristics a quadrilateral (see Figure 1). This offers readers an opportunity to engage in character development, perhaps coming up with a list of characteristics of quadrilaterals before any formal definition is stated. This example reveals cracks in the traditional story form that are open to more interpretations and offer opportunities to stimulate curiosity.

Another difference is found following a definition of an angle bisector of a triangle, where the authors raise a question that remains unanswered: "*How many angle bisectors does a triangle have?*" (p. 168). This question is not contained in the problem set and readers are not obligated to answer it. While the query arguably has one answer that would be deemed by the authors to be “correct,” the presence of this question could work to encourage readers to raise similar questions and form hypotheses outside of the official set of exercises.

Examples of quadrilaterals Non-examples of quadrilaterals

Figure 1. Defining diagrams adapted from *Moise and Downs Geometry* (1982, p. 167)

Perhaps the most prominent distinction is that while Euclid provides step-by-step directions for constructions, readers of this textbook are primarily responsible for their authorship. For example, after the unanswered question described above, an exercise is printed, “Construct a large scalene triangle. Using a ruler, construct its three medians. Using a protractor, construct its three angle bisectors” (p. 168). Interpreting this text as an assembly manual construes the mathematical definitions as tools along with the traditional forms of tools, such as the ruler and protractor. The textbook essentially communicates, “Here are the mathematical tools you need and this is what you are supposed to build. Figure out how to use these tools to build it.” Also, in exercise 3 (“Given $\triangle ABC$ with median $\overline{AD}$ perpendicular to $\overline{BC}$, Prove: $\overline{AD}$ bisects $\angle BAC$ and $\triangle ABC$ is isosceles” (p. 168)), the form of court drama is present, where the characters are provided and the student’s role is to write a script for a court scene that convinces the audience that the given claim is true. Therefore, this mathematical story reflects a similar form of the *Elements*, yet the authorship role of the reader is strikingly different.

Holt Geometry

In *Holt Geometry*, again most lessons included the *definitions – background – claims with justification* sequence or a close variation. For example, Lesson 5-3 starts with background information to be taken as true (“Sculptors who create mobiles of moving objects can use centers of gravity to balance objects” (p. 314)) and a definition of a median of a triangle (“A median of a triangle is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side” (p. 314). Then, like Euclid, the authors make a claim (“Every triangle has three medians, and the medians are concurrent, as shown in the construction below” (p. 314)). The authors follow this claim with step-by-step instructions for performing a construction that demonstrates (although does not prove) the claim.

This narrative form differs from *Elements* in that the authors allow a character (the centroid) to first emerge through the construction before it was introduced or named. Thus, when the readers read the construction, the focus may be on the medians of the triangle, not on the point at which the medians meet. This variation no longer allows the readers to assume this narrative acts as a play script or instruction manual, where all characters are identified at the beginning.

CME Geometry

In the third contemporary geometry textbook, *CME Geometry*, again the familiar pattern of *definitions – background – claims and justification* is found. For example, the beginning of

Lesson 2.17 starts with a discussion about how special characteristics distinguish some quadrilaterals from others and then presents definitions for a *trapezoid* and *kite*. Background information that is taken to be true is also present in a side-bar comment which explains that trapezoid is sometimes defined differently. Then an "example" is provided, which contains a claim that is then proved for the reader.

However, in a move very distinct from Euclid's, what follows the proof is a narrative form called "Minds in Action, episode 4" not found in any of the other texts surveyed here. In this portion, a script between two fictional students, Tony and Sasha, is presented in which the students discuss the logic behind *always*, *sometimes*, and *never* statements. This dialogue is offered to support exercises later in the lesson that ask students to complete a sentence with the words *always*, *sometimes*, and *never*, such as "The diagonals of a trapezoid are ___ congruent" (p. 149). The information it provides could have been stated in abstract propositional claims such as, "If the statement is sometimes true, then there must exist an example for which it the statement is true and an example for which the statement is false." Instead, the authors embedded this narrative within the lesson, forming a play within a play.

A second lesson further illuminates differences in narrative style of the *CME Geometry* (2009) text. Lesson 4.2, which develops the notion of scale factor as a ratio of enlarging and reducing shapes, opens with contextual background information about maps and blueprints. The text then offers an informal non-specific definition of a new character (*scale factor*), and indicates that students will "develop a more precise definition" throughout the lesson. The readers are then presented with a square and encouraged to generate through discussion possible multiple meanings of the expression "to scale the square by a factor 1/2." While the Moise and Downs (1982) textbook encouraged readers to develop characteristics of a character inductively given examples and non-examples, the CME text instead offers only a statement that scale factors "describe how much you reduce or enlarge" a diagram. However, does that mean that the reduced square has ½ the side length? Or ½ the area? The text encourages this ambiguity to arise and does not immediately provide an answer.

CONCLUDING THOUGHTS

This analysis demonstrates some of the ways mathematical story forms in geometry texts vary. While this exploration was not an exhaustive analysis at all narrative story forms within the texts considered, differences in the story forms were found that raise questions about how readers (including both teachers and students) who may be used to one form of mathematical story may respond when that form is changed. Although the traditional story form is still found in contemporary textbooks, some stories are being told in new ways, which may have ramifications for those in our classrooms. For example, when a teacher is used to a geometric story that provides directions for constructions, as Euclid's text does, how do they adjust to stories that do not? And when a teacher encounters ambiguity in a textbook, as found in *Moise and Downs Geometry* and *CME Geometry*, do they tend to reduce ambiguity? Future inquiry could explore whether teachers tend to revert geometric stories to a form closer to that in the *Elements*.

This analysis may also provide a new explanation to why a teacher or student may resist curriculum revision. In fact, a teacher whose view of what is mathematical stems from one story may not even recognize another as mathematical. In comparing two stories about the creation of the earth, King (2003) points out each culture has stories that frame the way they view the world and which may prevent looking at world in other ways. That is, changing mathematical stories may change mathematics, both in substance and nature.

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READING GEOMETRICALLY: THE NEGOTIATION OF THE EXPECTED MEANING OF DIAGRAMS IN GEOMETRY TEXTBOOKS

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This paper applies reading theory to examine the challenges present in the interpretation of geometric diagrams in mathematics textbooks and identifies the different expectations of elementary and high school students when it comes to interpreting diagrams. We motivate this analysis by exploring how a task with a geometric diagram can appear in both elementary and secondary mathematics textbooks in the same form and still have substantively different mathematical answers. Thus, it starts to address the questions: (a) What strategies of reading diagrams are younger and older students expected to have? (b) What implications might this have for student reasoning with diagrams and its development? We demonstrate, through the contrast of diagrammatic positioning in tasks found in textbooks at different grade levels, how students are expected to develop sophisticated ways to interpret diagrams using context and convention. Based on a survey of elementary and high school U.S. textbooks, eight dimensions of reading geometric diagrams are identified, illustrated, and discussed. The textbooks used in this analysis were selected to reflect a variety of pedagogical and philosophical commitments in case these commitments altered the relationship between diagram and text. Also, to understand the role of cultural context, we included a grade 6 textbook from Turkey in the analysis. As a result of this analysis, we introduce multiple dimensions of diagram negotiation that are expected in textbooks as represented by their tasks and answers. Using these dimensions, we propose a definition of "reading geometrically," a notion inspired by Pimm (1995), and offer a reading heuristic to support student geometric reading.

Keywords: geometry, geometry curriculum, geometric diagrams, reading theory, task analysis

INTRODUCTION

What is the name of this shape?

The answer to the above "rectangle" task, which can be found in almost any U.S. textbook with geometric content, surprisingly differs by grade level; in first grade, the answer given is "rectangle," yet in high school, it is "quadrilateral." Thus, some students are expected to interpret this as a rectangle, while others are expected to interpret this as not necessarily a rectangle. This variance reveals that there are multiple valid ways of reading the same diagram and that differently-aged students are expected to read the same diagram differently. At root, this example exposes a challenge of reading mathematics textbooks: how are students to know how to interpret these geometric diagrams and what assumptions can (or cannot) be made? We conjecture that students are expected to alter the ways they interpret geometric diagrams (i.e., develop new reading strategies) as they progress throughout school. Yet we know of no explicit curricular focus to support this change in the interpretation of diagrams. Our aim is to expose the subtle challenges of reading geometric diagrams and to propose curricular strategies that may support student reasoning with geometric diagrams.

Before designing ways to support student reasoning with diagrams, we argue that it is important to understand what it means to read a geometric diagram (hereafter "diagrams"). Pimm (1995) proposes that diagrams "need to be 'read' rather than merely beheld" (p. 41) and theorized geometric interpretation as "seeing as;" that is, taking a diagram of a circle (which is necessarily imperfect) and seeing it as a circle. Attending to the meaning, context, and purpose of diagrams, Pimm provocatively asks, "What do we ask a mathematical diagram? What do we ask of a diagram in mathematics?" (p. 181). We were inspired by Pimm's insight as we consider the questions: What is "reading geometrically" and what strategies of reading diagrams are younger and older students expected to have as represented by the diagrams found? What implications might this have for student reasoning with diagrams and its development?

THEORETICAL FRAMEWORK

Rosenblatt (1988) explains that a reader of a text "'composes,' hence, 'writes,' an interpreted meaning" (p. 2). This resonates with de Freitas and Sinclair (2012), who reinterpret a diagram not as a static image, but as a trace of its generation, the "gestures" of its reader. Pimm's (2006) notion of "seeing as" also interprets a reader as one who generates a mental construction that, while related to the given object, is different. Thus, for this study, we assume that reading a diagram is, in part, a generative and creative act by a writer of meaning, that is, its reader.

Diagrams can be distinguished using Peirce's three forms of signs: an *icon* (a representation of an object by its appearance), a *symbol* (a representation based instead by an association of concepts), or an *index* (a signal or representation pointing to another object or quality). Netz (2010) also distinguishes two qualities of diagrams, which he refers to as "topological" and "metrical." Metrical properties are those related to marked and unmarked lengths, angle measures, perpendicular/parallel markings, congruence markings, and other symbols that represented aspects of diagrams that were measured. The topological properties included the number of sides, intersections, the openness/closedness of figures, the orientations (inside/outside), adjacency, collinearity, and other relative relationships of the parts of the object. Netz notes that readers may assume the topological properties of a diagram since, "it is hard to draw a diagram such that it is topologically false" (p. 432). Also, drawing from Yerushalmy and Chazan (1990), we assume that diagrams for proofs can be interpreted as a *particular* or *general* case.

METHODS

This paper represents an analysis of the negotiation of meaning of student textbooks where theory about the reading of diagrams was generated through iterating between existing literature and the analysis of diagrams found in textbooks. We analyzed 3 U.S. early elementary textbooks (*Investigations*, *Everyday Mathematics*, and Scott Foresman/Addison Wesley *Mathematics*), 3 U.S. high school geometry textbooks (UCSMP *Geometry*, Holt

Geometry, and CME *Geometry*), and a grade 6 math textbook from Turkey. As this is not a study of a particular textbook design but rather to understand their use of diagrams, the specific texts from which examples were adapted are not named. As we analyzed the tasks with diagrams, we considered, “what information from this diagram is necessary to answer this task (according to its answer) and what am I expected to ignore?”, “What ways are students expected to interpret this diagram?”, and “What other ways might students interpret this diagram?”, focusing on what the student should assume to be true while responding to the prompt. After individually interpreting diagrams, we reconciled as well as drew from additional theory to explain differences in interpretations.

THE DIMENSIONS OF READING GEOMETRIC DIAGRAMS

In our analysis, we identified multiple dimensions of reading geometric diagrams. One dimension of reading diagrams involves recognizing the type of sign (icon, symbol, or index) the diagram takes in the text. The answer “rectangle" in the opening task suggests that young elementary students are not expected to reason from the presence or absence of marked properties of the figure but instead read for what the figure resembles. Thus, they are expected to interpret the diagram as an icon, assuming that the depiction resembles the object it represents. By high school, our analysis indicates that students are mostly expected to read diagrams as symbols, using conventions and markings without which the meaning changes. Surprisingly, however, both iconic and symbolic diagrams were found in the high school textbooks.

Another dimension involves the assumptions the reader is expected to make regarding the topological and metric aspects of the diagram. Although younger students are expected to make both topological and metrical assumptions (as shown in the “rectangle” task), older students are sometimes expected to assume the topological properties of the figure while withholding assumptions of unmarked measure. For example, in Figure 1, adapted from a high school geometry textbook, student are expected to assume that the figure is a triangle (assuming topological representation) with side lengths of 8 and 10 units (assuming marked lengths), and yet are not expected to assume anything based on visual appearance about the length of the third side or the measure of the included angle.

In the triangle at right, which of the following CANNOT be the length of the unknown side?

a) 2.2
b) 6
c) 12.8
d) 17.2
e) 18.1

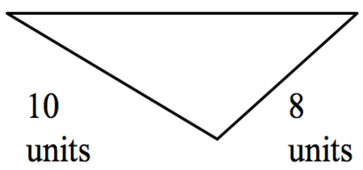
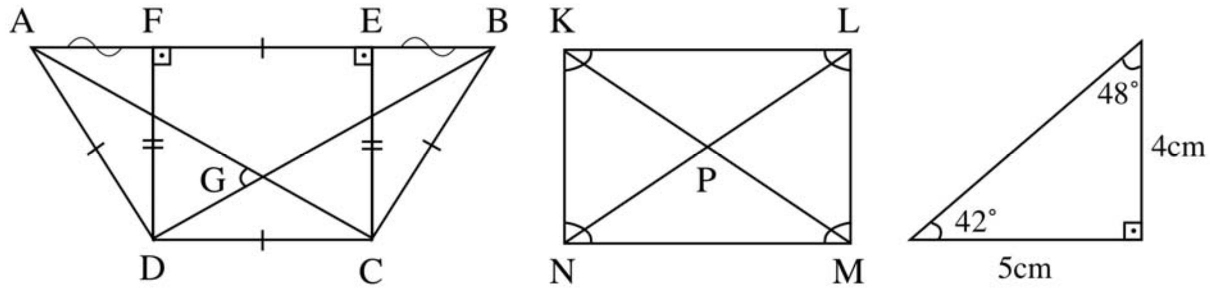


Figure 1: A Task Adapted From a U.S. Geometry Textbook

The metrical and topological properties found in diagrams of textbooks we analyzed varied. Across age groups, the topological properties expected to be assumed included closure (or non-closure) of figures, orientation (inside/outside), adjacency, collinearity, coincidence, and the number of sides. Students were expected to assume the validity of any given measured property, whether it was a numerical measure (such as 34°) or a symbolic measure (such as a right angle mark), including assumptions regarding relationships, such as marked congruence, parallelism, and perpendicularity. However, the older students were expected to ignore visual appearance of these metrical qualities when unmarked while younger students were expected to assume them.

A third dimension of reading geometric diagrams involves understanding and following cultural conventions. That is, rather than being a "true" object with singular meaning, a diagram is a system of signs dictated by cultural conventions with which a reader must negotiate. These conventions include the proximity of markings, capitalization (or not) of letters, use of Greek letters, and the specialized symbols to represent relationships. Consider the interpretation of geometric diagrams from a Grade 6 textbook from Turkey (see Figure 2) by readers familiar with U.S. conventions. It might be assumed that because the angles in (b) are all marked the same, they represent angles of equal measure. However, this would make them all 90° and yet they are not marked with a square and a dot as is done in the right angle in (c).



(a) (b) (c)

Figure 2: Diagrams Adapted From a Turkish Textbook for Grade 6

In our analysis of textbook diagrams, we found that the negotiation of meaning of a diagram involves deducing properties that may not be explicitly marked. Thus, students must also learn to read diagrams with anticipation of missing information and possible extraneous information. This is not a criticism of textbooks and we even suspect that the pedagogic nature of diagrams in textbooks sometimes leads to the purposeful limitation of marking. The act of deducing information about the objects in diagrams was a dimension that became apparent when we began analyzing high school texts that was not apparent in our analysis of elementary texts.

Therefore, there is a temporality to reading a diagram during which assumptions about it may change or its veracity may be challenged. We found that in some cases, geometric diagrams in textbooks either intentionally or unintentionally misrepresent a geometric object or break assumed spatial rules. Thus, a dimension of reading a geometric diagram involves accepting or rejecting information interpreted from the diagram. When a reader assumes that the geometric object in a diagram *could exist*, it requires deductive reasoning to recognize that the object is not accurately represented in the diagram or that the reader's initial assumptions about the diagram were misguided. For example, we expect that it may surprise some readers that Figure 2(c), with the measures indicated, is not possible (in Euclidean geometry). This temporal aspect to reading diagrams can therefore be recognized as a narrative experience for a reader and the misrepresentation of geometric objects is tantamount to an unreliable narrator.

At times, a dimension of reading involves re-construing a particular diagram as a collection of geometric objects (by mentally drawing them). That is, some tasks in textbooks appear to expect students to respond not based on a diagram in the text but by a diagram that is imagined. For example, in Figure 1, to get an answer of (e) with diagrammatic reasoning, a student needs to mentally redraw a set of triangles that have side lengths 8 and 10 units. Although some tasks begin with phrases such as, “For any triangle” or “For all quadrilaterals”, explicitly extending the focus beyond the given figure in the diagram, the example in Figure 1 (with the text "in the figure at right") indicates other times where the reader must make that extension.

Another dimension of reading a diagram negotiated through context is reading its geometric dimensionality. For example, without the text that accompanies the diagram in Figure 2(b), there are few clues about whether the figure is a quadrilateral or a pyramid. In this case, the focus of the textbook/course matters. This diagram would be read quite differently if found in a non-Euclidean geometry textbook than in a typical Euclidean textbook. Therefore, the axiomatic assumptions are also negotiated through contextual clues.

Finally, a dimension of reading geometric diagrams involves deciding if one is reading *as* a geometric object. In many cases, the context helps to determine if the geometric properties of the object are even the focus of study; the diagram in the “rectangle” task was found to represent an element in a pattern in a different task (see Figure 3). Changing the question from “what shape is this?” to “what shape comes next?” prompts the reader to read for differences between shapes rather than identifying any particular shape

What shape comes next in the pattern?

Figure 3: Pattern Task Adapted From a U.S. Elementary Textbook

READING GEOMETRICALLY

This study was inspired by Pimm’s questions, “What do we *ask* a mathematical diagram? What do we *ask of* a diagram in mathematics?” (2006, p. 181). Our analysis offers many potential heuristic questions in response. Reading diagrams geometrically involves asking many questions of diagrams, including “What, if anything, can be assumed? What evidence should I ignore?”, “Is it an icon or a symbol?”, “What more can I figure out about the geometric object?”, "Is it possible?", “Is it representing a single object or multiple?”, and “In what ways is this diagram limiting my vision?”, among others. That none of these questions actually appear in the textbooks in relation to mathematical work involving diagrams strikes us as especially remarkable. It is reasonable to assume that including questions such as these in the curriculum could help students develop sophisticated ways to interpret geometric diagrams.

Based on our analysis, we now define *reading geometrically* as a reader's negotiation of meaning with what is, is not, and may be misleading in a diagram. We propose that reading geometrically entails a re-drawing of the diagram in the reader's mind, taking into account its context (and thus, its purpose) while drawing upon prior experiences with conventions of drawing, marking, and labelling of diagrams. Although it is tempting to make claims about "the" reading of a diagram, we question the ability to make such a claim. Instead, we propose that reading diagrams is much more complex than originally acknowledged. We now stand back and wonder, with the challenges of convention, context, categories of signs, deductive possibilities, assumptions that are deemed acceptable and others not acceptable, and misrepresentations that readers confront in U.S. textbooks, how do students make sense of it all? Particularly because of messages of "this is a rectangle and, yet, it is not necessarily one," it is not surprising that learning mathematics appears to many students as a game where the rules seem arbitrary and hidden.

Finally, we conclude by noting that there are many questions that remain unanswered about how diagrams are used by texts, teachers, and students. Possibly, some of the potential confusion of reading diagrams experienced by students when transitioning from elementary to high school may be overcome by explicitly supporting the different ways students read geometrically. We recommend future studies explore the questions students of different ages bring to diagrams. We also encourage future studies that investigate the reasoning used by students during their interactions with diagrams in textbooks, the strategies used by teachers regarding the reading of diagrams, and a more detailed analysis of the form and function of diagrams found in textbooks across grade levels.

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A COMPARISON OF TWO GRADE 7 MATHEMATICS TEXTBOOKS: ONE UK, ONE SINGAPORE

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This study involved the comparison of two Grade 7 mathematics textbooks. The first book was The School Mathematics Project Book 1 *(SMP), which was first published in the UK in 1965, following the New Math Reform of the 1960s. The second book was the* New Express Mathematics 1 *book, which was first published in 2007 by Multimedia Communications in Singapore and approved by the Ministry of Education for use in Singapore schools. The textbooks were compared using a set of six criteria based on a framework developed by the Virginia Council of Teachers of Mathematics in 1980. The following six criteria were used: (1) Readability, (2) Organisation of Student Textbook, (3) Physical Characteristics, (4) Content, (5) Auxiliary Materials, and (6) Special Considerations. A composite score, based on a five-point scale, was calculated for each book: 3.43 for the* SMP *book and 4.045 for the* New Express *book. The scores basically informed the comparison of the textbooks in a general sense and are supplemented by a more detailed view of the six criteria over the two textbooks.*

Keywords: textbook evaluation, textbook comparison, grade 7 textbooks, England, Singapore

INTRODUCTION

This study involved the comparison of two grade 7 mathematics textbooks. The first book is The School Mathematics Project Book 1 (SMP Book 1), which was first published in the UK in 1965, following the New Math Reform of the 1960s. The second book is the New Express Mathematics 1 book (NEX Book 1), which was first published in 2007 by Multimedia Communications in Singapore and approved by the Ministry of Education for use in Singapore schools. The two books were selected precisely because they come from two different curricular contexts and cultural traditions and they are no longer in use in the contexts that they were produced. Also, these two books were published more than 40 years apart and the analysis will help to shed light on what were valued in the two textbooks and, to some extent, about the teaching and learning of mathematics in general, at about the time of publication. On a personal, I have used the SMP book as a student and the NEX book as an educator. The textbooks were compared using a set of six criteria based on a framework developed by the Virginia Council of Teachers of Mathematics in 1980. Basically, the study looked at the question: How do the two textbooks compare on the set of evaluation criteria developed by the Virginia Council of Teachers of Mathematics in 1980?

THE TWO BOOKS

SMP Book 1

The SMP Book 1 is the first book of a series of four textbooks leading up to the O-level first published in 1965 by the Cambridge University Press. About 18 authors collaborated in the writing of this large-scale textbook series. Writing about the SMP project in SMP Book 1, Bryan Thwaites wrote: "The project was founded on the belief, held by a group of practicing school teachers, that there are serious shortcomings in traditional school mathematics syllabuses, and that there is a need for experiment in schools with the aim of bringing these syllabuses into line with modern ideas and applications." (p. i)

New Express Mathematics 1

The NEX Book 1 was a textbook published by Multimedia Communications and distributed by Panpac Education Private Limited. The textbook series was approved for use in Singapore secondary schools by the Ministry of Education for 2007-2011. The authors were mathematics educators and mathematicians (14 in all) based at the National Institute of Education in Singapore. The textbook runs to over 368 pages and contains 13 chapters with four progress self-tests and answers to selected exercises.

Table 1: SMP Book 1 and NEX Book 1

SMP Book 1	NEX Book 1
Brought about by school teachers with support of university mathematicians.	Written by mathematicians and mathematics educators at NIE.
Initiated by shortcomings of the traditional syllabus.	Just a textbook for a new syllabus.
Emphasis on a new pedagogical approach.	Pedagogical approach not emphasised.
There seems to be a de-emphasis on computation.	Use of calculator advocated.
New content areas such as sets, bases, functions.	No major change in content compared to previous syllabus.
An emphasis on structure in mathematics.	Emphasis on the acquisition of basic concepts for problem solving.
Leading to O-levels. Tried to link school mathematics with higher (pure) mathematics (university mathematics).	Main objective to lead students to O-levels.
Explanations and instructions are quite wordy.	Explanations and instructions are less wordy.
Collaboration with schools. Teachers in the writing team.	No such collaboration – no teachers on panel.
Students learn by exploration and discovery. Directions for teachers. Teacher's Guides available.	No claims about how students are to use the textbooks. No instructions for teachers. No Teacher's Guides.

The evaluative criteria

The Virginia Council of Teachers of Mathematics (VCTM) produced a set of six evaluation criteria for evaluating mathematics textbooks. The following six criteria were used: (1) Readability, (2) Organisation of Student Textbook, (3) Physical Characteristics, (4) Content, (5) Auxiliary Materials, and (6) Special Considerations. For sake of brevity the six criteria are not described here. First, each of the six criteria had to be given a weight depending on how much each criterion was valued in the textbook evaluation. Under each of the six criteria were a set of descriptors. In this study the weights used are as follows: Readability (15%) (2) Organisation of Student Textbook (25%), (3) Physical Characteristics (15%), (4) Content (25%) Auxiliary Materials (10%), and (6) Special Considerations (10%). Second, each descriptor under a given criterion was rated on a scale from 1 to 5 points, with 1 being the lowest. Third, the total points from the descriptors were divided by the number of descriptors to obtain the average of the points. Fourth, the average of the section point was multiplied by the weight as a decimal to obtain the section rating (see Figure 1 below).

I. Readability (Weight 15%) — Rating

- A: 4
- B: 3
- C: 4

11 (total points) ÷ 3 (number of items rated) x .15 (weight) = .55 (section rating)

II.	Organization of Student Book (Weight 20%)	.82
III.	-----	.36
IV.	-----	1.10
V.	-----	.75
VI.	-----	.25
	Total of section ratings (final book rating)	3.83

Figure 1. Calculation of section a rating and final book rating (source VCTM 1980)

FINDINGS

The ratings for the two textbooks under each of the evaluative criteria were calculated using the procedure given above. The results are given in Table 2. The final book rating for the SMP Book 1 was 3.43 and that for NEX Book 1 was 4.045. It is to be emphasized that the ratings depend on several factors: inclusion or exclusion of particular descriptors in the rating process, the individual values of the rater on each of the descriptors, and the weights apportioned to each of the six sections.

Some general comments based on the use of the evaluative criteria:

- The SMP Book 1 is harder to read and the word count is higher per page - about 350 as compared about 200 for the NEX Book 1. As such the “Readability” section score for the SMP Book 1 is lower than that of the NEX Book 1 (0.65 for SMP Book 1 as compared to 0.725 for NEX Book 1).

- The content is organized better in the NEX Book1 as compared to the SMP Book 1. There is a more direct approach to the concepts in NEX Book 1. Also, there are sufficient practice exercises after the discussion of each subtopic to develop understanding. As such the score for NEX Book 1 on "Organisation of Student Textbook" is higher than the SMP Book 1 (1.00for NEX Book 1 as compared to 0.856 for the SMP Book1)

Table 2: Ratings on evaluative criteria for SMP Book 1 and NEX Book 1

Criteria	Weight (%)	SMP Book 1 Descriptor Rating	Section Rating	NEX Book 1 Descriptor Rating	Section Rating
Readability	15	4.5 + 4 + 4.5 = 13	0.65	5 + 4.5 + 5 = 14.5	0.725
Organization of Student Textbook	25	A = 3+ 4.5 + 3+ 4.5 + 3 + 4 + 4 + 5 + 2 + 2.5 = 35.5 B = 4 + 3.5 + 3.5 + 1 + 5 = 17 C = 2 + 5 + 5 + 2 + 2 = 16	0.856	A = 5 + 5 + 5 + 5 + 3 + 4 + 5 + 5 + 3 + 3 = 43 B = 5 + 4 + 4 + 2 + 5 = 20 C = 2 + 5 + 5 + 4 + 1 = 17	1.00
Physical Characteristics	15	5 + 4.5 + 5 + 3 + 4 + 5 + 5 + 5 = 36.5	0.684	4 + 5 + 5 + 5 + 4.5 + 4.5 + 5 + 4 = 37	0.694
Content	25	A = 3 + 3 + 1.5 + 1 + 5 + 4 + 4 = 4 + 1 + 1 = 27.5	0.6875	A = 5 + 4.5 + 4 + 3.5 + 4 + 5 + 4 + 5 + 1 + 5 = 41	1.025
Auxiliary Materials	10	A = 1 + 3 + 1 + 2 + 1 + 3.5 + 1 = 12.5 B = 3 + 4.5 + 1 + 5 = 13.5	0.236	A = 1 + 4 + 1 + 3 + 1 + 2+ 1 = 13 B = 3.5 + 3 + 3 + 5 = 14.5	0.25
Special Considerations	10	4 + 2 + 3.5 = 9.5	.317	4.5 + 2 + 4 = 10.5	0.35
			3.43		4.045

- Both books have a reasonable size. While SMP Book 1 is a hard-cover book, NEX Book 1 is a soft-cover book. The NEX Book 1 being more recent, it has benefitted from modern printing facilities and is well-enhanced with different colours, unlike the SMP Book 1. The ratings for "Physical Characteristics" is also higher for the NEX Book 1(0.684) as compared to the SMP Book 1 (0.694).
- The content in the NEX Book1 is sequenced better and there is an emphasis on problem solving. The use of a calculator is advocated. The SMP Book 1 adopts a more

conceptual approach by focusing on an inquiry-*cum*-discovery approach. The NEX Book 1 is given a higher score of 1.025 on "Content" as compared to a score of 0.6875 for the SMP Book 1.

- The SMP Book 1 is supplemented with a teacher's guide unlike the NEX Book 1. However, the NEX Book 1 discusses students' errors, has more worked examples and caters for individual differences when compared to the SMP Book 1. The score for SMP Book 1 is 0.236 and that for NEX Book is 0.25 on the criterion "Auxiliary Materials".
- The NEX Book 1 caters for a diverse population and is sensitive to issues such as gender in a much more significant way as compared to the SMP Book 1. The score for the SMP Book1 on "Special Considerations" is 0.317 compared to 0.35 for the NEX Book 1.

It should be noted that the scores for the NEX Book1 is higher than those of the SMP Book1 on each of six evaluative criteria. The ratings are very much dependent on who is the rater, accordingly different stakeholders would rate the textbooks differently. A score for rating the textbook can provide a general view but in no way is a definite answer to the question: "Which textbook is better?" The VCTM 1980 evaluative criteria were developed much later than the publication of the SMP textbook. A textbook is not the curriculum although it is a product of the curriculum. As such any judgment about the textbook is not and should not be the only criteria for an evaluation of the curriculum. Textbooks are products of their times. There are things which are valued at one point in time in a textbook and not so much at another - applicable to the two textbooks discussed in this study.

CONCLUSION

The focus of this study was not so much on the rating scores for the textbook but more on the instrument from VCTM published in 1980. This instrument is quite simple to use and depending on what is valued the weights can be adjusted to suit the preferences of particular raters. Unfortunately, a textbook cannot be rated by only one person (as carried out by me in this case). A textbook that comes out of the curriculum development process is looked at differently by different stakeholders which includes not only teachers and students but also others such as school leaders, inspectors and significant others such as parents who use the books to help their kids. Some questions worth looking at in the evaluation of a textbook include the following: How well does the textbook represent the curriculum? How good is it as a teaching resource? How good is it as a learning tool? How good is it as a resource for other stakeholders who wish to help the students? Do we have the same set of rating criteria for all stakeholders? Can we create a composite score based on the evaluation by each of the different stakeholders?

The VCTM 1980 is not a perfect instrument for rating textbooks, however, it is simple enough to be used easily by several stakeholders for some quick feedback on textbooks published at about the same time (unlike in this case where the publication dates differed by more than 40 years). There are several criteria that might be considered in the review of a textbook. Howson (2013, pp. 653-654) has proposed the following: (1) Mathematical coherence, (2) Clarity and accuracy of explanations, (3) Clarity in the presentation of kernels, (4) The range, quantity and quality of the exercises, (5) The connections with real-life and with other curricular subjects displayed in the explanations and the exercises, (6) Gender, racial and other social balance, (7) The use of appropriate language and the development of language reading skills, (8) Evidence that research results and accrued professional experience have been taken into account, (9) Provision for the differing abilities of the students who will use the text, (10) The physical attractiveness of the texts: format, type, colour, illustrations, (11) Some signs of originality in material, examples or form, and (12) The provision of teachers' guides that go beyond answer books and balance the twin demands of developing the teachers' mathematical understanding and assisting the management of lessons.

The textbook continues to be a major influence in the classroom: in many cases it still effectively determines the curriculum. (Howson, Keitel, & Kilpatrick, 1981). In trying to make a valid judgement about textbooks, stakeholders can use the criteria described above by Howson (2013) to supplement those mentioned in the VCTM 1980 rating instrument. It must be noted here that this may not be a simple process.

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RME AS A TEACHING APPROACH: A CASE STUDY OF ELEMENTARY GEOMETRY IN A SERBIAN INNOVATIVE 4TH-GRADE TEXTBOOK

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The aim of our research paper is to present the results of an experimental research on the effects of learning geometry with an innovative model of 4th-grade textbook as our practical implementation supporting the constructivist approach to teaching RME (Realistic Mathematics Education). We have examined the ways in which the innovative elementary textbook influences students' achievements (according to the levels of knowledge) and reasoning. Results confirm the general hypothesis that introducing an innovative textbook positively influences 4th-grade students' achievements in geometry and in this way we confirm the effect of this teaching approach to RME. We give an overview of the statistical analysis of the knowledge tests results. Comparing the experimental (N= 73) and control (N=75) groups with respect to the achievement in the two tests, we found the expected increase in the mean values of student achievement in the experimental group. This difference indicates the presence of a positive effect of the innovative textbook on students' achievement in elementary geometry. The results of variance analysis for repeated measures (ANOVA) showed that there was interaction between the pre-test–retest groups when we look at the overall test achievement (F (1,123) = 36.42, p <0.05). The difference between the pre-test and retest for both control and experimental groups was statistically significant. Appropriate structuring of a textbook can yield the effect of activating students' potential for learning geometry. Diversity applied in the textbook elicits students' opinion, encourages the development of students' geometry skills and helps students reach higher levels of knowledge and forms of geometry reasoning.

Keywords: innovative textbooks, Realistic Mathematics Education (RME), geometry, elementary geometry, levels of knowledge, mathematical reasoning, Serbia

INTRODUCTION

As a resource that provides support to the teaching process, textbook has recently become the centre of attention among researchers in mathematics education. In that sense, the traditional teaching approach is recognised as adjustment to the requirements of student testing instead of serving study purposes, just as a textbook should (especially because the textbook is considered one of the main carriers of educational changes). Apart from the results and analyses in international studies of student achievement in mathematics (for example, TIMSS, which points to the importance of textbooks and their use in the teaching process), textbooks too represent the basis for planning teaching and exerting a strong influence on the content of learning and the way it occurs. Fan et al. provide an overview of research on mathematics textbooks in scientific publications in the last six decades, particularly focusing on the idea of textbook development and future directions of its research and emphasizing the need for experimental research on how students use textbooks (Fan et al., 2013).

We follow the ideas of the great classic Poincaré related to human experience and the establishment of geometry, the relation between geometrical and physical space (Poincaré, 1905), and an understanding of space through a balance between the knowledge of geometric space and intuitive understanding of space (Djokic, 2006). An increasing number of mathematics curricula focuses on the development of spatial reasoning through instruction supported by the mathematics textbook. Therefore, the important issues we are dealing with are *the concept of space and spatial reasoning* (Clements & Battista, 1992), which also implies an important standpoint on the teaching of geometry. Researchers in mathematics education emphasize the importance of textbooks from the perspective of research on geometrical thinking as an important tool in teaching and learning. It is, therefore, important to encourage the use of textbook for the development of geometrical thinking, i.e. for the development of children's 'mental routine' in geometrical reasoning which leads toward systematic thinking (Gutierrez et al., 2005, Hershkowitz, 1998, Diezmann et al., 2002).

BACKGROUND

The research focuses on the levels of students' knowledge applied to tasks: 1. facts and information (from 1.a. recognition and reproduction of knowledge to 1.b. understanding of concepts / procedures) and 2. application of knowledge (in 2.a. mathematical context, and 2.b.1. semi-real and 2.b.2. real context or real problem tasks). The systematization and classification of requirements or levels of knowledge basically included Bloom's revised taxonomy (Krathwohl, 2001), which we reduced for primary instruction from six to three levels and where we incorporated three different paradigms of practice exercises in mathematical, semi-real and real contexts (Pepin, 2008). By analysing new textbooks in Serbia in the content domain geometry we found great inconsistency in the tasks related to knowledge application. There are frequent errors in real-context tasks, which leave an open question regarding students' preparedness for this level of knowledge (Djokic, 2013). The mainstay of the chosen teaching approach lies in the theory of Realistic Mathematics Education RME in which the learning process is based on the idea of mathematics as a guided process of (re)discovery of mathematical ideas with the main objective of understanding the procedure of mathematization (Van den Heuvel-Panhuizen 2002, Micic, 1999). An innovative model of textbook is seen as a practical implementation supported by the constructivist approach to teaching in the 'realistic context' (Milinkovic et al., 2008). Appropriate structuring of textbook units can help achieve the effect of activating students' potential for learning mathematical concepts / procedures and problem solving (Djokic, 2013). Differentiation reached in the textbook develops students' thinking, encourages the development of mathematical skills and helps students reach higher levels of knowledge and shape their mathematical reasoning. We have proposed structuring of textbook units through the following segments:

- activation of attention and a motivational task - at the beginning of each chapter in the textbook there is an announcement of what students will learn and every chapter in the book begins with a problem situation, or the so-called motivational task; solving the task itself comes only at the end of the chapter when students have sufficiently expanded their knowledge;

- concept / procedure problematization - the student is directed to discover a phenomenon (idea or procedure) or is getting guidance in the way of thinking in problem situations; this approach is based on introducing mathematical contents through analyzing and solving selected model tasks (problem situations), which are the basis of cognizance;
- practising and memorizing through problem solving - a variety of contents that new knowledge is related to in order to allow a more flexible application of knowledge in new contexts; exercises that are intended for practising and memorizing are diverse in complexity, in the variety of choice of representation and in the form;
- differentiation - content that is compulsory for all is highlighted (clearly labelled parts) with a vast variety of exercises for which no additional content learning is necessary; the structure of differentiated exercises helps students reach higher levels of knowledge;
- broadening and enriching knowledge - activities are initiated that go beyond the textbook and the school situation; this is how specific textbook sections were introduced, such as: Did you know ..., This is math too, From the history of mathematics, Research task, Learning from the Internet;
- self-evalutation - at the end of every chapter there is a summary of the lessons learned and an opportunity for students' self-evaluation, where students can check their solutions to the exercises and find periodical highlights of the crucial textbook contents, these being combined into a system of knowledge.

METHODS

We conducted the three-month survey in a city school that uses the traditional textbook of mathematics, and the sample consisted of fourth grade students (aged 9.5-10.5 years). In the experimental group, there were 73 students in the control group 75. *Data acquisition* was carried out by means of tests, questionnaires, and classroom observations (more details in: Djokic, 2013). After the conducted testing of students' geometrical skills for the purpose of balancing the experimental and control groups and the knowledge pre-test, six students were excluded from subsequent stages of research for the experimental group (N = 67). After this, a two-week experimental programme began. After two weeks, a retest was done on the basis of a test similar in form to that used in the pre-test. The aim of the research is to investigate how a 4th-grade textbook innovative in geometry can affect students' achievement (the cognitive aspect includes student achievement and reasoning). In the experimental group the innovative textbook was introduced as the independent variable. The dependent variable is the students' performance expressed through the difference in their achievement on the knowledge tests (pre-test and retest), stated upon the teaching cycle devoted to the textbook topic *Cuboid and Cube*. The data analysis was done based on the knowledge test results with the aim to compare students' achievement according to the levels and structured as task types 1. and 2. (i.e. 1.a. and 1.b., and 2.a, 2.b.1. and 2.b.2.). Based on the above said, the following *research questions* were formulated: 1) How does the introduction of an innovative textbook affect student achievement in geometry? 2) What will be the average level of geometry knowledge shown by the learners using the innovative model in comparison to students who are learning the traditional textbook? In the experimental study, we started from the following *hypotheses*: 1) The innovative model of math textbook will have a positive impact on student achievement in fourth grade geometry and 2) The difference in student achievement found between these two groups will be statistically significant.

RESULTS

When we compare the arithmetic mean values of the pre-test average results according to task types, we see that they are relatively similar ($M_{E(p)}$=0,37 and $SD_{E(p)}$=0,13 and $M_{C(p)}$=0,38 and $SD_{C(p)}$=0,15), while the values for the retest are quite different – for the experimental group the arithmetic mean is significantly higher ($M_{E(r)}$=0,43 and $SD_{E(r)}$=0,19 and $M_{C(r)}$=0,28 and $SD_{C(r)}$=0,15). We notice that the mean value of the retest averaged results according to the task type for the control group decreased compared to the pre-test. Comparing the two groups with regard to the overall achievement on the two tests we obtained the expected increase in the mean values of student achievement for the experimental group. The difference suggests that there is an effect of the applied innovative textbooks model on student achievement in geometry. Table 1 shows the results of *analysis of variance for repeated measuring* (ANOVA), which showed that there was an interaction between the pre-test and retest factors and groups when the overall student achievement on the complete test is observed ($F (1,123) = 36.42$, $p <0, 05$). The found difference is statistically significant.

Table 1: The significance of differences between the two groups on the pre-test and retest for the overall student achievement

	Sum of Squares	df	Mean Square	F	Sig.
Intercept	31,93	1	31,93	940,19	0,00
Group	0,49	1	0,49	14,52	0,00
Error	4,18	123	0,03		
Pre-test_retest	0,02	1	0,02	1,26	0,26
Pre-test_Retest*Group	0,46	1	0,46	36,42	0,00
Error	1,57	123	0,01		

Therefore, there is a statistical confirmation of the effect of the independent variable – innovative textbook on the dependent variable – student achievement. The experimental group made significant progress. Figure 1 of the same analysis shows what kind of effect was achieved through the experimental programme. It is noticeable that the experimental group improved significantly, while the students from the control group showed poorer results on the retest compared to the pre-test.

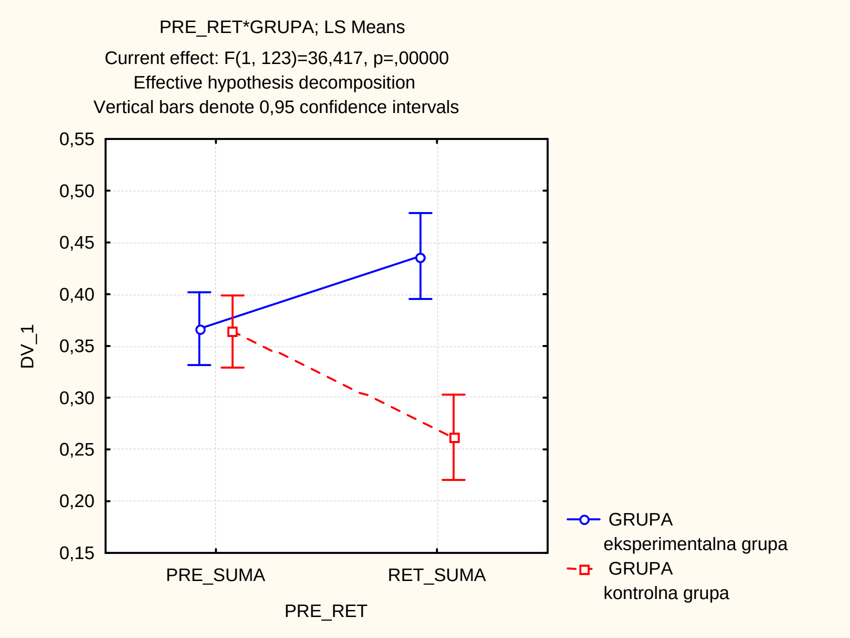

Figure 1. The effect of the experimental programme - average values of student achievement on the pre-test and retest for the experimental and control groups

DISCUSSION

Speaking about the benefits of the innovative textbook applied, we can say that students achieve better outcomes. Classroom observation recorded notably higher student motivation for learning geometry from the innovative textbook. The assumption remains that, if applied on a long-term bases, such a programme could yield even better effects, since learning occurs gradually as a result of active student engagement structured through activities that support the development of geometrical thinking, from informal to more formal ideas in a process of geometrical reasoning. We raise new questions for further research in geometry teaching, which is increasingly turning its focus to the development of spatial reasoning skills, and improving one's experience in measuring length, area and volume, especially at the initial levels of teaching. We also raise interest in the question of studying space as a model for teaching geometry in textbooks (throughout the primary cycle of education) following the great ideas of Poincaré through the idea of spatial reasoning in geometrically structured activities.

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TECHNOLOGICAL RESOURCES THAT COME WITH MATHEMATICS TEXTBOOKS: POTENTIALS AND CONSTRAINTS[1]

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Current textbooks come with technological resources (CD-ROMs and web portals) containing materials addressed to teachers and students. Some of these materials are independent of a specific textbook and can support a planned sequence of instruction designed by the teacher. They are currently available in web sites where the teacher, on his or her virtual workspace, can create and store lesson plans that incorporate these resources. These CD-ROMs and web portals are essentially composed of videos, applets, quizzes, texts and audio texts. This paper aims to discuss the role that technological resources, which come with textbooks, play in teaching. Based on the activity theory we aim to describe how these resources, as mediating artefacts, promote students learning. Empirical data were collected in classes of secondary school students focusing on topics related to functions and geometry. The results point to the occurrence of significant learning despite some technological constraints related both, to schools and to the proposals designed within the technological resources environments.

Keywords: technology, activity theory, functions, geometry

INTRODUCTION

The use of technological tools in the context of the classroom should take into account aspects related to curriculum development, learning and use of technology (Domingos Carvalho Costa, Teixeira, & Matos, 2008). In curricular dimension we assume that the curricula may be appropriate in different ways. We start from the categorization of Gimeno-Sacristan (1998) which defines six levels of curricular decision: the prescribed curricula (the curricula decided by the central government), the curricula presented to the teachers (through mediators, mainly textbooks), the curricula modelled by teachers (which is the result of the representations of teachers about different levels of curriculum decision), the curricula enacted in class (taking place in the classroom, operationalising the perception of teachers on the curricula modelled), the curricula realized (witnessed by outsiders) and the curricula evaluated (subject to external assessment) . Based on the curriculum presented to teachers, through textbooks and electronic materials that accompanying them, the aim is to explain how the modelled and in action curricula become learning tools. In Portugal there is a strong tradition of using textbooks that are accompanied by proposals for integrating technological tools presented on CD-ROMs, web pages, learning platforms, among others. These materials consist of videos, applets, quizzes, texts and audio texts and integrate recently textbooks in digital format. The curriculum modelled by teachers is based mainly on the use of the textbook adopted in paper format, and sometimes uses some of the technological tools that are associated with it, especially those that do not involve handling by the student (eg. ppoints). It is in this context that we resort to one of the technological mediators available by one of the publishing houses, “Escola Virtual” in order to understand the curricular dimensions related to modelled and in action curricula on student’s learning. It should be noted that the technological mediator used presents itself highly structured, following the sequence of learning presented in the textbook reproduced on paper.

THEORETICAL FRAMEWORK

The teaching and learning that takes place in environments using technology often involve a complex mathematical thinking. Sometimes this kind of thinking is seen from the cognitive point of view and has been designated by advanced mathematical thinking (Tall, 1991, 2007; Dreyfus, 1991). The processes of representation and abstraction allow us to move from one level of elementary to advanced mathematical thinking and when used in this sense are often mathematical and psychological processes at the same time.

The use of symbols is a key feature in the development of this kind of thinking and can be viewed with a double meaning, introducing some ambiguity between the procedure and the concept. This combination of procedural and conceptual thoughts is called proceptual thinking (Gray & Tall, 1994), and it is characterized as the ability to manipulate the symbolism flexibly as process or concept, freely interchanging different symbolisms for the same object. It is proceptual thinking that gives great power through the flexible, ambiguous use of symbolism that represents the duality of process and concept using the same notation. Activity Theory initiated by Vygotsky and developed by Leont'ev, assuming its system of collective activity (object oriented and mediated by artifacts) as the unit of analysis, has been developed over three generations. Was initially based on the idea of mediation introduced by Vygotsky in his triangular model that becoming the triad subject - object - mediator artifact, leaving behind the separation between the person and the social environment (Engeström, 2001). In the second generation, centred Leont'ev the unit of analysis is no longer individual and now includes links to other areas involved in a collective activity system, focusing now on the interrelationships between individual objects and communities.

Based on the collective activity system and giving special attention to the role that the mediator artefacts play in the relationship between subject and object, it becomes crucial to address the concepts of instrumental and documental genesis. The instrumental genesis (Rabardel, 1995) involves two processes, instrumentalization and instrumentation, and these are processes that enable the development and evolution of the instruments. The documentational genesis is a construct that expands the concept of artifact by defining the notion of document as building utilization schemes in teachers' action mediated by didactical resources (Gueudet and Trouche, 2012).

In summary we use as theoretical support the social and cognitive dimensions of learning, mediated by technological artifacts framed in curricular dimensions of the curricula modelled by teachers and curricula in action.

METHODOLOGY

This study follows a methodology of qualitative nature and is based on two case studies. The case studies relate to two groups of secondary school students, whose textbook used belongs to the same publishing house who owns the technological materials under study. In both cases students use textbook daily and this was the first time that they used the electronic resource as a learning tool. Teachers consider that are technologically competent, but develop this type of activities sporadically. Each of the case studies involves several work sessions with the tool. One case is centered on the theme of functions and the other is in geometry.

The exploration of the content is based on the manipulation of the tool by students. These contents have not been addressed previously by the teacher. Thus, the process of instrumental genesis is started simultaneously with the learning process. Given the lack of personal computers, students are organized in groups of two, so that the entire class may have access to handling the electronic tool in use. The working sessions of the student groups were recorded by special software that records sound and all actions carried out on the computer screen.

LEARNING ENVIRONMENT

The case studies discussed here refer to the subject of Cartesian geometry and quadratic functions. The technological tool used has a similar structure in both topics. It begins by introducing the topic from a problem in context (Figure 1).

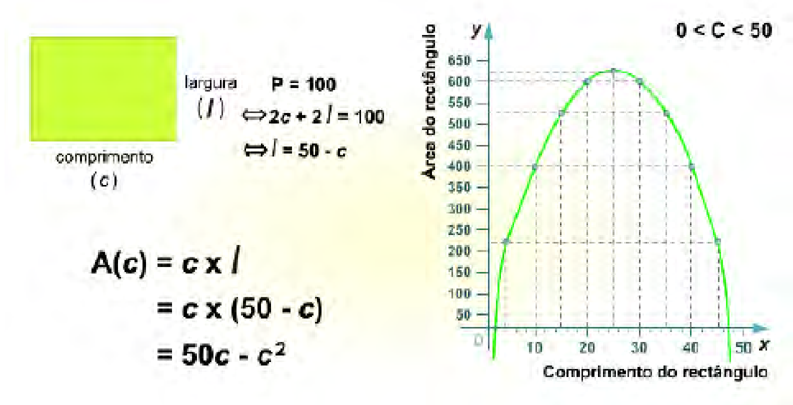

Figure 1 - Example of initial task.

This introduction is based on an audio presentation and accompanied by images that emerge on screen or even a video animation. Thereafter definitions of the concept under study are presented in audio format (Figure 2) together with visual representations, which are linked to the formal symbolism of the concept and sometimes followed for concrete examples. Audio texts are presented in formal language similar to that one reproduced in textbooks. This narration is matched by symbolic representations on the screen.

Figure 2 - Example of a task that involves definitions of concepts.

After defining the concepts application tasks are shown, where students can observe specific representations (graphs, plain domains, etc..) to which should match the respective algebraic representation (often chosen from a set of given representations (Figure 3).

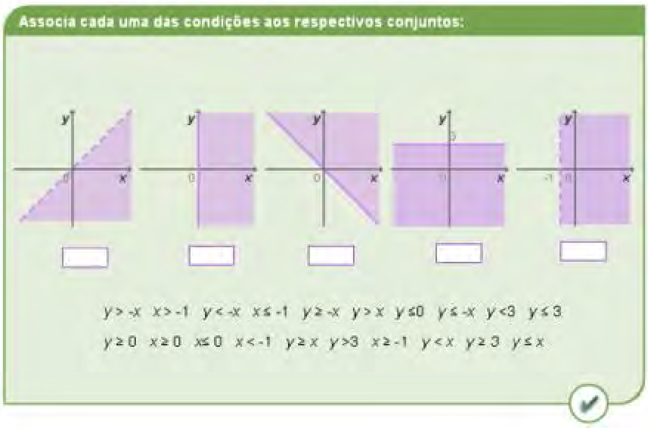

Figure 3 - Example of task into geometry.

There are also situations where students can carry out manipulations selectors in order to induce the role of the parameters involved (Figure 4).

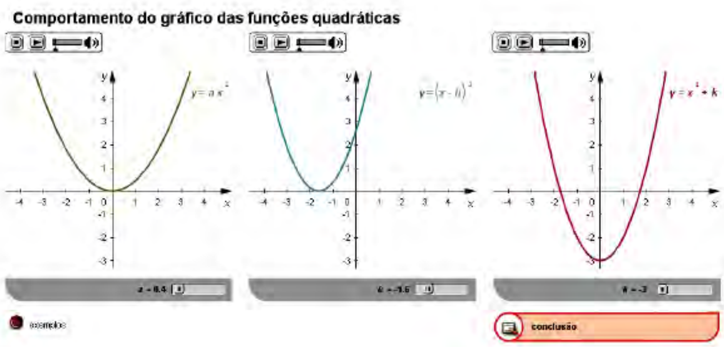

Figure 4 – An example of a task with functions.

The learning sequence presented in this technological environment reproduces a similar format to that which is transcribed in the student textbook, highlighting by the existence of audio texts, videos and interactive applets and exercises to consolidate the studied concepts. These exercises are self-correcting or allow a check of the answers without teacher intervention.

IMPLEMENTATION OF THE LEARNING ENVIRONMENT

In both case studies, students worked in groups of two per computer using a CD-ROM installed in the computer. As the tool used was very structured, teachers initially proposed that students would follow the structure presented on the CD and followed the interaction between students and tool, and between the tool and the group members. Since this is a first approach to the tool, the issues related to instrumental genesis were quickly overcome and the instrumentalization process was relatively short. Students have appropriated quickly the use of the tool proving to be more and more efficient in their handling.

With regard to the understanding of the concepts, students expressed some difficulties that were based primarily on the language used in audio texts and videos, which reproduced all the formalities involved in the concept and called to the symbolic representations that students sometimes did not dominate. These difficulties in understanding the concepts have led students to repeat several times the same video or audio text. Often times this situation was only overcome with help from the teacher. In order to overcome these difficulties the teacher had to intervene on the proposal didactical sequence through two approaches: changing the sequence of proposed learning designed and preparing intermediate proposals based on documents produced specifically for this purpose. This need for intervention in the curricula modelled, induced from the curricula enacted in class, shows us how monitoring and integration of several curricular dimensions can be crucial to promote learning and lead students to develop their mathematical thinking. It should also be noted that even when computational tools are used the documentational genesis becomes a powerful artefact to develop schemes that promote student’s learning.

The role of instrumentation was also revealed in the approach to tasks involving the manipulation of different representations of the same concept. This is the case of the study of parameters in the quadratic function (figure 4), where the manipulation of selectors provided a better understanding of algebraic and graphical representations, helping students to develop their proceptual thinking. The resolution of quizzes and other self-correcting tasks provided to the students a larger interaction with this kind of tasks, and sometimes its resolution was addressed in competition between groups or group members. The existence of such tasks, with the possibility to get a score depending on the hits performed, became an added value for the establishment of a competition between groups and thereby improved performance of algebraic procedures even when these are routine.

The documentational genesis was even developed by the teacher in order to provide students with resources and materials that could be retrieved later, in situations that they could not use the electronic tool. This approach proved to be an added value for the way students completed their written documentation, but essentially as an element of reflection and systematization of the concepts covered. It was thus possible to observe several learning situations involving the transition from a procedural thinking to a proceptual thinking. This transition was mediated both, by technological artifact or by documents produced by the teacher.

CONCLUSIONS

The interaction between students and mathematical concepts mediated by technological tools can constitute a rich learning environment. In this paper we have shown the way a technological tool constituted a mediating artefact in learning geometric and functional concepts.

We highlight the curricular dimension of the artifact that appears highly structured and close to the teaching sequence presented in some textbooks. We found that the relationship between the presented curriculum (tool) and the curriculum enacted in class are central to the teacher's intervention in the definition of the modelled curricula through an approach based on documentational genesis.

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PEDAGOGICAL AND CURRICULAR DECISION-MAKING AS PERSONALISED TEXTBOOK DEVELOPMENT

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The increase in expectations of teachers in England and Wales to differentiate mathematics learning for students and to personalise mathematics progression trajectories for students, either through the Teachers' Standards (DfE 2011), the inspection regime in England and Wales (Ofsted 2012) or recent initiatives from the Department for Education such as the 'Pupil Premium' (IFS 2010), can result in teachers in both primary and secondary schools utilising a range of resources to develop a 'personalised mathematics curriculum' for students in schools in England and Wales. Drawing on evidence from Remillard's (2005) study on the use of curriculum materials in differing contexts in the US and Clements' and Samara's (2004) theoretical examination of learning trajectories in mathematics education, we examine evidence from two experienced teachers who utilise established banks of mathematics resources for teaching the mathematics curriculum, present a conceptualisation of the curriculum as a 'personalised learning trajectory' for students and suggest this as an increasing trend in mathematics teaching in both primary and secondary schools in England and Wales. In presenting this evidence, we argue that this type of use of established curriculum materials by experienced teachers, in which there are successive implementations of the curriculum with individual students or groups of students, results in refining of the 'personalised textbook'.

Keywords: curricular decision-making, learning trajectories, personalised textbook, England

INTRODUCTION

This paper argues the case that the selection and use of specific mathematics resources from established resource banks by two teachers in localised classroom settings in England, and the repeated refinement of these resources, lends itself to the design of 'personalised textbooks' in which learning trajectories for individual students or groups of students are created. In England and Wales, it is increasingly usual for students to be grouped by attainment level in both primary (5-11 years) and secondary (11-16 years) schools (Boaler, Wiliam and Brown 2000, Hallam 2002, Hallam, Ireson, Lister, Chaudhury and Davies 2003, Wiliam and Batholomew 2004). This is partly as a response to government initiatives which place pressure on both teachers and schools to provide evidence of individual progression in mathematical learning for their students (Ofsted 2012), to respond to individual needs of each student under the Every Child Matters act (DfES 2004), and to provide evidence of additional provision for children deemed to be in particular need through the Pupil Premium initiative (DfE 2011). While some teachers adhere to the curriculum offered by commercially-produced textbooks, many teachers choose to design their own curricular routes through learning for the students in their classes (Remillard 2005) in response to these initiatives.

Remillard (2005: 212) also claims that findings from studies relating the high levels of availability of new resources to teachers (usually electronically) coupled with increasing regulation of mathematics teaching practices through a single curriculum "have not been consolidated to produce reliable, theoretically grounded knowledge on teachers' interactions with curriculum materials that might guide future research or the design or implementation of curricula". This study builds on Remillard's argument that examining curriculum materials and the implemented curriculum in classrooms requires an understanding of how teachers use resources to construct the implemented curriculum at a local level.

METHODOLOGY

We used a case study (Yin 2009) and narrative approach (Moon 2010) to this research, since accessing evidence from teachers' direct accounts in an exploratory study allows us to examine the reasons underpinning these teachers' choices of materials in their classrooms. The data is taken from semi-structured interviews with two teachers who regularly use established commercial mathematics resources for teaching mathematics. One of the teachers is a primary teacher in the 7-11 age range and the other is a secondary teacher in the 11-16 age range in schools in the south of England.

FINDINGS

Using resources in the 7-11 age range in a primary school

The curriculum learning outcomes used by this primary teacher are predetermined by the medium or long term plan constructed collectively by the team of teachers teaching a specific age group in the school. These medium and long term plans are developed from the national curriculum. The Abacus (Pearson n.d.) scheme is available in the school and this is one of the sources the teacher draws upon in searching for appropriate resources for teaching and learning mathematics. The teacher uses the Abacus textbooks mainly for the investigation-type and application activities they provide rather than for consolidation of procedural practice in exercises of similar questions. However, the teacher was highly aware of the importance of reinforcing procedural learning and expected students to complete such exercises, but expressed a keenness to avoid these as they had a high "boredom factor", although on some occasions, she acknowledged that completing these exercises was something that the students enjoyed and could "get on with" while she attended to the learning needs of others or undertake some teaching with a focus group of students.

The range of resources drawn upon by this primary teacher is narrow but familiar. As well as the Abacus scheme, she uses the Hamilton Trust (n.d.) resources (weekly plans that support Abacus in more detail), familiar computer programmes and mathematics websites, and any resources that have been suggested by others, such as a mathematics advisor or a colleague. The Hamilton Trust is a UK charity which provides planning materials for primary teachers for most curriculum subjects. The main reason given by the teacher for using the Abacus scheme and the Hamilton Trust support is the pressure of time to resource creatively both the mathematics learning for students in her class and the learning in a range of other curriculum subjects. Hence, the teacher tended to draw constantly upon familiar mathematics resources.

Often, she knew precisely the activity to be used, as it was familiar to her and she was aware of how successful it can be in supporting the students' understanding and achievement of certain learning outcomes.

Student motivation was the teacher's highest priority in choosing a resource, as engaging the students in mathematics was very important to her. Once a resource was identified as motivational for the students, she checked it for suitability to meet the learning outcome for the planned lesson before including the resource in the lesson plan. Developing the range of resources used appears to be incidental to ensuring high levels of student engagement. New resources often came to her attention from outside sources, such as colleagues or a mathematics advisor, rather than the teacher actively seeking them. The impetus for the teacher proactively seeking a different resource was her own motivation to change her teaching. In this case, sometimes she skimmed the textbook and the sample lesson plans in order to find something more appropriate for student engagement or better suited to meeting the learning outcomes.

Using resources in the 11-16 age range in a secondary school

The mathematics resources used by the teacher in the 11-16 age range were initiated during the middle to late 1970s by a collective of mathematics teachers who had a shared political view of teaching and learning and schooling in general which was based (but not overtly expressed) in sociocultural learning theory, an understanding of the nature of mathematics which reflected the social construction of knowledge, and a vision of mathematics learning which encompassed the success of all learners. The resources were devised in the context of a particular setting in a very large inner city, with areas of significant disadvantage, in which motivation for learning mathematics in schools was depressed. The collective of teachers continued developing these resources during the decade of the 1980s with support from the regional authority responsible for education until these totalled approximately 2500. The design of tasks reflected the beliefs of individual teachers in the collective about the importance of specific aspects of mathematics. The tasks were trialled widely within the classrooms of teachers in the collective, and modified in response to these trials following meetings to agree these modifications. With the introduction of the national curriculum in 1988, each of the individual tasks was analysed with respect to its place in the national curriculum, with some tasks being modified, some being discarded and others retained, despite their lack of direct relevance to the national curriculum, because of the perceived importance to learning mathematics *per se*. Now made available electronically by the National STEM Centre in the UK (http://www.nationalstemcentre.org.uk), these resources are freely available to all teachers.

As stand-alone resources which reflect the scope of the national curriculum, these can be used in a classroom in a variety of ways. The case study teacher selects resources to form a set of related activities which reflect her connected view of mathematics learning. Students work through this set of resources either individually or as a group activity involving three or four students working collaboratively. For example, a set of activities might involve 1) an activity which exposes students to relative ratios (which may not necessarily involve calculation of these), 2) an activity on enlarging shapes, which draws on knowledge of ratio in using scale factors, and 3) an activity on relative ratios of lengths of sides of a triangles, as a precursor to exploring the ideas of trigonometry. These activities are selected using the teacher's knowledge of students' prior achievement in the particular areas of the mathematics curriculum, so groups of students may be working on varying sets of tasks at any one time, depending on their prior achievement, interests and knowledge. Individuals and groups are able to determine their own trajectory through the set of tasks, making connections, exploring ‘wrong routes’ and learning from these.

Instead of choosing a particular learning objective from the national curriculum and measuring students’ achievement of this, assessment of students becomes a record of their achievement in recognising and connecting the relationships underpinning the activities and how these are used and built upon in future mathematical learning. A summative record is kept, for individual students, of observed applications of specific learning objectives from the national curriculum.

DISCUSSION

In the case of the primary teacher, there is, inevitably, a learning trajectory intended by the authors of the Abacus resources which may or may not be followed by the class teacher. As Clement and Samara (2004: 84) argue, “The main theoretical claim is that such tasks will constitute a particularly efficacious educational program”. However, this may not be the best route in a particular context at a particular time and the primary teacher involved in this research recognises when changes to an activity or to a set of activities in a learning trajectory is required in her setting for her students. Utilising these understandings of the learning context, the teacher is able to alter, in either small or significant ways, these learning trajectories to better effect mathematical learning. Similarly, the secondary teacher has the freedom to determine particular learning trajectories at a local level and specific to individual students, which may be different to ways described by Clements and Samara (2004). Her use of resources to design learning trajectories enables the secondary teacher to provide situations for students in which they experience connected and multiple solutions to problems encountered. The collaborative interactions generated between students are extended to the teacher which, in turn, impact on the teachers’ knowledge of the students’ learning, with the outcome of altered learning trajectories. Thus, “the realised learning trajectory, the taken-as-shared practices and understandings, are emergent” (Clements and Samara 2004: 85).

In defining the ways in which teachers engage with the mathematics curriculum, Remillard (2005: 217) describes four types of teacher-curriculum activity, using factors such as teachers’ conceptions of the curriculum, teachers’ conceptions of their role, their view of the teacher-curriculum relationship, and epistemological influences. These four types span the range from “*following or subverting*”, through “*drawing on*” and “*interpreting*”, to “Participating with”. Although the two teachers in our study appear to have very different rationales for choosing curriculum resources in the way that they do to implement the national curriculum, there are underlying similarities in their actions and a consequential overlap in their categorisation within these types. In the case of the primary teacher, she aligns most closely with both the *drawing on* and *interpreting* categories because she uses pedagogical and logistical influences, such as behaviour management and students’ enjoyment, to determine her choice of resources for mathematics teaching and because she actively took into account her personal knowledge of her setting and her students with the intention of enhancing the implemented curriculum. Similarly, the secondary teacher interpreted and analysed the curriculum materials, using these flexibly to “unlock much of the curriculum potential embedded in the materials” (Remillard 2005: 220), hence demonstrating *interpreting* qualities. Additionally, though, the use of resources by the secondary teacher suggests a highly flexible approach which necessitates a dynamic interrelationship between the teacher, the resources, and the students' work with these. The teacher's engagement with the sociocultural underpinnings of the design of the resources and her responsiveness to students' work with the resources suggests a *participating with* category.

Remillard (2005: 234) argues that the relationship between teachers and curriculum resources, especially how teachers "read" these resources, is a "highly interactive and multifaceted activity". The accounts of the two teachers in our study underline how teachers utilise their perceptions of their students' needs and potential to design localised mathematics learning trajectories which may not have been considered by the authors of the resources. Refining and adapting resources for a specific learning setting is a natural activity for these two teachers. As we move to a new national curriculum in England and Wales, which places greater emphasis on potentially unfamiliar areas of the mathematics curriculum, and how to assess this, for primary teachers and the development of new national assessment structures for secondary school students, we envisage an increase in what Kincheloe and Steinberg (1998: 4) refer to as "unauthorised methods" for teachers' interactions with curriculum resources which reflect Remillard's (2005) description of teachers' *participatory* engagement with the curriculum.

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PRE-SERVICE MATHEMATICS TEACHERS' USE OF TEXTBOOKS IN CLASSROOMS IN ENGLAND

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The use of mathematics textbooks by pre-service teachers in English schools is an under-researched area. While systematic research exists in other countries (see, for example, Nicol and Crespo 2006), there is only anecdotal evidence, albeit significant, to suggest that the use of mathematics textbooks in English secondary schools is underpinned by a complex set of rationales. A purposive sample of 42 pre-service teachers on a one-year secondary mathematics postgraduate course in southern England, some on a 70% school-based route to teaching and others on a 90% school-based route to teaching was surveyed using a questionnaire with both closed and open-ended questions about their use of textbooks while on school-based teaching placements. Responses confirm the limited use of textbooks by these pre-service teachers with a range of reasons for choosing, or not, specific textbook examples. Arguing that use of textbooks is an outcome of cultural and political activity (Apple 1992, Pepin, Gueudet and Trouche 2013), this data is examined in the light of the English context in which mathematics textbooks are written in a commercially competitive environment, the cultural context of personalised and differentiated curricula, the demands of the inspection regime of teachers in England, and the rapidly changing governmental expectations of teachers of mathematics. In this paper we present the survey findings about the more detailed rationale for decision-making in relation to this limited use of the mathematics textbooks available to pre-service teachers.

Keywords: pre-service teachers, textbook use, cultural context of curricula, England

BACKGROUND

The introduction of a national curriculum in England and Wales in 1988, and the associated assessment structure, began a wave of revisions of this curriculum in response to varying political agendas over more than 25 years. Although originally promoted as the means to support students transferring between schools, in subsequent years the assessment-driven system lent itself to a means of assessing teacher quality. This extended to the framework of a central government-funded inspection agency, the Office for Standards in Education (Ofsted), which reports on quality of both teaching in schools and overall school achievement, with the authority to recommend school closures. Aligned with these assessment structures is a system of criterion-referenced Teachers' Standards (DfE 2011), against which both pre-service and in-service teachers are assessed.

These emphases are driven by both reactionary political motives and neoliberal market-focused principles. Such central control echoes Apple's (1986) claim that teachers' day-to-day work is being linked more often to structures which expect particular behavioural outcomes on their part and to the measurement of specific curricular outcomes which are considered to represent good management of learning in classrooms. Underpinning all this is the recognition that such central control of a curriculum is micro-managed at school and classroom level by teachers themselves. Indeed, Pepin et al (2013:685) argue that "Textbooks are commonly charged precisely with the role of translating policy into pedagogy".

There are no state-determined textbooks in England and Wales, as in some other countries, nor at regional level as described by Apple (1996). Textbooks in the UK are written by commercial organisations, often in partnership with examination boards, with competitive profit-making motives. Hence, the design of these is essentially standardised with the national curriculum, or a reflection of the examination syllabus, thus making them a representation of political ideology in an economic market (Apple, 1991). Indeed, each politically-driven revision of the national curriculum, or revision of examinations, results in a plethora of 'new' textbooks purporting to reflect these revisions. Therefore, the direct link identified by Apple (1986) between the use of textbooks and the teacher as a cultural and economic product of testing and competence measures is less overt in England and Wales. However, a review of textbooks used locally in the south of England suggests that they are uniform in providing a structure which reflects the political focus on, for example, a 'back-to-basics' curriculum or on applied (or 'functional') mathematics. This textbook structure follows a pattern of:

- explanation or description of the mathematics in question (for example, a pictorial diagram of representations of fractions);
- an example of how to present the mathematics (for example, how to represent a fraction diagrammatically);
- some questions in increasing difficulty to practice the mathematics described;
- occasionally, a context-situated problem to summarise learning.

Such a structure contradicts Pepin's (1999) claim that mathematics education in the UK is influenced by Mason et al (1984), Lerman (1986) and Ernest (1991) (as cited in Pepin, 1999) in promoting problem-solving and investigational activities as a means of 'doing' mathematics rather than learning mathematical content. The structure of more recently published textbooks suggests that Pepin's claim is of its time and her arguments may well be different in analysing textbooks written as a product of the promotion of a more rigorous approach to mathematics curriculum content.

There is little systematic research on the use of mathematics textbooks in English 11-18 schools, unlike in other countries (Nicol and Crespo, 2006). Given, also, the lack of direct connection between the political motivations for the prescribed national curriculum and examinations and the commercial and market-driven motivations for the development of textbooks in England and Wales, we were interested in how teachers used textbooks to teach the national curriculum. Our research question is thus: How do teachers use textbooks resources available to them to teach the national curriculum?

METHODS

This study used a small-scale purposive sample of post-graduate pre-service teachers, totalling 42 in number, who were training to teach mathematics for the 11-18 age-group at a higher education institution in the south of England. We analysed qualitative data from a survey undertaken by our pre-service teachers from two distinct cohorts, a group of 25 pre-service teachers undertaking a University-based route to teaching, spending two thirds of their course based in schools and 17 school-based pre-service teachers who accessed University teaching for approximately 15 days during their course. The survey was undertaken two thirds of the way through the course for both cohorts and hence represented the more generic use of textbooks in the mathematics departments within which each of the pre-service teachers was training at the time of the survey.

As an analytical framework, we used Pepin and Haggarty's (2001) description of the use of textbooks by teachers and pupils. From a review of the literature, they identify six main themes, namely:

- whether textbooks are actually used in the process of teaching and learning;
- how the authority of the textbook is viewed in the context of the classroom;
- where textbooks are used, who makes the decisions about use and whether these are used by the teacher or students;
- who makes the decisions about what textbooks are used for and what aspects of textbooks teachers value;
- how teachers mediate the textbook for learners;
- the culture of the classroom or educational institution and how this affects how textbooks are used in classrooms.

FINDINGS

These pre-service teachers indicated, in both the quantitative and qualitative elements of the survey, that their use of textbooks covered a wide range of purposes, in particular:

- to support their planning for teaching;
- as a source of example problems;
- for determining the sequencing of topics;
- to provide activities;
- as a resource for individual students.

Despite these surveys being undertaken with pre-service teachers with only approximately 24-36 weeks teaching experience, we found evidence of all six categories from Pepin's and Haggarty's (2001) analysis of experienced teachers in these pre-service teachers' responses, across both the school-based route to teaching and the University-based route. We elaborate on some of these here.

Use of textbooks

Pre-service teachers spanned the full range of possibilities within this theme. Some reported little textbook use with classes, for example: "only used once"; "have not used a textbook to plan any lesson so far"; "not seen them being used within teaching". Others reported more extensive use: "commonly used in lessons"; "great for lower ability classes"; " I don't use textbooks much but the more established members of staff use them for most lessons"; "work is based on these textbooks"; "I was told to use textbooks to help plan and to use as a resource".

The authority of the textbook in the classroom

The findings in this category indicate that, for many of these pre-service teachers, the textbook is seen as a source of authority, but with a narrow remit. Hence, many report using textbooks "for ideas on the order, the level and type of questions" and that they are "good for sublevels" indicating that they see the textbook as a source of authority in reflecting assessment of learning in the national curriculum. Many also report using it to "make the progression through the topic clearer" and that "books say what within ratio I need to do" and that textbooks "break down topics" to support their planning. The quantitative data indicates that 67% of pre-service teachers never follow the textbook page by page and a further 24% reported that they rarely do so. The qualitative data also reveals little support for a view of the textbook as having pedagogical authority and pre-service teachers are often critical of textbooks, "questions are not always appropriate/well-planned….level of differentiation … is not supported by textbooks". There were several comments that textbooks were "boring" and about a perceived lack of coherence in textbooks: "jumps about a lot"; "topics separated". One pre-service teacher was highly critical, suggesting that "the approach of most textbooks denies the fundamental interconnectedness of mathematics …. dull, bland, generic …. culturally narrow, hard to read". Seventy-three per cent of pre-service teachers indicated that they never followed the suggestions in, or even looked at, the teachers' guide to the textbook when planning lessons.

Decisions about use and whether use is by the teacher or students

Eighty-five per cent of the pre-service teachers reported that students never or rarely used their textbooks for homework. This is supported by the qualitative data which further explains that, generally, textbooks are kept in school and students are not allowed to take them home. It is very clear throughout all the responses that the use of a textbook, and the decisions about use, are made at the individual level of the teacher rather than at departmental level. This is evidenced through comments such as, "I use pages from different books to produce own resources", "tasks are selected for particular classes based on their skills/preferences/prior knowledge" and "varying use of textbook depending on topic and class".

Decisions about what textbooks are used for and the value of textbooks:

The qualitative survey data asked the pre-service teachers to indicate what they thought were two strengths of a textbook they used. These comments indicate that value is given to aspects of the structure and overview of the curriculum for both teachers and students, "pupils can see progression" and "books say within ratio what I need to do" as well as being a source of questions, "good for questions to practice algebra" and "good range of examples".

How teachers mediate the textbooks for learners:

The quantitative data indicates that the pre-service teachers span the full range of responses in terms of selecting what they see as important from the textbook and in terms of supplementing textbook material with other resources. Qualitative responses also suggest that pre-service teachers clearly see themselves as 'gatekeepers,' making choices as to what, how and when materials from textbooks are presented to learners: "no one textbook covers every topic [the] best way"; "I use pages from different books to produce [my] own resources";

"tasks are selected for particular classes based on their skills/preferences/prior knowledge"; "not a 'ready to use' resource as it stands".

Culture of the classroom and how this affects how textbooks are used

The culture of the classroom is implicit throughout the responses from these pre-service teachers. They appear to have a strong sense of responsibility for providing interesting, motivating and engaging lessons for students and for providing work that differentiates and often structures progression for the range of attainment within a class. This kind of classroom culture is supported by qualitative comments suggesting that pupils find textbooks boring, that they "do not provide enough differentiated tasks", that there are "not enough examples" and that "examples progress too quickly".

DISCUSSION

While this sample is very small and undertaken in a localised setting, it reflects the findings of a much larger, less localised, and more experienced, sample of teachers in Pepin's and Haggarty's (2001) study and therefore probably reflects the situation in schools in England more generally.

The under-use of mathematics textbooks by these pre-service teachers, in general, raises the question of how the national curriculum is transferred to the implemented curriculum in the classroom. The most frequent use of the textbook was as a source of consolidation of class work as assigned homework and, hence, not a direct function of the classroom curriculum. Most pre-service teachers worked in a mathematics department which used long-term and medium-term schemes of work for each age group, based on the mathematics national curriculum. Short-term planning (lesson-by-lesson) was usually the responsibility of the teacher and required differentiation for individual student achievement. Some pre-service teachers therefore used the textbook as a source of information about how to teach elements of the scheme of work. Others used them as a source of teaching ideas or for example lesson plans or of learning resources, either for activities or questions. Such varied use reflects the findings of Haggerty and Pepin (2002:572) who argue that teachers often use textbooks in ways not envisaged by their authors:

> The mediatory role of teachers extends beyond that of content selection and includes decisions about wider pedagogical issues. Indeed, the teacher may act as mediator of the authority of the text; mediator or provider of the meta-discourse of the text; mediator of the language and explanations of the text.

The self-determining practice of pre-service teachers in developing their own pedagogical approaches to teaching the national curriculum through a cursory use of textbooks raises the question of the extent to which textbooks are, indeed, the mediators between the policy-driven national curriculum and the implemented curriculum in the classroom, as suggested by Pepin et al (2013). However, Apple's (1986) argument that the use of textbooks is linked directly to the teacher, as a cultural and economic product of testing and competence measures, is just as likely to be evident in this under-use of textbooks, since these pre-service teachers are as equally a product of the evaluative regime as teachers who regularly use textbooks. The justifications of these pre-service teachers for their use of textbooks lie in aiming to develop pedagogical approaches which enhance student progress in learning and hence their own assessments as a beginning teacher. The textbook appears to be a means to an end for these pre-service teachers in mathematics departments in the south of England.

Ball and Cohen (1996) suggest that the use of textbooks and teachers' guides, as described in our study, implies that the nuances of the strengths and weaknesses of specific lesson plans and learning activities, and the modelling of mathematical processes, are concealed from the pre-service teachers. Therefore, as they select particular elements of a textbook, omit others, or supplement learning with additional materials, they impose their own pedagogical judgments over those of the textbook author. While some may argue that, in the case of specific English mathematics textbooks, this may be a good thing, in general we have to question whether pre-service teachers have sufficient experience to question the pedagogical implications of textbooks they reject in favour of their own approaches. The solution may lie, as Ball and Cohen (1996:7) suggest, in textbooks using a discussion of approaches to teaching and learning, so that teachers can make informed decisions and "more thoughtfully examine ways to present content and consider students' understanding in tandem, and learn about both". In light of the findings of this study, an approach we will take with our pre-service teachers in future is to examine textbooks for their strengths in the hope that the authors' pedagogical intentions become more overt.

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REFLECTIONS ON TRENDS IN MATHEMATICS EDUCATION IN BRAZIL SET IN THE CONTEXT OF TEXTBOOKS FOR TEACHING MATHEMATICS

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In this article we bring some reflections related to trends in mathematics education and their possible interrelationships with textbooks of the mathematics teaching into secondary education. To analyze the data we used content analysis by form themselves into a research methodology used to describe and interpret the contents of all kinds of documents and texts. In this analysis, we seek to describe qualitatively the data which helped us to reinterpret the messages as well as to achieve an understanding of their meanings at a level that goes beyond a common reading. Having the works listed, we put them according to the Trends highlighted by D'Ambrosio and Borba (2010): Mathematical Modelling, Use of Technology in Mathematics Education, Ethnomathematics, Philosophical Aspects, Historical and Political Perspectives of Mathematics Education. Those trends, in our understanding may be presented as supporting mathematical discipline working as a strategy in which students and teachers can foster discussions about the theories and practices.

Keywords: textbook analysis, secondary school, Brazil

INTRODUCTION

This article is inspired by a research conducted during the development of a discipline entitled “Special Topics in Mathematics Education: Classics of the North American Mathematics Education”. This discipline was offered to students of the Graduate Program in Mathematics Education UNESP, Rio Claro, São Paulo, Brazil, by Professor Beatriz D'Ambrosio University (Miami University of Ohio) who worked as a visiting faculty member of the programme during the first half of 2010. And, in view of the results in the above research, we bring some reflections related to trends in mathematics education and their possible interrelationships with textbooks of mathematics teaching at secondary level.

To analyze the data we used the content analysis by form themselves into a research methodology used to describe and interpret the contents of all kinds of documents and texts. Having at hand the results of the data collection, we determined a priori the categories constituted by the Trend pointed out by D’Ambrosio and Borba (2010): Mathematical Modelling, Use of Technology in Mathematics Education, Aspects of Philosophical, Historical Perspectives and Politics of the Mathematics Education. These trends, in our understanding may be presented as supporting mathematical discipline and should be further discussed at the level of secondary education and when possible could be included in the proposals of textbooks functioning as a strategy in which students and teachers could promote discussions about the theories and practices.

TRENDS IN EDUCATION MATHEMATICS IN THE CONTEXT OF MATHEMATICS TEXTBOOKS

The different comprehensions concerning to the term "Trends in Mathematics Education" guided the form to writing this article, in which we present reflections related to trend and their possible interrelations with the textbooks of Mathematics in Secondary School level, highlighting that textbooks through of time reveal themselves as an important instrument of debate and research for teaching mathematics.

According to Valente (2008), the history of mathematics education has always been linked at textbooks "From the Origins of their teaching as knowing technical-military, undergoing their ascendancy, namely, of scholastic general knowledge, the historical trajectory of constitution and development of school mathematics in Brazil can be to read in the textbooks" (Valente, 2008, p.3). In this debate, D'Ambrosio and Borba (2010) show that distinct areas of research in Brazil have followed the trends of the development of other countries and therefore were adapted and contributed to the emergence of new methodologies that were appropriate to specific conditions in level of Brazil. still, All historical experience that pervaded the educational process contributed to that different forms of work were been thinking and would culminated in Trends in Mathematics Education (ME), which has guided and must underpin the way teacher working his classes when they use or even when do not utilize the textbooks. Speaking of trends, is worth highlighting that the Trends in Mathematics Education have had their roots in the trends inherent in education, meaning to say that the trend term was extended to the field of MS characterized forms of work relating to the discipline of mathematics by helping with the work of the teacher and the processes of teaching and learning (Fiorentini, 1995).

We understand, therefore, that such forms of work has influenced and increasingly influencing the organization and proposed mathematical content for mathematics textbooks at secondary level. However relationship between what is learned and how if learns have generated and continues to generate debate, giving rise to the controversial curriculum proposals at with regard to teaching and learning of mathematics in Brazil. This dichotomy causes uncertainties regarding practice of the teacher, because the textbook it is not any reading, it is a source of research, a construction of the historical way of mathematics education, an acquis that can mobilize trends about methodologies for mathematics teaching. In this context some questions proposed by Valente (2008), make us speculate on how we can use the textbooks in favour of a more meaningful mathematics education?

These issues lead us to think the textbook as a source of innovative production, source of trends, transformational proposals that can be studied and practiced by the school community such as: Mathematical Modelling, Use of Technology in Mathematics Education, Philosophical Aspects, Historical Perspectives and Policy Education. Trends and teaching methods that which could help the teacher understand the derived of new connections established didactic and pedagogical changes, significantly influencing their way of teaching.

Mathematical Modelling, in this scenario, is characterized as a trend both at international and national level. Of a restricted way we might setting it to a method of research of the mathematics applied. However, we defend be this more than a teaching method, broadening its meaning to a socio-historical perspective. In that sense, researchers such as Barbosa (2001), D'Ambrosio and Borba (2010), argue emphatically that modelling in the educational context broadens the way the subject / thinking individual perceives the role of mathematics in society.

In that chaining, we defend the idea that modelling enables the student to make links with other areas of knowledge, having mathematics as a basic tool, the approach to this topic is as a pedagogical alternative to the teaching of mathematics, allowing the teacher to work with your student, problems that portray the reality, in addition to promoting discussions during all step the modelling process of the problem to be studied.

However, often, this aspect modeller does not comprise a common practice in the sphere of secondary education, because there are few books which bring a proposal for inclusion of new themes in the curriculum and these are generally considered supplementary textbooks. E.g. Búrigo et. al (2012) entitled "Mathematics in School: new content, new approaches".

We believe, therefore, that Mathematical Modelling could be better acknowledged and explored according with the proposal of textbooks , thus becoming a means for the student can understand how the concepts were constructed, how these relate to the real world and to other areas of science.

Technology, in turn comprising the scenario of trends, interacting intimately with other trends, is assuming diverse functions in research and society. Specifically to mathematics teaching, this has increasingly become as a major ally, but unfortunately has not been explored effectively in schools. In Brazil, many public schools today have a computer lab, but commonly it is not often used. The reasons are several: unprepared teachers, few machines, lack of support from school management and textbooks that do not include the use of technology. The fact is that the worst off are the students that needs to use technology, although some already do, but not in a school setting, which could contribute to the understanding of mathematical concepts and for their integration labour market, with a view to understanding the importance of using technology in school.

In the Mathematics textbooks, the political aspect is often ignored, forgetting the power it has mathematics in the context of teaching of the subject; in relation to this, Giroux (1997) notes that education is involved with social issues. It reflects the socio-historical moment in which we live and therefore we should not imagine that it is a language unrelated to educational scenario. in other words the political-pedagogical relationship with the mathematical is not neutral, it should be reflected in the ways that travels. These ideas meet with the Critical Mathematics Education, Trend that cares about the processes of teaching and learning mathematics in which the dialogue between teachers and students are situated in political and social context. Important fact, because arouses both students and teachers about the importance of mathematics in society. Both D'Ambrosio (1984) & Freire (1970) understand that an important task of Education focuses on promoting the involvement of students and teachers for a critical consciousness in the technological world in which they are inevitably compromised. In this perspective Skovsmose (2000, 2001) argues for the importance of creating conditions in the school environment for today's students can make decisions and acting on this world “mathematically formatted”.

As soon, it's possible can see that the textbook of mathematics and perspective of ME are intimately connected the trends associated with the conception of teaching and learning, with the purposes and values assigned to teaching math and with the teacher-student relationship. Conceptions which represent important elements for research and implications for teaching and learning mathematics discipline revealing how the teacher can come compose their classes.

METHODOLOGY AND DATA ANALYSIS

The methodology excelled the qualitative aspect in view of documentary analysis, considering this to be an important source in qualitative research, by supplementing information obtained by other techniques, in addition to revealing new aspects of a topic or problem.

Data collection was based initially on an interview conducted with some members of community academic (teachers and students) programme of the graduate in Mathematics Education at UNESP, Rio Claro city, Brazil. In a second step, we conducted a survey with teachers, students and former students of the Graduate Program in Mathematics Education UNESP Rio Claro and teachers linked to different Post-graduate programmes in Mathematics Education of Brazil, it is noteworthy that these teachers work both at level of secondary education and in higher education.

Having in hand the works listed by interviewees we started the process of to allocate them according to the trends pointed out by D’Ambrosio and Borba (2010).

Table 1 exemplifies the loom that moves and produces large tapestry of trends which constitutes the Brazilian Mathematics Education. The lines show each work related to Trends in Mathematics Education pointed out by D’Ambrosio and Borba (2010). The columns indicate the number of works by Trend.3.1. Confluências que se levantam a partir das Obras indicadas pelos entrevistados

Analysis of the corpus of the research conducted, represented in Table 1, we observe that of the fifty-three works listed by interviewees, the predominance of the historical perspective over others trends is notable, coming in second Policies of Mathematics Education, third the Philosophical Aspects and at the end, tied for indications works that discuss Technology and Modelling. All these trends in the opinion of the interviewees reveal themselves important in the educational setting, which seek to highlight the current proposals of the textbooks of mathematics, understanding that ME constitutes a field of consolidated knowledge in interdisciplinary environments, whose objective aims at seeking alternatives that walk toward innovation and improvement of the teaching and learning of mathematics, however not reveal significant changes in the reality of classrooms and effectively in the proposed textbooks.

Table 1: Separation of the works in accordance with trends in ME

Trends	MODELLING	TECHNOLOGY	PHILOSOPHICAL ASPECTS	HISTORICAL PERSPECTIVE	POLICIES OF MATHEMATICS EDUCATION
Modelagem Matemática no Ensino (BIEMBENGUT, HEIN, 2000)	■				
Modelagem Matemática (BASSANEZZI, 2002)	■				
Humans-With-Media and the Reorganization of Mathematical Thinking: information and communication technologies, modeling, experimentation and visualization (BORBA,VILLAREAL, 2005)	■	■			
Educação Matemática crítica: A questão da democracia (SKOVSMOSE, 2001)	■				■
Informática e educação Matemática (BORBA, PENTEADO, 2001)	■	■			
Pesquisa em Educação Matemática: concepções e perspectivas (BICUDO, 1999)	■	■	■	■	■
Computadores e Educação (PAPERT, 1985)		■			
Tecnologias da Inteligência (LÉVY, 1993)		■			
Os professores em tempo de mudança: O trabalho e a cultura dos professores na idade pós-moderna (HARGREAVES, 1998)		■			■
Etnomatemática: elo entre as tradições e a modernidade (D'AMBROSIO, 2001)			■	■	■
Na vida dez na escola zero (CARRAHER, CARRAHER, SCHLIEMANN, 2003)				■	
Etnomatemática: arte ou técnica de explicar ou conhecer (D'AMBROSIO, 1990)				■	
Relações Culturais entre Alemanha e Brasil: 'Imperialismo Cultural' versus 'Nacionalização' (SCHUBRING, 2003a)				■	■
Ethnomathematics and its place in the history and pedagogy of mathematics (D'AMBROSIO, 1985)				■	■
Exclusão e resistência: Educação Matemática e legitimidade cultural (KNIJNIK, 1996)					■
Matemática e Realidade (MACHADO, 2005)			■		■
Da Realidade à ação: reflexões sobre a educação Matemática (D'AMBROSIO, 1986)					■
Educação Matemática: da teoria a prática (D'AMBROSIO, 1996)					■
Fenomenologia e Relações Sociais (SCHUTZ, 1979)			■		
História, filosofia e sociologia da educação Matemática na formação do professor: um programa de pesquisa (MIGUEL, 2005)			■	■	
Social Constructivism as a Philosophy of Mathematics (ERNEST, 1998)			■		■
Pesquisando Práticas e Táticas em Educação Matemática (BOVO, GASPAROTO, ROTONDO, SOUZA, 2011)			■		■
Filosofia da Educação Matemática (BICUDO, GARNICA, 2001)			■	■	
Modelo dos Campos Semânticos e Educação Matemática - 20 anos de história (ANGELO, BARBOSA, SANTOS, DANTAS, OLIVEIRA, 2012)			■	■	
Efeitos de poder e verdade do discurso da educação Matemática (BAMPI, 1999)			■	■	■
Pesquisa em Educação Matemática e mentalidade bélica (MIGUEL, 2006)			■	■	
Maravilhas da Matemática: influência e função da Matemática nos conhecimentos humanos (LANCELOT, 1970)			■	■	
A educação Matemática: breve histórico, ações implementadas e questões sobre sua disciplinarização (MIGUEL, GARNICA, IGLIORI, D'AMBROSIO, 2004)			■	■	■
Vidas e Circunstâncias na Educação Matemática (VIANNA, 2000)			■	■	■
Educação como prática de liberdade (FREIRE, 1967)			■		■
Pedagogia do oprimido (FREIRE, 1987)			■		■
Pedagogia da esperança: um reencontro com a pedagogia do oprimido (FREIRE, 1992)			■		■
Pedagogia da autonomia, saberes necessários à prática educativa (FREIRE, 1996)			■		■
History in mathematics education (FAUVEL, VAN MAANEM, 2000)				■	
História na Educação Matemática: propostas e desafios (MIGUEL, MIORIM, 2005)				■	
O uso da história no ensino da Matemática reflexões teóricas e experiências (MENDES, 2001)				■	
Desenvolvimento histórico do conceito e do processo de aprendizagem, a partir de recentes concepções matemático-didáticas (erro, obstáculos, transposição) (SCHUBRING, 1998)				■	
A Noção de Multiplicação: Um "obstáculo" desconhecido na História da Matemática (SCHUBRING, 2002)				■	
Alguns modos de ver e conceber o ensino da Matemática no Brasil (FIORENTINI, 1995)				■	
Introdução à História da Educação Matemática (MIORIN, 1998)				■	
O homem que calculava (MELLO E SOUZA, 1994)				■	
Análise histórica de livros de Matemática: notas de aula (SCHUBRING, 2003)				■	
Alguns modos de ver e conceber o ensino da Matemática no Brasil (FIORENTINI, 1995)				■	
Rumos da pesquisa brasileira em educação Matemática: o caso da produção científica em cursos de pós-graduação (FIORENTINI, 1994)				■	
Conceitos Fundamentais da Matemática (CARAÇA, 2000)				■	
Educação Matemática: pesquisa em movimento (BICUDO, BORBA, 2004)				■	
Uma História da Matemática Escolar no Brasil (1730-1930) (VALENTE, 1999)				■	
A Matemática na Escola Secundária (ROXO,1937)				■	
Formação docente e profissional: formar-se para a mudança e a incerteza (IMBERNÓN, 2000)					■
TOTAL NUMBER OF INDICATIONS OF WORKS BY TREND	6	6	17	29	19

SOME CONSIDERATIONS

In this paper, we try to make some reflections related to the Trends in Mathematics Education and its interrelations with the textbooks. These reflections have emerged from research conducted in the discipline group Special Topics in Mathematics Education: Classics of Mathematics Education North American, which directed our attention to other perspectives, among these the importance of discussing such tendencies in the context of textbooks for teaching mathematics. With emphasis, the aim of this paper consists in to conjecture about the results found in the analysis of research about the importance of introducing topics related to mathematics education befitting to methodologies and didactic and pedagogical approaches in the context of targeted textbooks for secondary education.

One explanation is that in general, the mathematical discipline aims at training the student about the mathematical content, including mathematical problem solving, mathematical demonstrations, among other , while the interrelationships historical, social, economic and cultural perspective are almost entirely absent, in addition to the teaching methods discussed at both masters and doctoral research proposals are out of the textbooks. In this scenario, we believe in a closer relationship between mathematics and mathematics education by understanding that this fellowship makes possible students to form a broad and critical view of scientific knowledge in the context of teaching and learning mathematics.

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OPEN TASKS IN JAPANESE TEXTBOOKS: THE CASE OF GEOMETRY FOR LOWER SECONDARY SCHOOL

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From the early 1970s Japanese mathematics teaching has put particular emphasis on designing and implementing lessons in which students can explore different approaches and ways to solve given problems. This is generally known as the open-ended approach because the tasks tackled by students are 'open' to different solution strategies and approaches. The purpose of this paper is to report on the extent to which such an open approach is realised in current mathematics textbooks in Japan. Our focus is geometrical reasoning in lower secondary school, as this is one of the important topics in mathematics. In analysing the topic of angles in polygons, we found that open problems were utilised by Japanese textbook authors as worthy approaches which all teachers could take in everyday lessons on this topic. We further found that while each of the seven textbook series had undergone the same official authorisation process, the textbooks showed different approaches for the same geometry topic. This illustrates the variety of ways in which the open-ended approach can be enacted in the teaching of mathematics.

Keywords: open-ended approach, geometry, secondary school, Japan

INTRODUCTION

Beginning in the early 1970s Japanese mathematics teaching began putting particular emphasis on designing and implementing lessons in which students can explore different approaches and ways to get given problems. This has become generally known as the open-ended approach because the tasks tackled by students in such lessons are 'open' to different solution strategies and approaches (Shimada, 1977; Becker & Shimada, 1997). The pedagogical value of such an open approach is widely recognised in mathematics education research (e.g. Silver, 1997).

School mathematics textbooks are important objects for analysis as they represent the 'potentially implemented curriculum' (Valverde, et al, 2002) that influences the ways of teaching and learning of mathematics in everyday lessons. In Japan, textbooks may be published by private publishers but the textbooks need to reflect the official 'Course of Study' and the accompanying 'Teaching Guide', both published by the Ministry of Education and Science. What is more, all textbooks must pass through a textbook authorization process overseen by the Ministry of Education and Science, a process that can take about three years from initial development to classroom use (Shimizu & Watanabe, 2010). In practice, there are usually around seven different textbook series on offer from different publishers. The use of textbooks by teachers can vary, but textbooks are one of the most influential resources for planning and implementations in daily lessons in Japan (Sekiguchi, 2006), and it is therefore important to study textbooks in order to understand the complexities of mathematics lessons.

The purpose of this paper is to address the following research questions: To what extent are open approaches realised in current school mathematics textbooks in Japan?; Can we observe any different approaches among the textbooks published by the seven publishers? In this paper, we particularly focus on open approaches in angles in polygons, because a) it is one of the common geometrical topics in lower secondary schools internationally, and b) our preliminary analysis suggests that this is one of the topics in which open approaches are evident compared to topics such as proving.

OPEN APPROACHES IN GEOMETRY REASONING-AND-PROVING

By elaborating the original ideas proposed by Shimada and his colleagues between 1971-6 (Shimada, 1977), Becker and Shimada (1997) described “incomplete” or “open-ended” problems as those in which “students are asked to focus on and develop different method, ways, or approaches to getting an answer to a given problem” (p. 1). In this situation, methods for arriving at answers are seen as just as important as the actual answer. The approach has been found to be effective in not only raising the general level of students’ performance, but also in cultivating students’ mathematical thinking and creativity (e.g. Kwon, Park & Park, 2006). Our focus is geometrical reasoning in lower secondary schools, as this is one of the important topics in mathematics. In particular, in Japan geometry is used to introduce ideas of formal proving (Jones & Fujita, 2013; Fujita & Jones, 2014).

In order to conceptualise activities involved in proving in geometry, we refer to “reasoning-and-proving” (Stylianides, 2009, p. 259); that is, the classroom activities of “identifying patterns, making conjectures, providing non-proof arguments, and providing proofs”. We have already obtained an overview of G8 geometry content within this framework (see Fujita & Jones, 2014). Through our analysis, we reported that in G8 geometry lessons start from a problem solving situation, with the geometrical facts to be proved and learnt often coming later. A sequence from conjecturing to proving is prominent in the process of reasoning-and-proving in the textbook.

In this paper, we take a step further and consider how “open situations” are intended in the textbook in geometrical reasoning. By considering the activities identified by the reasoning-and-proving framework, ‘open’ approaches in geometry can be conceptualised as follows: (a) devising different ways to identify patterns, (b) devising different conjectures, (c) devising different methods of proving, (d) devising different methods of non-proof argument, (e) devising different new statements after proving a statement. We used this as an analytic framework and have conducted an analysis of the two chapters related to geometry in Tokyo Shoseki’s Mathematics G8 (textbook A), one of the most popular textbooks in among seven publishers. Table 1 summarises our analysis of 34 (+4 flexible) lesson chapters related to geometry (Ch. 4 and 5).

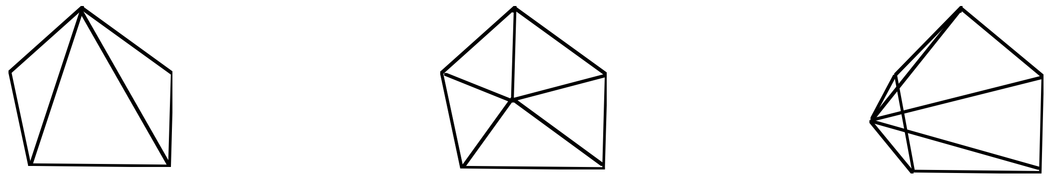
In Chapter 4 Section 1, students are particularly encouraged to devise different methods to identify the pattern (code (a)) or devise different ways of proof or non-proof arguments (codes (c) and (d)). For example, after the problem was introduced on p. 89, on p. 90, the following two different methods to find the sum are shown as examples (Fig. 1a & b). Also on p. 91 as an extension activity (Fig. 1c) is suggested.

Table 1: Suggested open activities in G8 textbook

	(a)	(b)	(c)	(d)	(e)
Ch. 4* Sec. 1	3		2	1	
Ch. 4* Sec. 2					
Ch. 5** Sec. 1		1			1
Ch. 5** Sec. 2		1	1		1

*Chapter 4. Parallelism and congruence (Section 1: Parallel lines and angles; Section 2: Congruent figures)

**Chapter 5. Triangles and quadrilaterals (Section 1: Triangles; Section 2: Parallelograms)



a b c

Figure 1: Different methods to find the sum of inner angles of polygons

For (c), different methods of proving the sum of inner angles of a triangle is 180 are shown on p. 99 and p. 100, and as a non-proof argument an alternative way of finding the sum of exterior angles of a polygon is shown.

OPEN APPROACHES FOR THE SUM OF POLYGON INNER ANGLES

We conducted further examinations of how the angles in polygons are taught across the seven textbooks. We found at least four different methods: (1) drawing lines from one vertex to the other (e.g. fig. 1a), (2) drawing lines from a point inside polygons (fig. 1b), (3) drawing lines from a point on one of the sides and (4) drawing lines from a point outside polygons (e.g. Fig 1c, although this method is unlikely to be considered by the majority of Grade 8 students).

Method (1) is used as an introductory example in the all seven textbooks. In these problem situations, cases for triangles, quadrilaterals, pentagons etc. are summarised in a table, and encourage students to inductively identify a pattern of the number of triangles in polygons (‘n-2’). Then ‘n-2’ is used as a premises to generalise the formula ‘180 x (n-2)’. This approach is used in the all seven textbooks. Also, this is an expected progression from primary school where students have already experienced investigating the angles in polygons. Their learning experience in primary schools is generalised through the case of the n-polygon, which is one of the aims of this lesson.

In contrast, treatments for (2)-(4) are quite diverse among the seven textbooks. For example, the method (2) appears as a main content except textbook D which treats the method (2) as an optional contents. This means a teacher using textbook D is not, on the one hand, expected to teach this method during the lesson. On the other hand, this does not necessarily mean that a learning opportunity with method (2) is missed (regardless the textbook design) because students can devise method (2) by themselves if the teacher encourage their students to think openly (Haneda, et al, 2001). The other treatments imply the method should be taught within a lesson. Also, textbook A uses a table so that students can inductively identify a pattern, but others do not use table but ask students to find the sum as an 'exercise'. The method (3) appears as a main contents in textbooks E, F and G, and as optional contents in A, C and D, but does not appear in textbook B. Also, textbook D asks students to examine the method (3) for only the case of hexagons, whereas A, C, E, F and G ask the case of n-polygon. Finally, the method (4) appears in only textbook A. When we refer to teacher's guide, this textbook tries to encourage students to see geometrical figures from a dynamic point of view. Also, the guide for textbook C suggests using dynamic geometry software as well.

In this problem, the number of triangles 'n-2' plays a key role to generalise the pattern, but the treatment of 'n-2' again differs in the textbooks. For example, some textbooks ask students to relate the numbers of vertices and triangles and to deduce the number of triangles when there are n vertices (textbook C). Textbooks B, E and F provide more detailed diagrams and ask students to explain why it is possible to divide an n-polygon into 'n-2' triangles by considering the number of diagonals drawn from a point. In contrast, textbooks A, D and G just show tables and ask students inductively to identify 'n-2' solely by looking at numerical values.

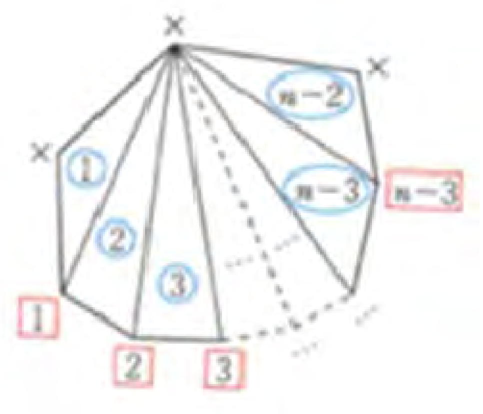


Figure 2: Deducing the number of triangles

Table 2: Open approaches for angles in polygons across the seven textbooks

	A	B	C	D	E	F	G
Method (1)	○	○	○	○	○	○	○
Method (2)	○	○	○	△	○	○	○
Method (3)	△	×	△	△	○	○	○
Method (4)	△	×	×	×	×	×	×
Deducing 'n-2' from diagrams	×	○	○	×	○	○	×

Table 2 summarises our findings in terms of approaches for the sum of the inner angles of triangles. ○, △ and × indicate 'appears in the main text', 'appears as optional', and 'does not appear', respectively. Note that we do not consider for example textbook D does not provide enough opportunities for open approaches for angles in polygons, but simply this table suggests that even the textbooks which have undergone the authorisation process show different approaches for the same topic in geometry. We discuss the implications of our findings in the next section.

DISCUSSION

Our analysis suggests that open approaches are particularly evident in identifying patterns. We then examined angles in polygons as an example of the open approach across the seven textbooks authorised by the Japanese Ministry of Education and Science. While all textbooks take the method to identify the formula for the sum of inner angles of polygons (method (1)), other methods are not always considered as the main topic for the lessons. However, at least the two methods (1) and (2) are 'visible' in the all textbooks. This shows how open approaches are recognised as worthy approaches which Japanese textbook authors consider that all teachers to take in everyday lessons.

One of the important purposes of G8 geometry is to introduce ideas of formal proving. Although in lessons which deal with the sum of inner angles of polygons students are not required to undertake formal proofs, it is important to provide learning opportunities to explain reasons why. Thus, some textbooks (B, E and F) explicitly ask students to explain why the number of triangles created by diagonals will be 'n-2' by relating to geometrical diagrams. Other textbooks just use tables and numerical values to find a pattern inductively. Which approach would be more appropriate for the foundation of geometrical reasoning? Again teachers who are aware of the importance of geometrical reasoning might relate the geometrical meaning of 'n-2' regardless the textbook design. In contrast, an explicit link between 'n-2' and geometrical diagrams in textbooks B, E and F might encourage teachers to teach this topic more 'conceptually' rather than 'procedurally' to find the formula.

For teachers it is always difficult to determine to what extent we should 'tell' to students. Chazan and Ball (1999, p. 10) argue the need for classroom-based research to identify and understand "what kind of 'telling' it was, what motivated this 'telling', and what the teacher thought the telling would do", together with ways of "probing the sense that different students make of different teacher moves".

CONCLUDING COMMENT

The main purpose of this topic is to find the sum of inner angles of polygons, and we want all students actively engage tasks and find different methods by themselves as much as possible. We, as educators, expect that textbooks give some insights for 'what kind of telling'. However, our analysis reveals even the authorised textbooks have varied views of 'what kind of telling' in open approaches in geometry. For textbook writers it is very difficult to decide to what extent textbooks should express their views how mathematical topics ought to be taught. Also, whereas some studies (e.g. Kwon, Park and Park, 2006) suggest that open approach might cultivate students' creative thinking, there is no guarantee teachers would follow the approaches suggested by the textbooks. For future research it might be particularly interesting to examine how this topic is actually taught in classrooms, and to what extent the ways in which teachers teach this topic are influenced by the textbooks that they use.

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THE DESIGN OF AND INTERACTION WITH E-TEXTBOOKS: A COLLECTIVE TEACHER ENGAGEMENT

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This article reports on an investigation of the design/re-design processes of a French grade 10 e-textbook (including its associated resources), which has been designed by the French Sésamath teacher association. These processes have been fostered by new 'digital' possibilities: platforms; discussion lists, etc. The object of the study has been the French Sésamath association, which mainly involved secondary school mathematics teachers designing various kinds of online teaching resources. The focus has been on their design of a Grade 10 e-textbook, more precisely the chapter on 'functions'. This design has been considered as a documentation work, and the analysis conducted with the theoretical perspective provided by Cultural-Historical Activity Theory. The authors studied the activity system of the e-textbook designer community, with particular focus on the resources, the rules and the division of labour in this system, their evolutions, and the factors of evolution. Three moments were identified, corresponding to three successive objects of the activity: a full-web e-textbook; interactive exercises; and finally a digital textbook. For lack of space, only one of these moments is presented, as an example. There is evidence that there were particular resources crucial for this collaborative work.

Keywords: e-textbook, teaching resources, textbook design, activity theory, collaborative work

INTRODUCTION

The traditional textbook has been recognized to play an important, perhaps crucial, role in teaching and learning (e.g. Pepin & Haggarty 2001). However, the e-textbook, as an emerging form of teaching material (including online materials and interactive functionalities), has been relatively sparsely investigated. One of the main issues is to determine, which specific features, different from the paper textbook, has an e-textbook.

In terms of teacher learning, textbooks are said to have a vital role to play. It has been found that textbook presentations are likely to influence teachers' beliefs and instructional practice (e.g. Nathan, Long, & Alibali 2002), and that they contributed to teachers' knowledge growth

(Ma 1999). Ball & Cohen (1996) emphasize the textbook's role in supporting teacher learning and professional development, yet studies exploring the role of e-textbooks in supportive teacher learning environments have been scarce.

In this paper we report on an investigation of the design/re-design processes of a grade 10 e-textbook, which has been designed by the French teacher association Sésamath. It is hypothesized that the digital means challenge in particular the usual divide between experts as designers of resources and teachers as users: design and implementation/use are now intertwined, with teachers intervening in both (Gueudet & Trouche 2009).

The research question studied here is: what are the design processes attached to the Sésamath e-textbook, and in which ways do the roles of group members evolve over the development of the textbook?

ANALYTICAL FRAMEWORK AND METHODOLOGY

In terms of context, our aim was to analyse the emergence of e-textbooks as new types of teaching resource systems, designed by a different kind of "authors" (i.e. mathematics teachers, whereas traditionally textbooks are designed by teacher educators, or inspectors), and to investigate the development of new kinds of interactions between authors and users. We chose the Sésamath association, whose members were developing such new types of resources. Sésamath is a French association of in-service mathematics teachers (mainly from middle schools, around hundred teachers, with a board of nine members) created in 2001, whose main goal is to "freely distribute resources for mathematics teaching": online exercises (Mathenpoche – standing for "mathematics in the pocket", MeP in the following); textbooks; and various software. These are designed by groups of teachers collaboratively working to develop/produce a given set of resources. Each project involves about fifty teachers, and the different project groups gather several thousand teachers sharing the same collaborative platform, who are not formally members of Sésamath, but interested in developing shared resources through the engagement with Sésamath.

We will focus in this paper on the design of a Sésamath e-textbook, choosing a moment of change between two edition models: the first one was a single static book, available both online (under a pdf, but also an odt format, allowing teachers to make modifications) and in hard copy, accompanied by separated animations online (in particular MeP exercises); the second one was a flexible and dynamic digital textbook, i.e. an e-textbook, which a teacher could organize according to his/her needs, with animations and extra exercises integrated in each chapter. After having successively published 'static' textbooks for grade 7 (2006), grade 8 (2007), grade 9 (2008) and finally grade 6 (2009),, Sésamath decided to design an e-textbook for grade 10 (first grade of high/upper secondary school in France), according to the second model, and gathered a group, named here *e-textcom*, for this purpose. We decided to follow, from June 2009 to December 2013, the design of a particular chapter, dedicated to functions, because this theme assembled diverse representations (graphs, tables of values, algebraic formulas) and offered possible links with the real world (in term of modelling), offering many opportunities for exchanging resources between e-textcom members.

Our investigations are anchored in the following data and their analyses: (1) for tracing the e-textcom activity, we have used its mailing list and the resources platform; (2) for tracing the interactions between the community and the members resources, we collected the designed resources shared by the members, during the realisation of the object; (3) for studying the interrelation between e-textcom and Sésamath board activity systems,, we used what we named a Small Agenda for Follow-up (SAF), filled in by two e-textcom members, chosen according to their role in the community (Benoît, Sésamath Board; Alexis, debates coordinator). Analysing these data, we identified different *moments* of the activity system transformation.

In terms of theoretical frameworks, we refer to Gueudet, Pepin, Sabra & Trouche (submitted) combining two frames: the documentational approach of didactics (Gueudet & Trouche 2009), in particular its meaning of resources, and the Cultural-Historical Activity Theory (CHAT) as defined by Engeström (2001).. Drawing on these we have analysed the evolution of the e-textcom activity system, focusing in particular on changes of the object of the activity (see Figure 1): we identified three successive objects of activity, each of them being associated to a particular *moment*. We show in the next section the activity system/s and its/their evolution during the first of these moments, and subsequently discuss the whole process.

THE FINDINGS: PRESENTATION OF ONEMOMENT IN THE EVOLUTION OF THE OBJECT

In June 2009, an initial group of 14 teachers, e-textcom, engaged in a Sésamath project, aiming to design a textbook "full Web" (flexible dynamic digital textbook) for grade 10. This project was the first Sésamath project for upper secondary school. During our observations from June 2009 to December 2013, we identified three moments. We present here in details the first moment, which took place between July 2009 and March 2010 corresponding to the object "designing a full web textbook for grade 10" (see figure 1).

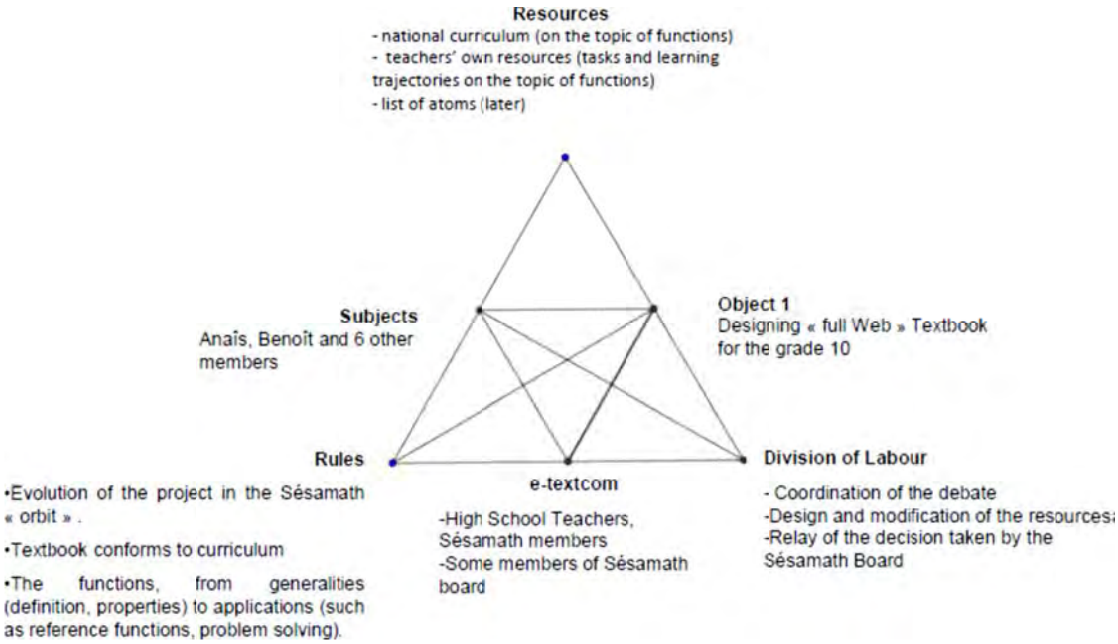

Figure 1. The "e-textcom" activity system at the first moment.

E-textcom gathered only high school mathematics teachers; some of them were also members of the Sésamath board. The project started with various kinds of resources: resources brought by members (in particular their own resources); resources provided by the association, in particular the collaborative platform, previous textbooks, MeP, a mailing list; and institutional resources, like in particular the grade 10 official curriculum. We observe that traditional (paper) textbooks do not seem to intervene as resources. The grade 10 official curriculum formed a central resource: it provided mediation between members at this early stage of the project. One important rule, 'inherited' from the middle school Sésamath projects, was to start with a list of elementary "tasks" identified from the official curriculum. This list played an essential role: each member chose to design exercises corresponding to a given task. A coordinator (see the roles below) checked if all the tasks were tackled by the e-textcom members.

For this textbook, a new vocabulary was introduced. A member of the Sésamath board proposed the term "atom", to describe the elementary units that would constitute the book. This term "atom" was then kept by e-textbook members; but they used it in fact as synonymous of "task". Thus, the first resource designed by the e-textcom members (drawing on the official curriculum, and on their own courses) was the list of "atoms".

The e-textcom members designed a new resource containing a list of 38 "atoms" for the chapter dedicated to functions in the grade 10 e-textbook Then e-textcom designed a "progression", i.e. a trajectory/order for the "atoms" to be taught/learnt. A "progression" is a very usual resource for teachers in France; each teacher, for a given class, has her/his "annual progression", the agenda of the contents to be taught during the year, sometimes designed with colleagues at the beginning of the school year. A "progression" can also be designed for a given theme; in the Sésamath middle school projects, for each theme the tasks were arranged according to a "progression". The multi-representational nature of functions (graphic, algebraic, table of values) offered several possible entries and orders, and made the choice of a progression especially difficult. This progression was developed by members through discussions mediated by the resource "list of atoms", the curriculum and suggested progressions of some members, shared with the community.

This structure could also be viewed as providing rules for the community, and these rules reflected the ways functions should be introduced to students. In particular one rule was explicitly discussed on the mailing list: the functions theme should start with general statements, i.e. definition and properties, before developing more specific notions, i.e. "reference functions" (well-known examples) and applications.

The division of labour firstly drew on the division or labour retained in the middle school textbook project. Several roles were chosen, from the beginning; we identified them via the discussion on the mailing list and the associated resources: resource designers (each designer having to take into account the reviewers' propositions for resources which s/he designed); resource reviewers; a debate coordinator; a Sésamath 'torchbearer' aiming to keep the evolution of the project in the "orbit" of Sésamath projects.

The application of this division of labour followed a process organised in successive steps explicitly proposed by the coordinator:

1) design of a resource (exercise, or course) by a subject/member;
2) review of the resource by other subjects/members;
3) modification of the resource by the designer and proposal of a new version.

At each step, the coordinator had a crucial role. He managed the discrepancies coming from the reviewers, organizing the discussion between members (reviewers and designers).

During the nine months of this first moment, many exercises for the different atoms (exercises corresponding to the same “type of task”) were written; and also a course corresponding to a part of the first chapter. Nevertheless, we observed structural tensions appearing in the activity system (tensions subjects – resources – object). E-textcom members were all teachers, and none were IT developers/specialists. But the design of a fully web-based textbook required technical skills, and the missing competence of an IT developer was an important obstacle.

In order to overcome these tensions, the e-textcom members contacted the Sésamath board: we observed here interactions between two activity systems, the Sésamath board and e-textcom activity systems. The contacts between the two systems led to the ‘arrival’ of IT developers as new members. They also led to change the object itself: since the present technical means and skills within e-textcom did not permit to reach the initial objective, the new object of the activity was the design of interactive resources for grade 10: this marked the beginning of the second moment

DISCUSSION AND CONCLUSIONS: COLLECTIVE DESIGN OF DIGITAL TEACHING RESOURCES- EVOLUTIONS IN A DYNAMIC ACTIVITY SYSTEM

In our analyses we identified three *moments* (exemplified by one in this paper) in the e-textbook activity system, corresponding to different objects of the activity. These evolutions, the changes in the object, were directly linked to the digital nature of the textbook to be designed: in line with the innovative nature of the resource, the members of the Sésamath board tried to develop new and more flexible learning paths, and new tasks that took advantage of the digital and flexible nature of the book, permitted by recent technological evolutions. Nevertheless, the initial project encountered technical obstacles, leading to a first change in the object, which evolved from a full-web textbook to interactive exercises. Conversely, unplanned technical means also brought evolutions.

From the evidence we present in this article (and other works on e-textbooks), it is clear that the collective design of an e-textbook is a complex matter. Our analysis proposes taking into account an evolving activity system (that of e-textcom), and its interactions with other activity systems (at least those of the Sésamath board, and of the middle school textbook developers), leading to the emergence of new roles and new rules. Specific roles were indeed required, not only authors and reviewers of resources (which exist for all textbooks) but also:

- a coordinator was crucial (also for an ordinary paper textbook) as soon as several authors intervened. S/he was especially needed in the case of e-textcom, because of:

- the large number of non-expert authors involved in the design of the textbook;
- the length of the design process (several years);
- the change of the actors involved all along the duration of the project.

- a 'torchbearer' was important, since the Sésamath association provided the frame for the e-textbook project. The 'torchbearer' had to take charge of the coordination between the Sésamath board and e-textcom.

The rules appeared to evolve in line with the processes: the initial design was followed by a review step, then by a new design. However, this cycle could take several forms; it changed along the different moments. More generally, it seemed that e-textbooks, as shared resources designed by communities of teachers, evolving through interactions between individual and collective resource systems, constitute a promising field for new developments.

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IN-SERVICE TEACHER EDUCATION AND E-TEXTBOOK DEVELOPMENT: AN INTEGRATED APPROACH

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MatDigital is a project conducted by the Brazilian Mathematical Society (SBM), aiming the development of a set of e-textbooks for elementary school (grades 6 to 9, ages 11 to 14). Due to the challenges (mostly related to the acceptance by teachers of a model of instructional materials they are not familiar with) faced during development process, it was clear to the team responsible for the project that it should be integrated with in-service teachers training programs. In this paper, we report results of a pilot study, in which preliminary versions of the e-textbooks were applied in classrooms from 17 public schools in Brazil, involving 50 teachers and reaching 2355 students.

Keywords: e-textbook, in-service teachers, MatDigital project, Brazil

THE *MATDIGITAL* PROJECT

The Brazilian Mathematical Society (SBM) has been running, since early 2012, the research project *MatDigital*, for the design and production of digital materials (e-textbooks) for the teaching of mathematics in the elementary school (Brazilian grades 6 to 9, corresponding to ages 11 to 14). *MatDigital* is under the *Klein Project for the 21st Century*, internationally conducted by International Commission for Mathematical Instruction (ICMI) and International Mathematical Union (IMU). The authors of this paper integrate the editorial board responsible for the project.

On his renowned work *Elementary Mathematics from a Higher Standpoint* (Klein, 1908), the mathematician Felix Klein points out a rupture between school and academic mathematics. According to the author, elementary and secondary school curricula lost any bonds with more recent scientific production in mathematics (at the time the book was written). He associates this rupture with a *double discontinuity* in teachers' education: the mathematics prospective teachers are presented to in undergraduate courses have insipient connection with, on the one hand, the one they had contact with as students, and, on the other hand, the one they will approach in their practice as school teachers. Another important aspect of Klein's view is the important role he assigns to the school in the development of mathematics. For the author, school is responsible for assessing education needs and establishing categories that will determine the production of new knowledge, rather than simply receiving and spreading knowledge produced at the university, (Kilpatrick, 2008; Schubring, 2014).

Despite Klein's ideas were situated in the context of German secondary education in early 20th Century, they highlight issues that are still on spotlight in current mathematics education research (e.g. Shulman, 1986; Even & Ball, 2009). For instance, as teachers often see poor relation between their undergraduate courses and their classroom practice, they tend to acknowledge their prior experience as school students as a major reference to build up their practice, as if these courses had to influence to shape them as teachers.

The *Klein Project* was launched in 2008 by ICMI and IMU as a celebration for the 100th anniversary of the first edition of the book *Elementary Mathematics from a Higher Standpoint*, and of the foundation of ICMI (of which Klein was the first president). The goal of Klein Project is to develop instructional materials, in different languages and media, accessible to any person with interest in mathematics, and, more specifically to teachers and other people involved with the teaching of mathematics in elementary and secondary school, and with teachers' education. With inspiration in Klein's ideas, the guiding principle of the Klein Project is to establish links between a comprehensive view of mathematics, the contents and approaches of the discipline in elementary and secondary school, and the curricula of undergraduate teachers' courses (Barton, 2008).

MatDigital was initially conceived as a project to develop a set of textbooks integrating digital tools (e-textbooks) for elementary education. The set of e-textbooks would be implemented in html and Android operational system versions. In line with Klein Project's guiding principle, the methodology of the project is structured upon the collaborative work of a design team of 60 members, including elementary school teachers and university lecturers, based on different parts of the country. This team members was organized into subgroups (of 4 or 5 members), coordinated by a central editorial board. Each chapter of the e-textbooks was assigned to a subgroup. All the subgroups as well as the central editorial board are formed by school teachers and university lecturers.

The subgroups were instructed by the editorial board to design the chapters in effective hypermedia structure. That is, rather than reproducing the linear structure of conventional texts on a pdf version, the chapters should incorporate different modalities of media in a such way that they would play an actual role in the approach of the concepts, and not just be garnishments for traditional models of teaching. The tools available in the devices allowed the use of videos, audios, games, interactive activities, and also to interact with external environment (by means of taking pictures, recording sounds, making measurements). These tools offered a wide range of possibilities for creating innovative approaches.

Therefore, it was clear from the beginning of the project that its aims, conception and methodological design posed challenges in different dimensions: (1) how to manage to collaborative work of a large team, including members with quite different (and complementary) backgrounds; (2) how to make the best possible use of the available digital tools, as intended by the project's conception; (3) how to incorporate e-textbooks into classroom, especially in the case of teacher who have little familiarity (and resistance) with digital tools.

Thus, the success of the project was clearly dependent on carefully examining these questions through the lens of scientific research. *MatDigital*, as a development/research, is still ongoing. In this paper, we briefly report partial results of a pilot study that tested preliminary version of the materials in actual classroom situations.

CONTEXT: ELEMENTARY EDUCATION IN BRAZIL

Compulsory education system in Brazil is organised in three sections: fundamental school I (grades 1 to 5, ages 6 to 10), fundamental school II (grades 6 to 9, ages 11 to 14), and middle school (grades 1 to 3, ages 15 to 17).

Textbooks are distributed for free to public schools, and are chosen by each school out of a list of titles previously approved by the Ministry of Education, through the National Textbook and the National Middle School Textbook Programs (PNLD and PNLEM, initials in Portuguese). Textbooks are submitted by authors or editors to PNLD and PNLEM, in full sets for each section of education system (5 books for fundamental I, 4 books for fundamental II, and 3 books for middle school). The assessment is mainly based on evaluations by experts.

Since the latest editions of PNLD Program, it is required that textbook sets submitted to PNLD and PNLEM programs must be accompanied by electronic versions. However, in most of the cases, these electronic versions are only digitalised conventional texts, showing little or none differences from the hardcopy versions. *MatDigital* aims to differ from this model.

DEVELOPMENT OF THE PROJECT: CHOICES AND SHIFTS

An important obstacle for incorporation of e-textbooks into classroom is expected to be resistance from teachers, who, in most of the cases, had little contact with digital tools during pre-service training, and have been working within a culture formed by schools and classrooms that ignore the use of these tools. This resistance is expected to be manifested in two levels (that overlap each other): (1) a *substantive level*, related to lack of familiarity or insecurity towards technical aspects of the tools; (2) a *subjective level*, concerning lack of preparation for the new classroom dynamics triggered by the use of digital tools, which can possibly drive students to a more independent attitude towards their own learning process, and seriously change the authority position established for teachers.

Therefore, the need to integrate the design of digital materials with in-service teachers' education initiatives was clear for the team from the beginning of *MatDigital* project. These initiatives had several goals: (1) to train teachers on the use the digital materials, from both the perspectives of technical knowledge of software, and devices and of the preparation for potential changes in the classroom dynamics brought into play by the emergence of digital instructional tools; (2) to create and consolidate an environment, involving school teachers, teachers educators and policy makers, for long- term joint discussion, integration the reflection on mathematical subject matter, classroom practices and use of instructional materials; (3) to enroll teachers in the development process of the digital materials, as a means to turn passive use to authorship, and to reverse the traditional top-to-bottom paradigm on instruction development; (4) to use the feedback from workshops and testing in actual classroom situations to develop successively improved versions of the digital materials.

Thus, *MatDigital* has evolved to a project with two-fold, integrated and equally important objectives: design of digital textbooks and in-service teachers' education. This perspective is grounded upon a conception of teachers' education as a permanent on-going process of knowledge and meta-knowledge construction, which is not restricted to pre-service and in-service formal courses, but also comprises the reflection to, in and on classroom practice, including, in particular, the design and critical use of instructional materials.

THE PILOT STUDY

The participants

During the year of 2012, a pilot study of *MatDigital* Project was conducted in Brazilan public schools. The full list of participant schools is shown on table 1.

Table 1: Schools participating on the pilot study

State	School	Number of teachers	Number of students
Amazonas	AM1	3	75
Minas Gerais	MG1	4	170
Minas Gerais	MG2	2	120
Minas Gerais	MG3	3	175
Minas Gerais	MG4	3	130
Minas Gerais	MG5	2	140
Piauí	PI1	1	40
Rio de Janeiro	RJ1	1	60
Rio de Janeiro	RJ2	1	140
Rio de Janeiro	RJ3	3	190
Rio Grande do Norte	RN1	2	125
Rio Grande do Norte	RN2	1	80
Rio Grande do Sul	RS1	3	210
Rio Grande do Sul	RS2	2	155
Rio Grande do Sul	RS3	11	250
São Paulo	SP1	6	115
São Paulo	SP2	2	180

The pilot study was broadly offered to teachers who were taking a professional masters course conducted by the Brazilian Mathematical Society in a countrywide network (for more information see http://www.profmat-sbm.org.br/). In order to participate, school had to be public, and available to apply the materials on the first semester of the grade 6. In response, the 17 school from 7 different Brazilian states enrolled for the pilot study. Thus, in total, the pilot study involved 50 teachers and reached 2355 students from grade 6.

Two chapters were distributed to the participant schools, approximately corresponding to the first three months of grade 6:

- Chapter 1 – Numbers in Daly Life
- Chapter 2 – Geometric Forms: First Drawings

Participant teachers were invited to a three days meeting to discuss the chapters. Their critiques and suggestions were taken into account for the versions of the chapters that were distributed to schools. During the pilot study, communication among the participant teachers, and between them and the project's design team was conduct through an online discussion forum.

Data collection and analysis

Data were collected from several sources: (1) written questionnaires from participant teachers, assessing adequacy and difficulties of each of the chapters; (2) interviews with participant teachers; (3) interviews with participant schools' pedagogical support teams; (4) students' answers to tests and selected tasks; (5) students' performance in exams; (6) written questionnaires from a selected sample of students, concerning their general experience and difficulties with the materials.

All the interviews were tape recorded and fully transcribed. Data concerning teachers, pedagogical team, and students was separately analysed. General impressions, advantages and obstacle pointed out by each group were identified and categorized by frequency. In the following section, we report results of schools from the state of Minas Gerais (MG1 to MG5).

RESULTS AND DISCUSSION

Students' learning

Initial resistance by students was reported from all the participant schools. However, this resistance decreased through the development of the study. Students' responses to questionnaires revealed that: 93% of the students claimed that they liked mathematics more after the experience than before; and 89% of they claimed they liked the pilot study materials more than their usual textbooks. A significant increase on students' performance on exams exams was also reported.

Teachers' practices

All the participant teachers commented that they faced difficulties on adapting to a new classroom dynamics. On the other hand, all of them claimed to be unable to resume previous practices after the pilot materials were used. They claimed that they "could not be the same teachers as they used to be before". Teachers also reported an increase on engagement, performance and a more inquiring attitude from their students. After the materials were used, they reported that most of the students showed resistance on fit back to usual approaches: “they no longer take things for granted, and want to know *why* everything is like that”.

FINAL COMMENTS

A detailed analysis of the results is in preparation, to be published on a longer paper. The positive results surprised to authors. Obstacle and teachers from both students and teachers were much less profound and serious than expected. In our interpretation, the engagement of the teachers with the project was more important to these results, than to any particular feature of the material. Therefore, we are aware that these results are very particular, and can hardly be generalized to a context in which teachers just receive the materials and do not participate on discussion about them.

Participant teachers were invited to discuss and criticize the textbook they were using in the classroom with the design team, and to exchange ideas with each other about the approaches and methodological challenges involved. This aspect seems to have created an environment that integrated collective discussion about mathematical content, pedagogical approach, use of textbooks and use of technology. This may have led to a shift on teachers’ role: from mere recipients of instructional materials to an attitude of authorship and a sense of ownership. The participant teachers’ engagement with project may be associated with a profound change in the classroom dynamics and in their own practices.

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WHAT CAN TEXTBOOK RESEARCH TELL US ABOUT NATIONAL MATHEMATICS EDUCATION? EXPERIENCES FROM CROATIA

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Analysis of textbook content may help in the identification and better understanding of the requirements of national mathematics education. This paper concerns the role of textbooks in mathematics education in Croatia. The study included a review of educational traditions in Croatia and of foreign research on mathematics textbooks, conducting a survey, interviews and classroom observations on the role of mathematics textbooks as well as analysis of textbook content of grades 6 to 8. The results show a traditional picture of mathematics education in Croatia. There is an emphasis on algorithms, closed answer form and simpler connections. These studies helped to determine the characteristics of mathematics education and generated new research questions on ways of improving the teaching of mathematics. This could be applied not only to Croatia but also to other countries with limited experience in research in mathematics education. In such environments the textbook may be an appropriate first step for research because it is a tangible artefact with text, it is part of the curriculum and it reflects national or regional traditions.

Key words: textbook analysis, textbook use, intermediate variable, Croatia

INTRODUCTION

The research of mathematics textbooks has made significant progress over the last few decades (Fan, Zhu & Miao, 2013). This progress has mainly been made by researchers from countries or regions with well-organized research of mathematics education. In Croatia, *Mathematikdidaktik* is still not fully recognized as a scientific discipline. This means that there is relatively little research on mathematics education and that such research is at the beginning of its scientific development (Čižmešija, Milin Šipuš, & Glasnović Gracin, 2013). In such an environment it is natural to pose questions such as: “Where should we start? How should the first steps be taken and what should they be? How can mathematics education in Croatia be improved?” This paper is concerned with why exactly textbook research is a good first step towards understanding and improving national mathematics education.

THE TEXTBOOK AS AN OBJECT OF RESEARCH

The textbook as an intermediate variable

Textbooks have an influence on other factors in education, and other factors have an influence on textbooks. Fan (2013) argues that “textbooks as the subject of research can be viewed as an intermediate variable in the context of education and, consequently, textbook research can be defined as disciplined inquiry into issues about textbooks and the relationships between textbooks and other factors in education” (p. 776). This is a good reason to start with textbook research. The textbook is connected to other educational issues as an independent as well as a dependent variable. This means that through research on textbooks we meet other variables concerning education, we learn and broaden and deepen our knowledge. This knowledge comes from various sources because textbooks are connected to many issues, such as the curriculum, classroom practice, students, teachers, parents, and educational and cultural traditions.

Why are textbooks suitable for research?

A thorough reading the available literature highlights the reasons why textbooks are suitable objects of research: a) Textbooks are artefacts and tangible media; b) Textbooks contain text to a significant extent; c) Textbooks are widely used by students and teachers; d) Textbooks are deeply embedded into the curriculum; e) Textbooks reflect cultural and educational traditions.

A textbook is an *artefact* which has “a major influence on the activity of learning mathematics” (Rezat 2006, p. 482). Traditionally, the textbook is a *tangible tool* because students, teachers and researchers can touch it and hold it. Also, textbooks contain *text* to a significant extent. Text and its structure have always been important in mathematics education and related research (Pepin & Haggarty, 2001; Van Dormolen, 1986). The characteristic of textbooks as tangible artefacts is useful as a starting point, especially in environments with no previous research.

The benefits of textbook research also lie in the fact that textbooks have always been deeply embedded into the *curriculum*. They can be seen as a link between the intended and the implemented curriculum, making them the potentially implemented curriculum (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). This means that textbook research provides a broader and deeper picture of curricular requirements and practices. In addition, they are heavily used by students and teachers and are influenced by other social and cultural aspects (Rezat & Sträßer, 2012). Textbooks reflect the *cultural and educational traditions* of a particular country or region (Pepin & Haggarty, 2001). Looking at textbooks as parts of the curriculum and as reflections of traditions is of great use in understanding mathematics education in its environment. Additionally, the results can be easily connected with other studies.

RESEARCH OF MATHEMATICS TEXTBOOKS IN CROATIA

The Croatian educational system and mathematics textbooks

Compulsory education in Croatia takes place in a standardized primary school and lasts for eight years. It is divided into two stages: grades 1 to 4 (lower primary education) and grades 5 to 8 (upper primary education). All pupils from grades 1 to 8 follow the same educational program according to the national curriculum outlines. In compulsory education pupils have mathematics lessons four times per week; one educational unit lasts for 45 minutes. All textbooks used in schools are authorized by the state board which ensures that the textbook follows the curriculum requirements. Mathematics textbooks are mainly written by a small group of authors consisting of teachers and a university mathematician. As of 2010, teachers jointly select authorized textbooks for their school for the period of 4 years. Before 2010, teachers chose a textbook individually each year to be used in the following year. In Croatia, textbooks are traditionally bought by parents. They are brought to every mathematics lesson and used at home for homework.

Research of mathematics textbooks

The study on mathematics textbooks in Croatia encompassed various research approaches. It included: a) a review of educational traditions in Croatia and of foreign research on mathematics textbooks; b) conducting a survey, interviews and classroom observations on the role of mathematics textbooks; c) analysis of textbook content; d) a comparison of textbook content and curricular requirements; e) analysis of the PISA 2009 mathematics items and their comparison with textbook and curriculum requirements; f) reflection on results and discussion on further research.

Textbooks and educational traditions

Textbooks reflect the goals of the national curriculum and the cultural and educational traditions of a particular country (Apple, 1986; Love & Pimm, 1996; Pepin & Haggarty, 2001). Research on the history of Croatian education showed that mathematics education in Croatia followed the educational and political trends of different epochs. Mathematics education in Croatia in the Middle Ages was influenced by European scholastics ideas, books and organization. From the 16th century until the end of the First World War the Croatian educational system, and textbooks, were directly influenced by the reforms of the Austrian Empire (Dadić, 1982). After the Second World War, education in communist Yugoslavia was influenced by Russian authors. This approach was also evident in curricula and mathematics textbooks. After the break up of Yugoslavia in 1991, the content and structure of mathematics textbooks in the Republic of Croatia did not change much from the textbooks of the 1980s. One area of change, however, was the free textbook market; there were more publishing companies, more than one textbook per generation, and more choice. This interesting aspect of cultural and educational traditions reflected in current Croatian mathematics textbooks needs to be further analyzed and researched in the future.

The role of mathematics textbooks in Croatia: survey, interviews, observations

The review of research in the field of mathematics textbooks all over the world shows that textbooks play a very important role in mathematics education. These findings raised questions about the role of mathematics textbooks in Croatia. A comprehensive survey on these issues was conducted (Domović, Glasnović Gracin, & Jurčec, 2012a, 2012b; Glasnović Gracin & Domović, 2009) which involved nearly one thousand mathematics teachers; about half of the total number of upper primary mathematics teachers in Croatia. The findings show that mathematics textbooks play an important role in mathematics education in grades 5 to 8 in Croatia. Mathematics textbooks are in use in Croatia to a great extent, especially in teachers' preparation, in practice exercises for students and in their homework. New material is mainly presented by the teacher at the front of the class followed by students working individually on textbook exercises. These results were confirmed through classroom observations. The survey was followed by interviews and classroom observations which provided a qualitative view on the use and role of mathematics textbooks (Glasnović Gracin

& Jukić Matić, 2014). The research showed that the content and structure of the textbook have a great influence on mathematics teaching. Other materials are also used by teachers in lesson preparation, but not to such a large extent.

Textbook analysis

Since textbooks greatly influence mathematics teaching, it is reasonable to research the content within the textbook. The textbook structure in Croatia follows the most typical model: “exposition – examples – exercises” (Love & Pimm, 1996, p. 386). The survey, observations and interviews showed that textbooks are used in particular for practising. Therefore the research encompassed all the worked examples and exercises from the three most frequently used mathematics textbook series in Croatia for grades 6, 7 and 8. The analysis of more than 38,000 textbook items involved identifying the contents, activities, complexity levels, context types and answer forms that are required from students in textbook examples and exercises (Glasnović Gracin, 2011). The results show that the researched textbooks mainly contain operation activities on the levels of reproduction or simpler connections. In addition, more than 96% of the exercises analyzed are given in closed answer form requiring short (numerical) answers. The geometrical content predominantly requires the ability to deal with numbers, formulas and terms rather than mastering geometric concepts. Modelling activities are not required to a significant extent in Croatian textbooks in compulsory education. The chapters on arithmetic do not include interpretation activities. Argument and reasoning skills, open answer forms and reflective thinking are not represented in mathematics textbook problems in Croatia. The results also indicate the usage of intra-mathematical and symbolic items to a great extent (mostly more than 90%). Only the research results for descriptive statistics, probability and proportionality show a predominance of real life contexts. The results showed a picture of traditional mathematics education in Croatia with an emphasis on algorithms and “mathematics as a tool” (Heymann, 1996).

Textbook content, national curriculum requirements and PISA requirements

The analysis shows that the textbook authors fully adhere to the current official requirements of the national curriculum (Glasnović Gracin, 2011). Examination of mathematics syllabi published by MZOS (Ministry of Science, Education and Sports) shows a predominance of procedural tasks with an emphasis on calculating and operating activities (MZOS, 2006). The syllabi do not include reflective thinking, open answers or argumentation activities, so these requirements are not present in the textbook exercises. It is only in relation to context that the textbook results do not meet the curriculum requirements. Namely, the current plan for mathematics encourages the implementation of school mathematics in a variety of everyday situations (MZOS, 2006). The textbooks do not correspond to this requirement as the results show that intra-mathematical and symbolic items are used to a great extent.

The textbook and curriculum requirements are further compared with requirements of PISA mathematics items. The results show that most of the PISA 2009 items are *not usual* for Croatian mathematics textbooks and for Croatian mathematics education (Glasnović Gracin, 2011). Although the PISA content requirements are all mainly included in the Croatian curriculum, a lot of text with context, interpretation activities and reflection requirements make the PISA items significantly different from the textbook exercises. These differences of textbook, curricular and PISA requirements are surely factors in the poor performance of Croatian students in the PISA assessment.

DISCUSSION AND CONCLUSIONS

Research on mathematics textbooks showed a traditional picture of mathematics education in Croatia. There is an emphasis on algorithms, closed answer tasks, reproduction and simpler connections. These requirements are predominant both in the current official plan and in mathematics textbooks, which are used to a great extent. These findings are considered to be a good starting point for the next steps in research. The findings not only generated new information, but also raised new questions on ways of understanding and improving the teaching of mathematics. For example, the results on Croatian mathematics textbooks and the issues raised are primarily descriptive. The next steps would involve experimental methods in order to research how textbooks, as an intermediate variable, affect and are affected by other factors in the context of education (Fan, 2013).

The experience from Croatia shows that we gained a clearer picture and a broader and deeper understanding of mathematics education from textbook research. Analysis of textbook content and requirements, studies on textbook use and other textbook research may help in the identification and better understanding of the requirements of national mathematics education. This could be applied not only to Croatia but also to other countries with limited experience in research in mathematics education. In such environments the textbook may be an appropriate first step for research because it is a tangible artefact with text, it is part of the curriculum and it reflects national or regional traditions. All these characteristics make it possible to connect the results with results from other studies, as well as with theory and practice in education.

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THE NORDIC NETWORK FOR RESEARCH ON MATHEMATICS TEXTBOOKS, EIGHT YEARS OF EXPERIENCE

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The Nordic Graduate School in Mathematics Education (NoGSME) organised a workshop in 2006 on research on mathematics textbooks. The researchers present created an informal network, which functioned as a supporting and inspiring group for the members. Among other things the network arranged a Discussion Group on research on mathematics textbooks in PME 30 in Prague, 2006. Another event that members initiated and took part in was a "Symposium on Mathematics textbooks, mathematical tasks and pupils' identity: An international perspective" at ECER in 2008 (which took place in Gothenburg). A number of master and doctoral students were supported on a voluntary basis by researchers in the network and could get advice and reading suggestions as a kind of informal supervision. Joint papers were prepared, intended for a book and a special issue of Nomad. For three years from 2011 NordForsk financed the network (see http://textbookstudy.wordpress.com). Opportunities were thus opened for more regular meetings and activities like seminars and workshops, and deeper collaboration among members could be developed. In the paper I give examples of scientific work done by members in the network and elaborate on some of the publications produced over the eight years the network existed. The scientific work was created at all levels: master, doctoral, postgraduate and senior researcher level.

Keywords: Nordic textbook network, informal supervision, collegial support, research groups

INTRODUCTION AND BACKGROUND FOR THE NETWORK

Research on mathematics textbooks, curriculum material and teaching resources has been carried out in most countries for many decades as one important part of mathematics education research (MER). In the Nordic countries there seemed to be a lack of such studies at the end of the 20th century. As I had carried out a couple of such studies in Sweden in the 1980ies and worked with curriculum development for more than 20 years up to year 2000 I saw the value of such studies and wanted to initiate more of them. Thus, I first of all encouraged a couple of my doctoral students to carry out studies in this area and secondly took the initiative to the NoGSME workshop on textbook studies. Textbooks are here taken in the widest sense including all kinds of teaching material and resources. As one outcome of this workshop a network for research on mathematics textbooks was created. In 2011 the network was given funding from NordForsk and thus could intensify its activities. In the application the aim of the network was described like this:

> The main aim is to increase the Nordic and Baltic collaboration in research on mathematics textbooks with implications for teachers' teaching, students' learning, and decisions by policymakers and publishing houses. In this way we aim for reaching better quality of ongoing and new studies and inspiring to comparative studies, longitudinal studies or

replication studies to get broader evidence of the situation in the Nordic and Baltic countries. The network will support the doctoral studies and help students to finish in good ways through offering new opportunities to learn about research on mathematics textbooks. Another aim is to produce anthologies on research on textbooks suitable for teacher education and to publish in scientific journals and journals for teachers. Such publications will build on the collective knowledge and insights of all members in the network. We also intend to build up cooperation with publishing houses in order to offer research based evidence for authors about valuable features, content and structures for mathematics textbooks. We want to use the advantages of shared Nordic history and culture, values and structures of society, educational systems and traditions and language bridges.

The aim of this paper is to give an overview of some of the outcomes of the network, some examples of the activities and critically discuss them in relation to the aim of the network.

THEORETICAL FOUNDATIONS

There is no single unified theoretical base for textbook studies but a great variation of theoretical perspectives and models are used. The TIMSS curriculum model is one well know and often used model (Valverde et al, 2002) using the intended curriculum, the implemented and the achieved curriculum. The textbook is often seen as the potentially implemented curriculum. In the network I created a model, which has been used by the members alongside with many other models and theories (for an overview see Grevholm, Fan & Rezat, 2013; Rezat & Strässer, 2012, 2013). The model we used presents crucial properties of textbooks and factors influencing and influenced by textbooks and relations in play in textbook studies:

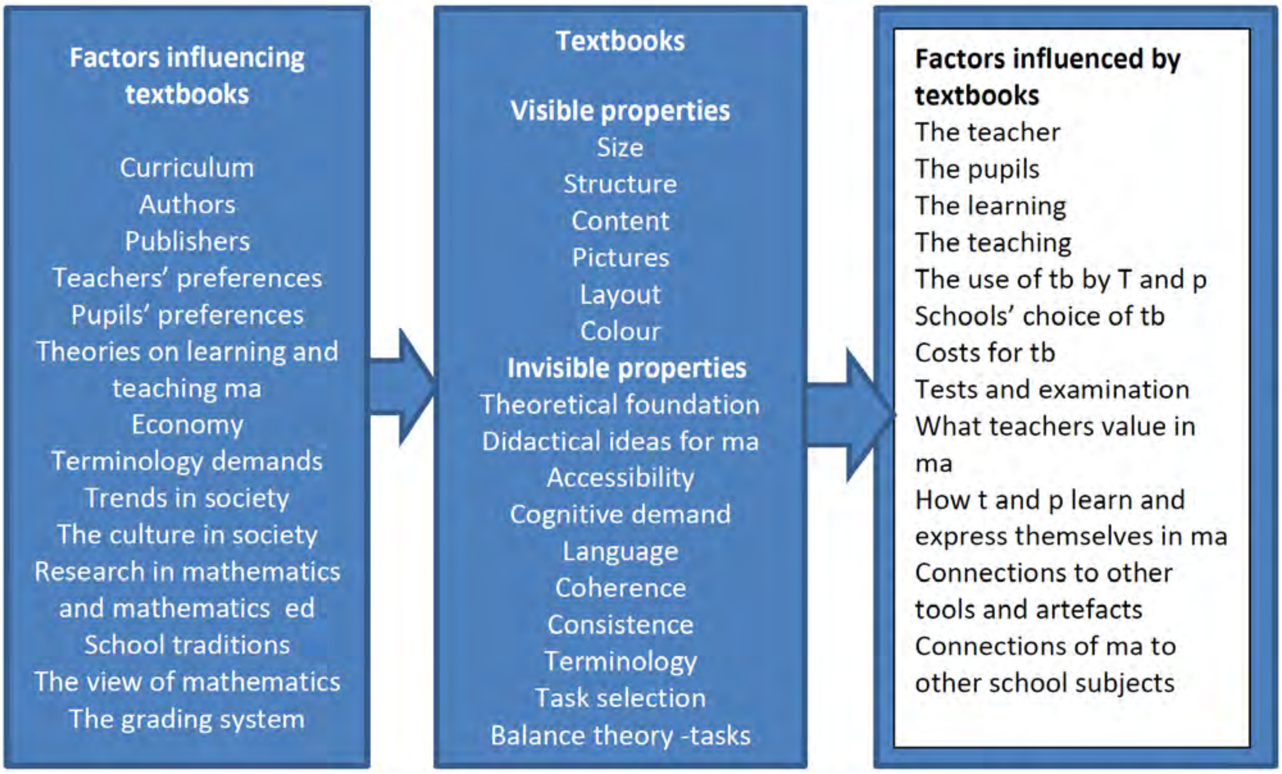

Figure 1: Theoretical model of factors related to issues on textbooks in mathematics and their influences (Grevholm, 2012)

In their overview of research Fan, Zhu & Miao (2013) are exposing the different kinds of studies that have been common in the area of textbooks studies. In order to respect page-limitations in this paper I refer to their study for further elaboration on theoretical aspects and foundations.

EARLIER RESEARCH ON MATHEMATICS TEXTBOOKS AND RESOURCES

An overview of studies in the Nordic and Baltic countries up to 2011 is given in Grevholm (2011). Fan, Zhu and Miao (2013) described in their international overview research on textbooks and teaching resources and gave a developmental status and directions for future studies. The main categories used are textbook analysis and comparisons, the use of textbooks in teaching and learning and textbook research in other areas. They point to the fact that important progress has been made over the last few decades. The major achievement has been in the area of textbook analysis and comparison. They claim that to advance research it is necessary to establish a more solid conceptualisation and theoretical underpinning of the role of textbooks and relation between textbooks and other variables. They also ask for more research on issues about the development of textbooks. The use and development of electronic textbooks will also be an area that is explored in the future. In the PME 37 working session we (Grevholm, Fan & Rezat, 2013) suggested finding a common and shared research agenda in order to gather efforts of research to be able to work in cumulative ways.

Earlier studies that have become classical in the area of textbooks research are the book by Howson (1995) and the chapter in the International Handbook ‘This is so’: a text on texts by Love and Pimm (1996). The TIMSS textbook study (Valverde et al, 2002) confirmed that teachers are highly dependent of textbooks in mathematics, especially so in the Nordic and Baltic countries. Textbooks are also the most common resource for pupils in the mathematics learning and often used in every lesson and for many purposes (Johansson, 2006). There are still many unexplored areas in textbooks research and focused studies may help improve a learning tool that is important for students at all levels.

THE NETWORK AND ITS ACTIVITIES AND PRODUCTS

In order to illustrate and exemplify what has been done in the network I give short accounts of some of the areas in which the network has been active.

The first five years of the network: After the first workshop in 2006 most of the participants wrote papers representing their research on textbooks. These are intended to be part of a book that documents the network activities. Although there was no funding available the network managed to arrange a *Discussion group on research on mathematics textbooks in PME 30* (Pepin, Grevholm & Strässer, 2006). It attracted many participants showing the interest for research in the area. Another event that members initiated and took part in was a *Symposium on Mathematics textbooks, mathematical tasks and pupils’ identity: An international perspective in ECER* in 2008. Here two doctoral students and two senior researchers from the network presented their studies. A number of master and doctoral students were supported on a voluntary basis by researchers in the network and could get advice and reading suggestions as a kind of informal supervision.

The seminars and workshops between 2011 and 2014: As soon as funding was available more regular work in the network could be planned. Starting in autumn 2011 the network has carried out 6 two-day seminars and 5 three-day workshops, where the main theme was methods and methodological issues concerning studies on textbooks. In each of the meetings we had at least one invited expert researcher, who gave lectures about their specialities on textbooks. Details about the programme and lectures during these meeting are available at the website for the network. (http://textbookstudy.files.wordpress.com)

Methods and methodological issues created the main interest for the network participants. We tried to analyse systematically a large number of papers and conference presentations in order to get an overview of often used methods. Different members took on the task to go through papers over the last 10 years in some major journals and in the international conferences. The findings were then presented in the whole group and discussed in some detail. In one of the latest workshops three lecturers contributed with recent models and theories and methods from the special issue on mathematics textbooks in ZDM 2013. The working session in PME 37 organised by the network also focused on methods and methodological discussions.

The historical group: One subgroup in the network focused on historical aspects of textbooks. Many papers have been published from this group and also one chapter in the recent international handbook of history in mathematics education (Bjarnadottir, 2013; Bjarnadottir Christiansen & Lepik, 2013). Christiansen has so many papers that they cannot be listed here.

The comparative studies: Another group focused on comparative studies and used data from the so called NorBa-project to compare and analyse how teachers in Estonia, Finland and Norway use textbooks (Lepik, Viholainen, Grevholm). Teachers in Finland and Estonia share many features in how they use textbooks, while teachers in Norway partly differ from this.

The context studies: Greimas' semiotics theory was used by four researchers to study from a narrative point of view the introduction of negative numbers in year 4 and 5 in compulsory school (Kudzma, Grevholm & Preidyte). It was found that in some cases the introduction did not contain a question or problem and the variation was great of how authors choose to start this new topic in the books. One way was via an equation and other authors referred to every day phenomena such as temperature, economy or water level. One doctoral student, who started his work in 2006 investigated heuristic approaches to problem-solving and published a major report (Kongelf, 2011). His second part-study concerns the introduction of algebra in school year 8 in Norway in all seven textbook series.

The teacher guide group: No earlier studies in the Nordic countries have been made on teacher guides, although they seem to play a major role for teachers in planning and carrying out their teaching. Several members in the network are exploring the use of teacher guides and at least two papers are being produced. One paper is about how elementary student teachers in Finland utilize teachers' guides for planning their mathematics lessons (Hautala, Manninen, Sarenius). The other is a comparative study about Swedish and Icelandic teachers' use of teacher guides in mathematics. Both these studies are using a framework developed by a group in Sweden (Hemmi, Koljonen, Hoelgaard, Ahl & Ryve, 2012).

The tertiary level group: As doctoral student Mira Randahl (2013) has belonged to the network since its start. Her study concerns prospective engineers and their use of the textbooks as a learning tool. She has published widely, also together with her supervisors Grevholm (Randahl & Grevholm, 2010). Rensaa and Grevholm carried out a study on prospective engineers' use of the textbook in linear algebra.

CRITICAL DISCUSSION, CONCLUSIONS AND IMPLICATIONS

Four Nordic and three Baltic countries participated in the network but from Denmark we managed only to get one doctoral student visiting the network once. Thus in Denmark there seems to be little going on in the area of textbook studies.

Several publications have been produced by members of the network, on their own or in groups, as can be seen above. The properties and the factors in the model (Figure 1) have all been part of one or other of all these studies, thus indicating the viability of the model. Although we started out by writing papers for an anthology on textbook studies we have not yet published a book with only contributions from the network, but it is on its way. This is partly depending on the fact that the network has been invited to contribute to a special issue of the journal Nordic Studies in Mathematics Education (Nomad) in 2015. The papers that are not used for this issue will become chapters in the anthology building on papers not only from the 7 first participating countries but also on papers from The Netherlands, UK, Germany, The Czech Republic and others, from where we had participants. On the other hand members of the network also contributed to chapters in anthologies (Bjarnadottir, 2014, Grevholm, 2014) and many of the members have written papers for scientific journals or books (Christiansen, Rensaa, Pepin, Gunnarsdottir, Palsdottir, Lepik and others).

Maybe the network could have been more influential in the public debate on the development and quality of textbooks or trying to have an impact on authors and publishers. Randahl (2013) carried out interviews with textbook authors and they are sensitive to messages from research on what creates a good book and what can improve a textbook. Generally the studies show that more can be done to improve the books and raise the potential for learning with the help of them. Further exploration of what we can learn from the studies is necessary.

In addition to all the publications from the network, the most convincing evidence of the need for and success of the network is the fact that in the Nordic conference on MER in Finland in June 2014 a working group was held in order to try to establish continued activity of the network, although the funding period is over and I am leaving as leader because of retirement.

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CROSSING THE BOUNDARIES: SWEDISH TEACHERS' INTERPLAY WITH FINNISH CURRICULUM MATERIALS

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Crossing the cultural boundaries provides a fruitful setting for investigating the dynamic interplay between teachers and the applied curriculum materials. In this paper, we report on the initial analysis of interviews and meetings with eight Swedish primary teachers regarding using translated Finnish curriculum materials, i.e. a textbook and teacher guide, for the first year. All teachers had chosen to use the materials voluntarily. Our analysis shows that despite some consistent experiences concerning using the materials, their ways of designing lessons vary greatly in terms of selecting ideas from the materials to be realized in the classroom. Most of the teachers seem to rely on the Finnish teacher guides more than the Swedish ones. We elaborate on both similarities and differences in relation to the teachers' experiences and the specific features of the current school context.

Keywords: primary school, cross-cultural study, Sweden, Finland

INTRODUCTION

A teacher does not work in a vacuum but rather in interaction with students and various kinds of resources, such as curriculum materials[1]. The use of mathematics textbooks has been a controversial topic in many countries, in both teacher education and public debate over school systems. On the one hand, for example in Sweden, for a few decades now teachers have been criticized for their dependence on curriculum materials. This has resulted in a movement discouraging the use of ready-made materials and instead composing one's own. Furthermore, the use of curriculum materials has been associated with an uncreative teaching method (Ball & Cohen, 1996). On the other hand, several researchers have recognized the potential that using curriculum materials has for teachers' professional development and even for improving education (e.g. Ball & Cohen, 1996; Remillard & Bryans, 2004). Studies on the use of curriculum materials have pointed at the complex relationship between teachers and these materials (e.g. Brown, 2009), and furthermore, paid attention to the complex impact on student learning (Van Steenbrugge, Valcke, & Desoete, 2013). For instance, teachers assimilate new materials into their current ways of teaching in different manners that do not necessarily follow the curriculum developers' intentions (e.g. Remillard & Bryans, 2004).

[1] The term *curriculum materials* refers to the printed or digitally published resources designed to be used by teachers and students before, during and after mathematics instruction. This includes the textbooks to be used by the students as well as the teacher guides and the additional materials like software or concrete material.

Since sociocultural dimensions play out in the complexity of the teaching context, we aim to contribute to a better understanding of the teacher–curriculum interplay in a special setting, with Swedish teachers working with Finnish curriculum materials. Our study sheds light particularly on the impact of the cultural context of the interplay between a teacher and curriculum materials. We follow the idea that the interplay is an essential contextual cross-cutting pattern (Herbel-Eisenmann, Lubienski & Id-Deen, 2006), and furthermore, bring up the relevance of regarding curriculum materials as cultural tools (e.g. Brown, 2009). These tools can be seen as both shaping and being shaped by human action (Wertsch, 1998).

There are many similarities between the school systems in Finland and Sweden, for example an inclusive compulsory basic education with no special tracking. Teachers in both countries are to follow quite general national curriculum guidelines. Furthermore, curriculum materials are produced by commercial publishers and there is no national inspection of school materials. However, there are also several differences, for example the way teachers view the curriculum materials. According to Pehkonen (2004), Finnish teachers perceive the available Finnish curriculum materials as support for teaching in a new way. Swedish teachers mostly use only student textbooks, whereas teacher guides are seldom used (Jablonka & Johansson, 2010). There are indications that Finnish teachers often organize whole-class instruction, with all pupils engaged in the same mathematical area, while ‘speed individualization’ and individualized teaching in different mathematical areas are common forms in Swedish mathematics classrooms (e.g., Jablonka & Johansson, 2010). All Finnish teacher guides seem to follow a rather homogeneous cultural script concerning the suggested activities and the focus on designing specific lessons. On the contrary, the Swedish curriculum materials vary greatly, with no focus on designing certain kinds of mathematics lessons (Hemmi et al., in press).

There has been a growing interest in applying Finnish curriculum materials in Swedish schools. The Finnish materials have been translated into Swedish, and lately some minor changes have also been made to adjust them to suit the Swedish national guidelines, which are quite general. The curriculum materials are always influenced by the norms and traditions of the context in which they have been written. Thus, the study design provides a special setting, where the interplay between a teacher and curriculum materials is embedded in the cultural context (Stigler & Hiebert, 1999); thus, the study highlights special features of the educational context of the dynamic interplay. The aim of this study is *to develop further conceptual understanding of the interplay between teachers and mathematics curriculum materials by investigating the case of Swedish teachers working with the Finnish materials*.

METHOD

The data were gathered during 2009–2014 from Swedish primary teachers with various teaching experience. The teachers were taking part in development projects financed by the Swedish Agency for Education and the municipality. The common feature of the teachers participating in the study was that they had started using the translated Finnish curriculum materials, i.e. a textbook and teacher guide, in their work. All teachers had chosen to use the materials voluntarily in order to improve their mathematics teaching, and received some background information about the ideas behind the materials. On the whole, the data of our longitudinal study following the teachers' development process consist of their responses to questionnaires about their work manners and relation to curriculum materials; interviews with them; classroom observations and video recordings of the mathematics lessons; and documentation of collegial meetings and seminars about teaching mathematics, which the teachers attended during the project. In this paper, we report on the initial results based on the analysis of the data from interviews and meetings with eight participating teachers. The teachers' teaching experience in our sample varied from one to 40 years at the time they started using the new materials. We used an open iterative approach when analysing the data in order to find recurrent items and themes.

RESULTS

We approach the interplay between teacher and curriculum materials through two aspects. We illuminate, on the one hand, how curriculum materials influence teachers' views and practice, and on the other, how teachers apply the materials in their work. The interplay is presented through three themes that the teachers mentioned in the interviews.

Teachers as users of curriculum materials

The most striking result is that all the teachers started using the Finnish materials intensively by reading and following the teacher guide, which differs from their previous practice with Swedish materials. Most of the teachers stated that they rely on the Finnish teacher guides more than the Swedish ones:

> Teacher 1: I've never used a teacher guide as much as now… earlier I just 'shut' it [the guide]; no, I won't look at this.

Several teachers wanted to follow the materials in detail in order to become familiar with them and understand their ideas for designing mathematics instruction. Despite some consistent experiences concerning using the materials, the ways of designing lessons vary greatly in terms of selecting ideas from it to be realized in the classroom:

> Teacher 3: I think it's [the guide] good, there're even suggestions about what to write on the board.
>
> Teacher 4: I think that the book itself, the central content and the lesson plans, I take everything as suggested and it's super. I want to follow this carefully … I feel so…It's a way also to get to know the materials, next time there might be more variation, you learn all the time.

The teachers expected that their individual needs as a teacher could be addressed by the curriculum materials, which were perceived as a sort of manual mediating norms and highlighting the importance of certain aspects of mathematics education. The teachers mentioned their needs related to planning and implementing mathematics classes, and also stated that both the teacher guides and a student textbook are more extensive than they are accustomed to. This seemed to become a problem if a teacher wants to implement everything suggested in the teacher guide, or if the pupils are to complete all the tasks included in the textbook:

Teacher 5: There's so much to choose from, and there's a danger that we talk too much, I think, as I lose them [the pupils] if we have an introduction, problem-solving and so on, then you lose them, we can't have overly long introductions, it's impossible to do so much.

Most of the teachers considered the Finnish materials to be easy to follow and clearly structured, but still they perceived this differently. Some experienced it as a support for professional development and for improving their mathematics teaching in individual ways, whereas some experienced its structure as inhibiting their creativity as a teacher (cf. Ball & Cohen, 1996). Some teachers with long teaching careers considered a possibility to incorporate their own ideas into the lesson plans of the materials as the most important aspect. These teachers viewed the materials as particularly suiting novice teachers, who might need extra support.

Presentation of the mathematical content

All teachers with further experience teaching mathematics paid attention to the way mathematical content is presented as well as descriptions of the progress students are expected to make during a certain time period, such as a single lesson or a teaching sequence. They also paid attention to the emphasis on different mathematical topics. For example, the Grade 1 teachers discussed the notion of presenting the number line (1-20) already at the beginning of the first year, which is contradictory to their previous experience. According to the teachers, the Swedish tradition has been to focus only on numbers 1-10 during the first school term. Similarly, they said, ten transitions are traditionally not included in the content of Grade 1 in Sweden like they are in Finland, which is another example of the differences teachers were confronted with when using the materials. Most of them experienced this as a positive challenge, and wanted to test it with their pupils.

The teachers also reflected on the mathematical progression they observed for both the pupils and themselves as teachers. Some of the teachers regarded the faster progression as a problem, as they experienced that this makes pupils' learning difficulties too visible already in the first grade. However, others felt that it allows them to better identify the pupils' difficulties at an earlier stage. Some teachers were concerned about difficulties pupils coming from other schools using Swedish materials might be confront with because of the faster progression with the Finnish materials. Two late-career teachers also felt that their pupils had too little practice in basic arithmetic when using a Finnish textbook. Thus, this forced them to give them extra homework aiming to enhance the automation of basic mental arithmetic.

Organization of mathematics teaching

The use of Finnish materials in Swedish schools also reveals differences between school traditions with respect to the organization of teaching, which the teachers reflected on in the interviews. All teachers reacted in the same way to the idea of giving homework in order to prolong the pupils' learning opportunity after the school day. They were afraid of what might happen if they let the pupils take the textbooks home, as they expected them to forget the books there. According to the teachers it was not common to give homework after every lesson. Some of them seldom gave homework, while others did so once a week at the most.

The Finnish materials include sections of homework mediating the Finnish tradition of having regular homework and some teachers in our study first tested how it would work at Grade 1, telling pupils that 'the textbook lives in your school bag'. The experience was good also because teachers noticed that taking the textbook home allowed pupils work in case of illness. In addition, some additional material from the textbook, such as audio files on the website, allows pupils to have the same activities that the class had at school. However, some parents demanded the possibility to complete homework on a certain occasion, mostly weekends, due to several time-consuming hobbies that pupils have on weekdays. This is against the idea mediated in the Finnish materials that homework is mainly for weekdays while weekends are for resting after the school week (cf. Pehkonen, 2004). Some teachers had difficulty resisting the pressure, although they themselves regarded the short homework after every lesson as effective.

Teachers tried to organize classroom activities to focus on the same learning object with all students, and thus make them work on the same pages of the textbook in accordance with the underlying idea of the curriculum materials. Most teachers understood the idea that the materials include both basic and additional tasks for every lesson. Still, after the first term one of them started using speed individualization, as she had previously done with the Swedish materials. Two of the teachers, who were ill for a longer period during the project, witnessed that the substitute teachers simply allowed pupils to continue to new mathematical content without providing an introduction. Also, some parents complained that their children were not allowed to proceed in the textbook even though the teachers explained the pedagogical idea behind the activities.

DISCUSSION

The interplay described through the three themes is obviously embedded in a specific sociocultural context, and our analysis reveals certain traditions that impact the teachers' interplay in different manners, for example concerning the mathematical content and organization of teaching (cf. Stigler & Hiebert, 1999). All the teachers were interested in the new materials and studied the teacher guides carefully, and most of them wanted to follow them; at least for the most part. Yet, the teachers who had previously worked with materials containing a great deal of skills training did not trust that their pupils would gain skills if they did not assign them extra homework. Further, although all the teachers wanted to avoid speed individualization, one of them gave up already after the first term as she experienced the content as difficult and did not trust that all her pupils could proceed at the same pace.

The school community also has an impact on the way teachers act with the materials. Parents can be influential in textbook adaptions (e.g. Herbel-Eisenmann et al., 2006). Indeed, some teachers had problems with parents resisting the new ways of organizing instruction, with respect to both the homework and holding the pupils within the same mathematical area. This kind of pressure seems to be more common in later school years (Grade 3 and 4) when certain routines, for example concerning homework, have been rooted. Some teachers who had difficulty resisting the pressure adapted the organization to certain parents' wishes. Further, it was difficult for the substitutes to work in the new manner, which might be because they were not among those who were working with the new materials.

This paper is the first step of an analysis of Swedish teachers' interplay with Finnish curriculum materials. Applying the materials to another context than the original one may reveal inconsistencies between teachers' views and the materials, such as conflicting views on mathematics education, teacher professionalism and classroom practices. Our study will contribute to the theorizing of the interplay between teacher and curriculum materials in a specific context, and deepen our understanding of essential issues that should be considered when developing high-quality materials for teaching, and furthermore, enhancing teachers' ability to utilize the materials for their professional development.

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THE ROLE OF TECHNOLOGY FOR LEARNING STOCHASTICS IN U.S. TEXTBOOKS FOR PROSPECTIVE TEACHERS

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Professional organizations have emphasized the use of technology in teaching and learning topics in stochastics (i.e., probability and statistics). Prospective elementary teachers need experiences with various types of technology in order to develop their technological pedagogical content knowledge. In the United States, such experiences may occur within a mathematics content course. Six textbooks commonly used for mathematics content courses for prospective teachers in the U.S. comprised the sample for this study. For each textbook in the sample, references to technology in the stochastics chapters were classified according to location (main text, auxiliary text, or activity) and also by type of technology. References located within activities and exercises were also coded according to whether technology was required or optional. The textbooks within this sample displayed considerable variation along the coded dimensions. A profile describing the use of technology for learning stochastics was developed for each textbook in the sample.

Keywords: pre-service teachers, primary school, textbook analysis, statistics, technology

INTRODUCTION

Professional organizations such as the International Society for Technology in Education (2007, 2008), the National Council of Teachers of Mathematics (2000), and the Conference Board of the Mathematical Sciences (2012) have advocated for the use of electronic technologies in the teaching and learning of mathematics. An ever-growing number of technological resources are available to both students and teachers. Some of these resources are content-generic (e.g., tablets, SMARTBoards), while others are mathematics-specific. Several different types of technology may be used in teaching and learning topics in stochastics (i.e., probability and statistics). For example, handheld calculators may be utilized or pre-programmed to compute measures of centre, spreadsheets have capabilities to quickly produce graphical displays, and some specially-designed software programs allow the user to design and run simulations.

Prospective elementary teachers need experiences with these various types of technology in order to develop their technological pedagogical content knowledge. In the United States, such experiences may occur within a mathematics content course. While a multitude of factors influence what actually occurs in the classroom, the textbook is a common feature in such courses. In this paper, I address the following research question:

> How is technology utilized and portrayed within chapters related to stochastics in mathematics textbooks for prospective elementary teachers in the United States?

CONTEXT AND RELATED RESEARCH

In the United States, an individual who wishes to become a teacher in a public school must earn a bachelor's degree and a teaching certificate. The majority of teachers in elementary school (kindergarten–grade 5, typically) have a degree in elementary education (Dossey, Halvorsen, & McCrone, 2012). Undergraduate programs in education vary by institution (National Council on Teacher Quality [NCTQ], 2008), but often prescribe courses in both mathematics content and methods of teaching mathematics. Content courses, offered by mathematics departments, have titles such as *Mathematics for Elementary Teachers I* or *Geometry for Elementary Teachers*; methods courses, offered in schools of education, have titles such as *Teaching Mathematics in the Elementary Classroom* or *Learning and Teaching Number and Operations*. While the Conference Board of the Mathematical Sciences (2012) recommends elementary teachers take 12 semester hours (540 classroom hours) of mathematics content from a teachers' perspective, a typical program requires 6 hours of content courses and 3 hours of methods courses (NCTQ, 2008).

One goal of teacher preparation programs – spanning both content and methods courses – is the development of pedagogical content knowledge. Shulman (1986) defined pedagogical content knowledge as "a second kind of content knowledge…, which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge *for teaching*" (p. 9). As such, it is the unique blending of content expertise and skill in pedagogy that forms a knowledge base to support robust instructional decisions.

Since the time of Shulman's initial work, electronic technologies have been developed at an ever-increasing rate. It is difficult to overestimate the impact of technology on teaching and learning, particularly in the area of mathematics. In order to investigate teachers' knowledge for teaching with technology, Niess (2005) expanded on the concept of pedagogical content knowledge and described the construct of technological pedagogical content knowledge as "the integration of development of knowledge of subject matter with the development of technology and of knowledge of teaching and learning" (p. 510). Applying this model to stochastics, Lee and Hollebrands (2011) represented teacher knowledge as a set of three concentric circles: the outermost circle of statistical knowledge contains technological statistical knowledge (TSK), which in turn contains technological pedagogical statistical knowledge (TPSK). Whereas teachers use statistical knowledge to recognize the need for data or consider variation, they may employ TSK to choose a technological tool to automate calculations, construct representations, or investigate real data. Teachers may engage TPSK when, for example, they consider the impact of technology on students' learning or evaluate the use of curriculum materials for teaching stochastics with technology.

Research on textbooks used in mathematics content courses for prospective elementary teachers in the U.S. is scarce. Three notable examples are McCrory's (2006) comparison of textbooks written by research mathematicians to those written by others, the NCTQ (2008) study of 77 U.S. teacher preparation programs, and the examination of the treatment of reasoning-and-proof by McCrory and Stylianides (2014). In each of these studies, the authors examined between 16 and 20 textbooks used in mathematics content courses for elementary teachers – nearly the entire population of such textbooks.

METHODOLOGY

Sample

Six textbooks comprised the sample for this study; they are listed in Table 1 along with the abbreviated name that will be used within this report. All of these textbooks (or other editions) were analysed by McCrory & Stylianides (2014) and NCTQ (2008). The first five textbooks listed were identified as the most commonly-used textbooks in the NCTQ study, and were also analysed by McCrory (2006). The sixth textbook, SSN, was also analysed by the NCTQ, although it was relatively new at the time. It was selected for this study, in part, because of its use at the authors' institution.

Table 1: Sample used in this study

Bas	Bassarear, T. (2008). *Mathematics for elementary school teachers* (4th ed.). Boston: Houghton Mifflin.
Bec	Beckmann, S. (2014). *Mathematics for elementary teachers with activities* (4th ed.). Boston: Pearson.
BLL	Billstein, R., Libeskind, S., & Lott, J. W. (2010). *A problem solving approach to mathematics for elementary school teachers* (10th ed.). Boston: Addison Wesley.
LDM	Long, C. T., DeTemple, D. W., & Millman R. S. (2012). *Mathematical reasoning for elementary teachers* (6th ed.). Boston: Addison Wesley.
MBP	Musser, G. L., Burger, W. F., & Peterson, B. E. (2011). *Mathematics for elementary teachers: A contemporary approach* (9th ed.). Hoboken, NJ: John Wiley & Sons.
SSN	Sowder, J., Sowder, L., & Nickerson, S. (2014). *Reconceptualizing mathematics for elementary school teachers* (2nd ed.). New York: W. H. Freeman and Company.

Coding

For each textbook in the sample, the chapters related to stochastics were identified from the table of contents. Each page within these chapters was examined for references to technology. References were identified when (1) the word "technology" appeared, (2) a specific type of electronic technology was mentioned, or (3) a portion of text was marked with a technology-related icon (e.g., a calculator or computer mouse). Internet addresses (URLs) were included when students were directed to view the content therein, but not when the URL was provided solely to cite the source of a dataset. These references were classified by type of technology (e.g., calculator, spreadsheet, internet application) and location (main text, auxiliary text, or activity). References within the *main text* were a part of the chapter prior to exercises, but not in a specially marked section; *auxiliary text* was contained within a sidebar or specially marked technology section. References within sections labelled as activities, problems, or exercises were coded as *activity* and further coded as to whether the use of technology was *required* or *optional* within the activity, problem, or exercise.

RESULTS AND DISCUSSION

The textbooks within this sample displayed considerable variation along the coded dimensions (see Table 2). The extremes were marked by Bas and SSN for both the proportion of pages within stochastics chapters containing references to technology and the total number of references to technology. Across the textbooks in the sample, technology was used to collect and manage data, generate random numbers to conduct simulations, create graphical displays, and compute values such as percentages, factorials, standard deviations, or equations of regression lines.

Table 2: Technology references within stochastics chapters of textbooks

Textbook	Proportion of pages referencing technology	Number of references to technology: Main text	Auxiliary text	Activity (required)	Activity (optional)	Total
Bas	4 out of 118 (3.4%)	1	2	1	0	4
Bec	6 out of 124 (4.8%)	2	2	4	0	8
BLL	9 out of 163 (5.5%)	3	3	3	4	13
LDM	6 out of 115 (5.2%)	2	0	7	0	9
MBP	21 out of 140 (15.0%)	0	5	24	0	29
SSN	27 out of 174 (15.5%)	6	0	20	20	46

Most textbooks provided specific directions for performing a task with a specific type of technology (e.g., calculating standard deviation with a scientific calculator). These types of references occurred within the main text of BLL and LDM and the auxiliary text in Bas, BLL, and MBP. (SSN offers such instructions on its companion website, which was not examined for this study.) Each textbook in the sample contained at least one activity, problem, or exercise requiring the use of technology. In BLL and SSN, about half of the activities or exercises that were analysed provided the option of using technology; in almost every case, these tasks were set in the context of conducting simulations, and included a table of randomly selected digits as a non-technological option.

With respect to the types of technology, each textbook in the sample addressed the use of calculators, and spreadsheets were mentioned by each textbook in the sample with the exception of Bec. Four textbooks (Bec, BLL, MBP, and SSN) directed students to use web-based applications, and three textbooks (Bas, Bec, and SSN) required the internet to download data.

Implications for the development of TSK and TPSK

These references primarily addressed the development of TSK. That is to say, students are provided with opportunities to use technology to perform tasks related to stochastics. As these textbooks are intended for use in content courses, this emphasis seems appropriate. Furthermore, three of the textbooks in the sample (LDM, MBP, and SSN) provided guidance in interpreting apparently different results that arise from using different types of technology, which may also impact a prospective teacher's TSK.

With that said, these six textbooks may not have similar influences on a teacher's TSK. To illustrate, I will provide a descriptive profile for each textbook in the sample. Within Bas, there were very few references to technology; the two references to specific technologies addressed downloading data and calculating percentages. In Bec, five out of eight references directed students to download data. The technology references in BLL emphasized using a calculator to generate random numbers and compute factorials, summary statistics, and data displays. LDM also referred to using a calculator for computing factorials and standard deviation, and used spreadsheets to conduct simulations. In MBP, the technology references focused on using *e-Manipulatives,* located on the book's companion website, to construct data displays and conduct simulations. SSN contained the greatest variation of types of technology, where prospective teachers (or their instructors) were given several options to conduct simulations, construct data displays, and compute summary statistics; these options included calculators, spreadsheets, dynamic statistical software packages, and open-access web applications from the National Council of Teachers of Mathematics.

On the whole, it was difficult to determine how or whether any of the references in these textbooks would affect the development of TPSK. Two candidates occurred in Bec, with links to reports from professional organizations that address the teaching and learning of stochastics.

Limitations of the study

One limitation of this study lies in the sample. While the most commonly-used textbooks were selected for this study, a number of other textbooks (at least ten) are available for mathematics content courses for prospective elementary teachers. Furthermore, an instructor may choose not to use any particular textbook. This leads to the second limitation of the study: textbooks do not dictate what occurs in the classroom. An individual instructor may supplement a textbook to enhance the use of technology, or he may choose to ignore some or all of the technology references. This study does provide a comparison of textbooks as they are written. As textbooks may influence an instructor's decisions, provide structure to courses, and serve as a reference to students, such a study serves as one measure of what may occur in classrooms.

CONCLUSION

Not all mathematics textbooks for prospective elementary teachers are the same. This study highlights the variation in the extent and nature of the use of technology in teaching and learning stochastics. It is important to note that these findings provide a description, but not an evaluation, of the textbooks in the sample. Nevertheless, if the goal is to develop technological pedagogical content knowledge within prospective elementary teachers, textbooks should provide multiple opportunities which utilize different types of technology within all content areas – including, but not limited to, stochastics.

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MODEL METHOD IN SINGAPORE PRIMARY MATHEMATICS TEXTBOOKS

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The model method has been widely recognised as a signature pedagogy of Singapore primary mathematics, and because of Singapore's outstanding performance in mathematics in both TIMSS and PISA, there has been an increasing interest in this method in mathematics education worldwide. Focusing on the design and development of the model method, in this paper we shall explain and discuss how the model method is presented in the textbooks, discuss the why and how of the model method, and make connections between the model method and the algebraic method. We argue that it is necessary to integrate the model method with the algebraic method to help students develop their confidence and competence in using the algebraic method which is a fundamental skill in higher mathematics. This approach has been incorporated in the current Singapore secondary mathematics textbooks. The paper ends with a discussion on the implications of the model method on mathematics curriculum and textbooks.

Keywords: primary school, model method, algebra, Singapore

INTRODUCTION

Since independence in 1965, the Singapore education system has undergone several reforms, of which the New Education System (NES) in 1979 is a major landmark (MOE & NIE, 2012, p.1). Under the NES, the Curriculum Development Institute of Singapore (CDIS) was set up in 1980 to take charge of the development of quality curriculum packages and teacher training in order to improve the teaching and learning in schools. Under CDIS, the Primary Mathematics Project team, led by Dr Kho Tek Hong, developed the CDIS primary mathematics textbooks and instructional packages in the 1980s and 1990s for all schools to use. The model drawing method, or simply the model method, was developed by the project team, and it was introduced in the 1980s and became a distinguishing feature of the CDIS primary mathematics textbooks. In fact, Fan conducted a survey and found that the model method was known by all Singapore mathematics teachers and mathematics educators in the survey and was viewed highly positively by them (Fan, 2007). The CDIS textbooks and their adapted versions have been used in schools in many countries because of Singapore students' outstanding performance in mathematics in both TIMSS and PISA, and there has been an increasing interest in the model method in mathematics education worldwide.

In this paper, we focus on the design and development of the model method in the textbooks, discuss the why and how of the model method, and make connections between the model method and the algebraic method. The integration of the model method with the algebraic method in the current Singapore secondary mathematics textbooks for solving word problems will also be discussed.

WHY THE MODEL METHOD?

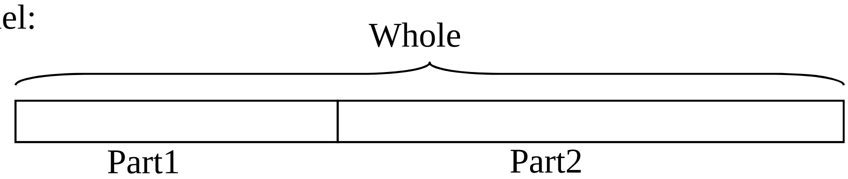
It was widely realised in the early 1980s that many students in various countries had difficulties in understanding and solving mathematics word problems, and Singapore students were no exception. It was for this reason that the model method was developed and introduced by the CDIS project team into the primary mathematics curriculum. The method entails students drawing a pictorial model to represent known and unknown quantities and their relationships given in a problem. The pictorial model helps students, especially the visual learners, understand the concepts of the four operations, fraction, ratio, and percentage, and solve word problems. Numerous training workshops were conducted to familiarise teachers with the model method, and intensive trials of the model method were carried out in classrooms in the early 1980s. The model method was well received, and was found to be a powerful tool for solving structurally complex problems (Kho, 1987).

DESIGN AND DEVELOPMENT OF VARIOUS TYPES OF MODELS

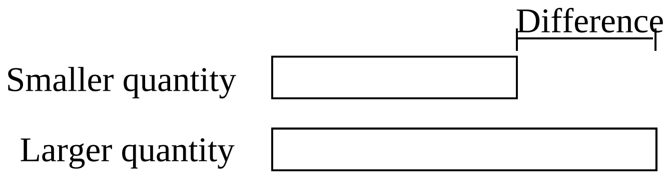
The model method, also known as bar modelling, is partly based on the part-whole and comparison models which are pictorial forms of Greeno's part-part-whole and comparison schemas[1] for addition and subtraction word problems (Nesher, Greeno & Riley, 1982; Kintsch & Greeno, 1985). These schemas represent the conceptual structures of addition and subtraction word problems. In the Singapore primary mathematics textbooks, the part-whole and comparison models were further developed to include multiplication and division, as well as fraction, ratio, and percentage. The models enable students to visualise the problem structure and make sense of the quantitative relationships in word problems (Kho, 1987; Kho, Yeo & Lim, 2009; Ng & Lee, 2009; Yeap, 2010). The various models are elaborated below.

Part-whole and comparison models for addition and subtraction

Part-whole model:



Comparison model:



Each model represents a quantitative relationship among three variables. In the part-whole model, the variables are *whole*, *part1* and *part2*. In the comparison model, the variables are

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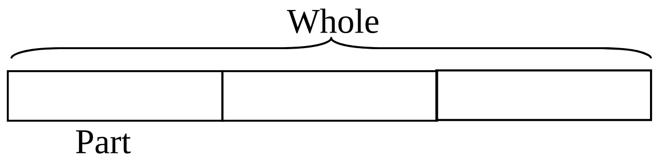
[1] In the cognitive theory of learning, schemas (or schemata) are building blocks of mental structures and cognitive processes. Students construct schemas in the process of learning and problem solving. They use the schemas to successfully comprehend and solve problems.

larger quantity, *smaller quantity* and *difference*. Given the values of any two variables, we can find the value of the third one by addition or subtraction. A part-whole model may comprise more than two parts, and a comparison model may involve three or more quantities. For a comparison problem, the sum of some or all of the quantities may be given, and this may be indicated in the model. The model method was first introduced in the primary mathematics textbooks for Primary Four (the 4th grade) in 1983. The textbooks in the 1990s introduced it at Primary Three. Since the 2000s, it has been introduced at Primary Two as the model method has become a common strategy in Singapore primary schools. When students solve a problem by drawing a part-whole or comparison model, they consciously make use of the problem schema to visualise the problem structure, make sense of the quantitative relationship in the problem, and determine what operation (addition or subtraction) to use to solve the problem.

Part-whole model for multiplication and division

The part-whole model for multiplication and division was first introduced at Primary Five in 1984. The textbooks in the 1990s introduced it at Primary Three. Some textbooks in the 2000s introduced it at Primary Two. The current textbooks introduce it at Primary Three.

The following part-whole model represents a whole divided into 3 equal parts:



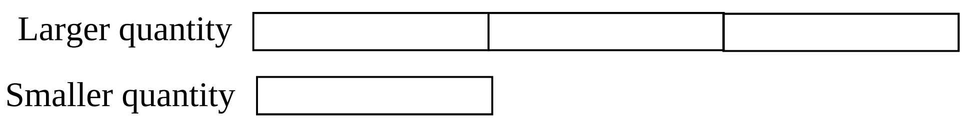
The model illustrates the concept of multiplication as:

$$\boxed{\text{One part}} \times \boxed{\text{Number of parts}} = \boxed{\text{Whole}}$$

The whole is the product of *one part* and *number of parts*. Knowing the two factors (*one part* and *number of parts*), we find the whole by multiplication. Conversely, knowing the whole and one factor, we find the other factor by division.

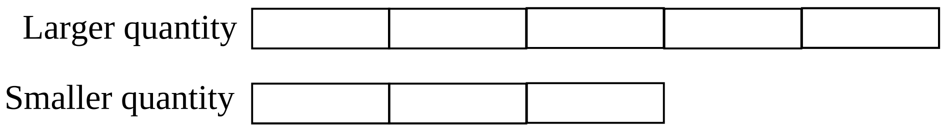
Multiplicative comparison models

The comparison model can be used to represent two quantities such that one quantity is a multiple of the other, e.g.



In this model, the larger quantity is 3 times as much as the smaller quantity, and the smaller quantity is equal to $\frac{1}{3}$ of the larger quantity. Here the larger quantity is 3 units, and the smaller quantity is 1 unit. The sum of the two quantities is 4 units, and the difference between them is 2 units.

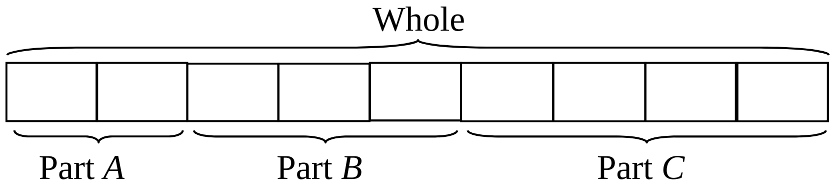
The following model shows two quantities which are 5 units and 3 units respectively.



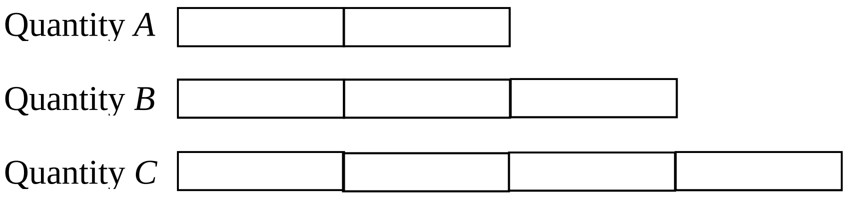
Here the smaller quantity is equal to $\frac{3}{5}$ of the larger quantity, and the larger quantity is equal to $\frac{5}{3}$ of the smaller quantity. The larger quantity is $\frac{2}{3}$ more than the smaller quantity, and the smaller quantity is $\frac{2}{5}$ less than the larger quantity.

Ratio models

In Singapore, students learn the concept of ratio at Primary Five, and they use the part-whole and comparison models to illustrate the concept. The following part-whole model shows a whole divided into three parts *A*, *B* and *C* in the ratio 2:3:4.



The following comparison model shows three quantities *A*, *B* and *C* which are in the ratio 2:3:4.



In these models, the ratio 2:3:4 means “2 units to 3 units to 4 units”. That is, *A*, *B* and *C* are 2 units, 3 units and 4 units respectively. The ratio language makes explicit the proportional relationship among *A*, *B* and *C*.

Model method and structurally complex problems

For structurally complex problems, the drawing of bar models may involve some variations of part-whole and comparison models, or a combination of both models. When students solve word problems using the model method, they consciously mark “1 unit” on the model, and solve the problem by finding the value of 1 unit first, as shown in the following example. The solution process varies.

> Devi and Minah have \$520 altogether. If Devi spends $\frac{2}{5}$ of her money and Minah spends \$40, then they will have an equal amount of money left. How much money does Devi have? (Kho, 1987, p.350; Kho, Yeo & Lim, 2009, p.80)

<u>Variation 1</u>

The following model represents the after and before situations of the problem. In the after situation, the two amounts of money are equal. These amounts are respectively $\frac{3}{5}$ of Devi’s money and \$40 less than Minah’s money in the before situation.

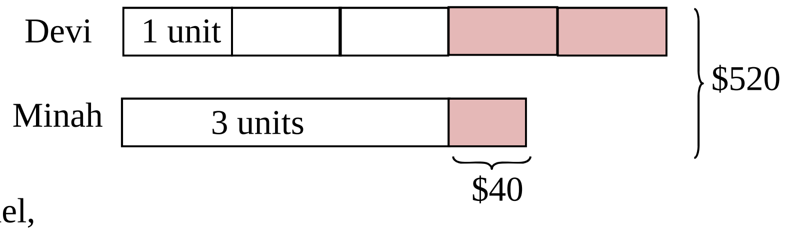
After:

Before:

From the model,
8 units = \$520 – \$40 = \$420; 1 unit = \$480 ÷ 8 = \$60
Devi's money = 5 units = \$60 × 5 = \$300

<u>Variation 2</u>

Here is another way of drawing the model:



From the model,
8 units = \$520 – \$40 = \$420; 1 unit = \$480 ÷ 8 = \$60
Devi's money = 5 units = \$60 × 5 = \$300

MODEL METHOD AND ALGEBRAIC METHOD

The following shows how the problem given in the previous section can be solved by the algebraic method. Variation 3 involves a pair of simultaneous linear equations in two variables, and Variation 4 involves a linear equation in one variable.

<u>Variation 3</u>

Let Devi's money be \$$x$, and Minah's money be \$$y$. Then
$x + y = 520$ and $\frac{3}{5}x = y - 40$.
Solving the equations for x and y, we have $x = 300$ and $y = 220$. Therefore Devi has \$300.

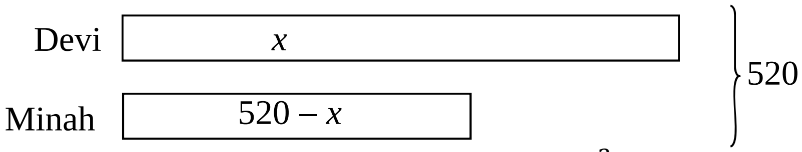
<u>Variation 4</u>

Let Devi's money be \$$x$, and Minah's money be \$(520–x). Then $520 - x - 40 = \frac{3}{5}x$.
Solving the equation for x, we have $x = 300$. Therefore Devi has \$300.

In Singapore, students learn linear equations in one variable at Secondary One (the 7th grade), and simultaneous linear equations at Secondary Two. Many of them have difficulty in formulating the equation(s) and continue to use the model method instead of the algebraic method to solve problems. Formulating algebraic equations is a key step that makes the algebraic method different from the arithmetic method for solving word problems. In the arithmetic method, students simply perform arithmetic operations to calculate unknown quantities from known quantities without formulating an algebraic equation. As the arithmetic and algebraic methods are procedurally different, many students encounter difficulties in transiting from one to the other. These students are familiar with using the model method to solve word problems in primary schools, and prefer to use the same method in secondary schools. It is therefore necessary to integrate the model method with the algebraic method to bridge the cognitive gap from arithmetic to algebra (Kho, 1987; Kho, Yeo & Lim, 2009, p.79; Fong, 1994; Ng, 2001).

In the following, Variation 5 illustrates how the model method can be integrated with the algebraic method above to help students formulate a linear equation in one variable, and Variation 6 is the algebraic equivalent of Variation 2 shown in the previous section.

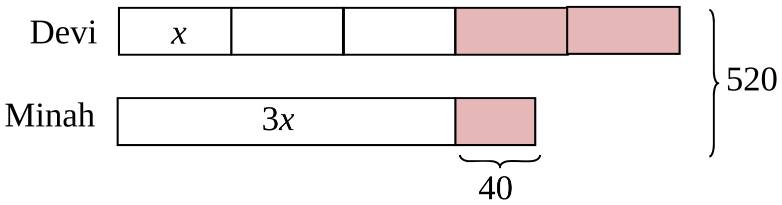
<u>Variation 5</u>



From the model, we obtain the equation $520 - x - 40 = \frac{3}{5}x$.

Solving the equation for x, we have $x = 300$. Therefore, Devi's money = \$$x$ = \$300.

<u>Variation 6</u>



From the model, we obtain the equation $8x + 40 = 520$.
Solving the equation for x, we have $x = 60$. Therefore, Devi's money = \$$x$ = \$300.

The equation in Variation 6 involves only whole numbers, and so is much easier to solve than the equation in Variation 5. Students should have opportunities to share and discuss the different ways to solve a problem.

IMPLICATIONS ON MATHEMATICS CURRICULUM AND TEXTBOOKS

The Concrete-Pictorial-Abstract (CPA) approach[2] is a pedagogy adopted by Singapore primary mathematics textbooks since the early 1980s. At the concrete stage, students are provided with the necessary learning experiences and meaningful contexts, using concrete objects such as picture cut-outs to talk about and tell stories about the concepts of the four operations, and to make sense of the part-whole and comparison relationships. At the pictorial stage, students' progress to using pictorial representations (set diagrams) to help them understand the concepts, followed by drawing bar models to represent the part-whole and comparison relationships. They use the models to help them understand the problem situation and work out the solution to the problems. Eventually, at the abstract stage,

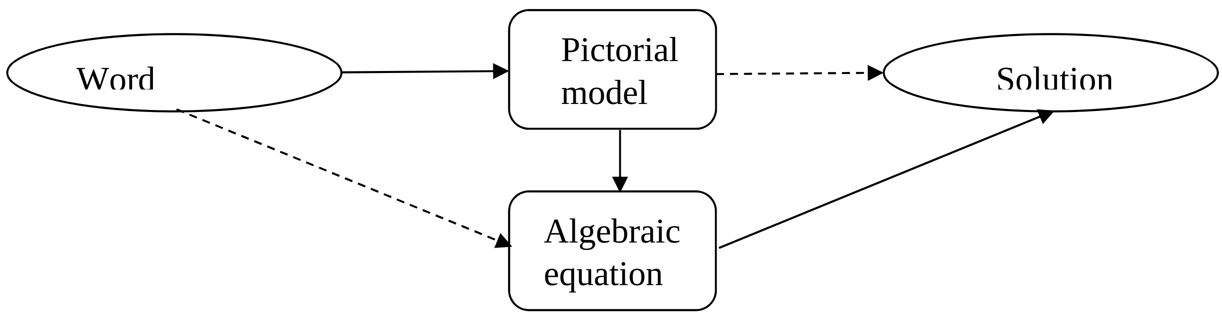
[2] In the CPA approach, students develop conceptual understanding of mathematics concepts beginning with concrete experiences and representations, followed by pictorial representations, and then abstract representations.

students may solve problems without drawing models. The bar model is a visual aid; students are not required to describe the model or to explain how it is constructed.

In this paper, we have illustrated that drawing a pictorial model as problem representation plays an important role in the problem comprehension and solution processes. As students draw, think, examine, reflect and discuss about the models, they engage in active construction of meaning, mathematical reasoning, monitoring their own thinking process as well as self-regulating their own learning. Model drawing also provides opportunities for problem posing. Students may work in groups to create word problems with similar problem structures (Kho, Yeo & Lim, 2009, Appendix B).

The use of the model method to solve mathematics word problems has been explicitly included as part of *Learning Experiences* in the latest national primary and secondary mathematics syllabuses (MOE, 2012a, 2012b & 2012c). This will be reflected in the new textbooks.

The model method may be regarded as a pre-algebraic method. The integration of the model method with the algebraic method can be shown diagrammatically as:



In Singapore, students use the model method before they learn to solve algebraic equations. In primary schools, when students use the model method to solve a problem, they draw a model and use it to work out the arithmetic steps to find the answer. In secondary schools, when students use the algebraic method to solve a problem, they formulate an algebraic equation from the problem text and solve the equation to find the answer. When the model method is integrated with the algebraic method, students first draw a model and use it to formulate the algebraic equation. It should be noted that, as a relatively new development, this approach has been incorporated in the latest secondary mathematics textbooks. It is hoped that the integration of the two methods will also help students at the secondary level develop their confidence and competence in using the algebraic method, which is a fundamental skill in higher mathematics. Further classroom-based research is needed to support and improve the integration of the model method into secondary school mathematics curriculum and textbooks.

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TEXTBOOK ANALYSIS: EXAMINING HOW KOREAN SECONDARY MATHEMATICS TEXTBOOKS SUPPORT STUDENTS' MATHEMATICAL THINKING

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This study aimed to examine how Korean secondary mathematics textbooks support students' mathematical thinking and learning. For the purpose mathematical tasks in selected textbooks for secondary students (grades 7-10) in terms of the levels of cognitive demand by using the task analysis framework suggested by Stein & Smith (1998). For the analysis, I first identified tasks in all the selected textbooks and coded each task as memorization (M), procedures without connections (PNC), procedures with connections (PWC), and doing mathematics (DM). The findings from the analysis revealed that 94 percent of the tasks from the secondary mathematics textbooks were at the low level of cognitive demand, that is, M and PNC tasks. In particular, the low level tasks mostly focused on PNC tasks that have no connections to concepts or meaning underlying the procedures, by requiring no explanations. Also, the findings suggested that the tasks in the textbooks hardly provide students with opportunities to use procedures for developing understanding of mathematical concepts and to explore mathematical relationships.

Keywords: secondary school, task analysis, cognitive demands, Korea

INTRODUCTION

School mathematics textbooks have been considered as a crucial medium in mathematics education. Textbooks are major resources for teachers in planning and implementing lessons (Grouws, Tarr, Chavez, Sears, Soria, & Taylan, 2013; Kim, M., 2013). Textbooks are strong determinant of what and how students learn; thus, school mathematics textbooks focus both on mathematical contents and processes in order for students to develop conceptual understanding and problem solving and reasoning skills. In Korea, mathematics textbooks are developed based on the national curriculum guideline, which is controlled by the government. The Korean National Curriculum has gone through several amendments since 1950s; as such, textbooks were developed or revised according to the amendments. Research on curriculum materials reveal that *Standards*-based curriculum materials dominantly consist of mathematical tasks that place high-level cognitive demands on students (Stein & Kim, 2009; Kim, Lee, A. & Lee, S., in preparation). Thus, this study aims to examine the mathematical tasks included in Korean secondary textbooks in terms of how the tasks support students' mathematical thinking and learning through the lens of the level of cognitive demand of tasks suggested by Smith & Stein (1998).

METHOD

In order to examine mathematical tasks in Korean secondary textbooks in terms of the level of cognitive demand, I selected function and geometry in textbooks for grades 7-9 and the whole contents in textbooks for grade 10. Those textbooks were published in 2010 according to 2007 National Curriculum. There were 18 different publishers in the middle grades and 25 in the high school level; among them, 5 middle school textbooks and 2 high school textbooks were chosen for the analyses. Also, I adopted the Task Analysis Guide by Stein, Smith, Henningsen & Silver (2000, p. 16) for analysing tasks. According to the guide, tasks can be categorized as memorization tasks, procedures without connections tasks, procedures with connections tasks, and doing mathematics tasks; the first two are considered as low-level and the other two as high-level. As an initial step, mathematical tasks in all the selected textbooks were classified; I identified about 397 tasks in function from 5 different textbooks, 2412 tasks in geometry from 3 textbooks, and 2565 tasks from 2 different textbooks for grade 10. Then, all the tasks identified were coded as: a) memorization for relying on memorization and reproducing facts, rules definitions, or algorithms [M], b) procedures without connections to concepts, meaning or understanding [PNC], c) procedures with connections to concepts, meaning or understanding [PWC], and d) doing mathematics by engaging in complex and non-algorithmic thinking, problem solving, reasoning, and justification [DM].

FINDINGS

The findings from the analyses suggested that 94 percent of the tasks in all the selected textbooks were at low level of cognitive demand (details shown in Tables 1, 2, 3).

Table 1: The level of cognitive demand of the tasks in function

	M	PNC	PWC	DM
A	5% (4/77)	88% (68/77)	7% (5/77)	0% (0/77)
B	4% (3/74)	90% (67/74)	3% (2/74)	3% (2/74)
C	2% (2/114)	90% (103/114)	6% (7/114)	2% (2/114)
D	3% (2/63)	97% (61/63)	0% (0/63)	0% (0/63)
E	4% (3/69)	92% (65/69)	1% (1/69)	0% (0/69)
Total	3% (14/397)	92% (364/397)	4% (15/397)	1% (4/397)

Table 2: The level of cognitive demand of the tasks in geometry

	M	PNC	PWC	DM
B	4% (35/845)	92% (774/845)	3% (25/845)	1% (11/845)
E	10% (88/886)	85% (749/886)	3% (31/886)	2% (18/886)
F	9% (59/681)	86% (589/681)	3% (22/681)	2% (11/681)
Total	7% (182/2412)	88% (2111/2413)	3% (78/2412)	2% (40/2412)

Table 3: The level of cognitive demand of the tasks in the textbooks for grade 10

	M	PNC	PWC	DM
B	5% (74/1376)	88% (1207/1376)	6% (86/1376)	1% (9/1376)
E	3% (41/1189)	91% (1085/1189)	5% (59/1189)	1% (4/1189)
Total	5% (115/2565)	89% (2292/2565)	5% (145/2565)	1% (13/2565)

In particular, PNC tasks were dominant in function (92%), geometry (88%), and contents in grade 10 (89%). All the selected textbooks included limited number of high-level tasks (5 percent); there were very few DM tasks (1 percent) in the textbooks.

CONCLUSION

The findings show that the mathematical tasks in Korean secondary textbooks focus on simple computation through repeated use of procedures and producing correct answers rather than developing mathematical understanding. The results also suggested that the tasks in Korean textbooks hardly promote students to engage in conceptual understanding and non-algorithmic thinking so that students have little opportunities to think non-algorithmically and explore and understand mathematical processes and relationships. Interestingly, this does not align with the 2007 National Curriculum of Korea which puts strong emphasis on developing students’ problem solving and reasoning skills.

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KOREAN STUDENTS’ USE OF MATHEMATICS TEXTBOOKS

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This study aims to investigate Korean secondary students’ beliefs of their use of mathematics textbook and to make a contribution to further study by providing information on the use of textbook in Korea. For the purpose of this study, we develop a survey asking questions about textbooks’ role in students’ learning and questions about mathematical concepts, principles and formulas, and problem solving in Korean mathematics textbooks. The results show that Korean students highlight not only the importance of textbooks but they also put great emphasis on their teachers’ explanation in studying mathematics. Korean students use their textbook mainly for their learning; however they selectively work through mathematical tasks suggested in their textbook.

Keywords: student beliefs, secondary school, Korea

INTRODUCTION

There has been growing interest in the differences of learning from different countries since international comparative studies such as the Trends in International Mathematics and Science Study (TIMSS 2007, 2011) and Programme for International Student Assessment (PISA 2012). Korean students have showed great performance in the results of the international comparative studies. Recently, Hong & Choi (2014) compare Korean and American secondary school textbooks, in particular the case of quadratic equation. Their results show that the problems in American textbooks required more explanations and various representations than the problems in Korean textbooks. These results seem to conflict with the performance in the international comparative studies. However, there is little information about how to teach mathematics using textbooks in Korea and how Korean students use their mathematics textbooks for their learning.

Research in mathematics education has been concerned with the utilization of mathematics textbooks and its effects on students’ learning. Although the mathematics textbook is one of essential resources for teaching and learning mathematics, studies have examined the use of mathematics textbooks mostly by teachers (Nicol & Crespo, 2006; Remillard, 2005; Richard, 1996). On the other hand, we have limited information on the use of mathematics textbooks by students (Love & Pimm, 1996; Rezat, 2010). In order to develop a better understanding of students’ learning influenced by textbooks, we pursue to examine how Korean students use their textbooks for their mathematics learning.

In Korea, secondary mathematics teachers select a mathematics textbook that they teach for a year, and students buy the mathematics textbook for the school year. In fact, since students use their mathematics textbooks at school and also at home for further study, it is hardly to measure the actual use of the mathematics textbook by students. In the present study, we focus on how students perceive their use of mathematics textbooks not measuring how much students use them. We develop a survey on how students think of their mathematics textbooks and how they use concepts, principles, and formulas in textbooks. The results of this study provide foundation information on Korean students' use of mathematics textbook in the secondary level.

METHODS

For the purpose of this study, we developed a survey about the utilization of mathematics textbooks. The survey consisted of 7 items of backgrounds and 37 items included four parts of questions of mathematics textbook use in terms of understanding the textbook's role, mathematical concepts, mathematical principles and formulas, and mathematics problem solving for students' learning. A five-point Likert scale survey format was used in the questionnaire.

Korean secondary schools began school year early March to late July for spring semester and early September to late December for fall semester. In order to investigate students' thinking about the use of their textbooks during the whole school year, data was collected during winter break. We surveyed 391 middle and high school students in two metropolitan areas of Korea. Middle school students (grades 7 to 9) and 10th grade students in high schools participated in this study. Since the 10th grade mathematics was common subjects and grades 11 and 12 took elective courses in Korean mathematics curriculum at the time of data collection, we decided to collect data from the 10th grades only in high schools. The valid data constituted of 380 respondents without incomplete response (See table 1).

Table 1: Validation of participants

	Middle school	High School
Number of respondents	250	130
Number of Female	58	71
Number of classrooms	6	4
Number of schools	3	4
Respondents' Grades	7, 8, 9	10

RESULTS

Results from the analysis of the survey suggested that Korean secondary school students consider their textbooks as a crucial material for their mathematics learning. In the survey, the students positively responded that they needed a mathematics textbook for learning mathematics. In addition, the students checked that they used workbooks (58%) and their textbooks (31%) most often for their mathematics learning. In specific, among the respondents checking mathematics textbooks most often, approximately 37 percent of the middle school students responded that they used it most often; however, approximately 18 percent of the 10th grade students responded most often on the same question. This result implied that the Korean high school students used often not only their mathematics textbooks but also highly relied on their workbooks to study mathematics.

Interestingly, Korean secondary students seemed to not work through all the tasks suggested in their textbook. For instance, the middle students responded the following questions with the lower scores than average: “I read extra reading materials relevant to the history of mathematics or mathematics in real life situations in mathematics textbook”(avg. 2.76), “The extra reading materials help me understand mathematics”(avg. 2.94), and “The extra reading materials make me have interests in mathematics”(avg. 2.83). Those responses inferred that Korean students seemed to select mathematical tasks in spite of a variety of tasks including readings relevant to mathematics.

Approximately 62 percent of the Korean secondary students in this study believed that teachers had a prior authority in making a decision on the accuracy of solving problems. Even though the participants regarded a textbook as essential for their learning of mathematics, they responded that they trusted their teachers’ explanation of mathematical concepts and principles more than ones in textbooks or other documents. This result confirmed that teachers had a great influence over their students’ mathematical thinking process. However, the Korean secondary students presumably supposed that teachers explained mathematical concepts and ideas based on the textbooks and put emphasis on principles and formulas in the textbooks.

FURTHER STUDY

For further study, we should examine the tendency and influence of mathematics textbooks on Korean students’ learning and also explicate the Korean students’ concerns in their textbook use. In addition, as Hong & Choi (2014) indicated, we need to study the link between written curriculum and enacted curriculum. The Korean secondary students in this study gave us hints of how Korean teachers use textbooks in their teaching. The further study on how teachers teach with mathematics textbooks would help us understand students’ learning mathematics.

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THE ANALYSIS OF TEACHERS' MOBILISATION OF THE TEXTBOOK AFFORDANCES

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In this paper, which forms part of a wider study investigating the relationship between the affordances of a textbook, and teachers' pedagogical design capacity (PDC) (Brown, 2002) in the mediation of the object of learning; we report on the analysis of teachers' mobilisation of the textbook. In the wider study, we have established the mathematics content, and the approach to the teaching and learning of the content as the major potential affordances (Gibson, 1977) of the textbook to the teachers' practice. In this paper, we focus on the mobilisation with respect to the content; and demonstrate that teachers' appropriation of the content of the textbook is more a function of insertions to, and omissions from the textbook, than it is of their offloading, adapting or improvising of the content of the textbook.

Keywords: textbook affordances, textbook insertions, textbook omissions, textbook use

INTRODUCTION

A recent review of research on textbooks by Fan et al (2013) indicates that textbook use makes up only 25% of this research; hence, a need for more research on textbook use, that is, "on studies focusing on how textbooks are used by teachers and/or students; in other words, how textbooks shape the way of teaching and learning of mathematics" (Fan et al., 2013, p. 635). In South Africa itself where the study originates, we have found five studies on textbooks, three of which are concerned with textbook use, albeit in different ways (Adler, Dickson, Mofolo, & Sethole, 2001; Ensor et al., 2002; Taylor & Vinjevold, 1999); and two studies that look at different aspects of use (Bowie, 2013; Fleisch, Taylor, Herholdt, & Sapire, 2011). In our view, five research studies in South Africa is a relatively small number considering how highly and importantly the textbook is regarded as a useful resource for teaching and learning in South Africa. Hence, the current study is a welcome addition and extension to this field of research in South Africa specifically, and more generally.

The study involves seven Grade 10 teachers from three schools in one township in South Africa forming part of three clusters of schools participating in the Wits Maths Connect Secondary Project (WMCS)1. All Grade 10 teachers in these schools were invited to participate in the study and they accepted the invitation. The participating teachers shall be referred to as teachers A1; A2; A3; A4; B1; B2; and C1, for reasons of anonymity and confidentiality. The data used for this current article involves transcriptions of twenty (20)

[1] Wits Maths Connect Secondary Project is a research and professional development project aimed at the improvement of the teaching and learning of mathematics in ten secondary schools in one education district in South Africa (relatively disadvantaged)

videotaped class observations comprising of three lessons each for six teachers and two lessons for one teacher whom we could not obtain a third lesson due to timetabling clashes. The prescribed textbook which is the same in the three schools is also used as a data source.

In the wider study, we investigate the relationship between teachers' pedagogical design capacity (PDC), that is, teachers' "capacity to perceive and mobilise existing resources in order to craft instructional episodes"(Brown, 2002); and the mediation of the object of learning (that which has to be learned) in the classroom. In the current paper, we confine the analysis of data to how teachers appropriate the content of the textbook, focussing specifically on the insertions and omissions which teachers make in the process, and their role in the mediation of the object of learning. The study aligns itself with views of textbook use as participation with the text wherein teacher and textbook enter into a "dynamic interrelationship in which each participant (teacher and text) shapes the other; together they shape instruction"(Stein & Kim, 2009, p. 39); hence we recognise Brown's (2002) *Design Capacity for Enactment (DCE) Framework* and Remillard's (2005) *Framework of components of teacher-curriculum relationship*, as foundations for the development of analytical tools for the study.

THEORETICAL GROUNDING

The study is theoretically grounded in socio-cultural theory (Vygotsky, 1978) wherein all humans are inherently social beings and grow from and through the use of tools; in mediated action (Wertsch, 1998) where the relationship of agents towards mediational means can be characterised "in terms of appropriation" (p.25); and where the textbook is considered as an artefact (Wartofsky, 1973) that can afford and constrain teacher's activity. This theoretical grounding points to the importance of attending to the context in which the teacher-text interactions occur; the need for determining potential affordances and constraints of the textbook to the teacher's practice, together with establishing the degree and manner of appropriation of the affordances by the teacher. While we background the context in this study, we are continually cognizant of contextual influences in how teachers mobilise their textbooks.

ANALYTICAL FRAMEWORK

According to Brown's (2002) DCE Framework, the teacher-tool interactions "can be understood in terms of different degrees of artefact appropriation: offloading, adapting, and improvising (emphasis in original)"(p.34). Furthermore, Brown (2009) suggests that the appropriation of the affordances may be depicted as a continuum with offloading and improvising at the two extremes of the continuum and adapting lying in the middle. These three types of textbook use depict the degree of appropriation of the affordances of the curriculum resource, with offloading referring to teachers relying to a very large extent on the curriculum resource for the delivery of the lesson. Improvising refers to when teacher improvises own materials in the lesson and uses the curriculum resource quite minimally; while adapting implies depending equally on the curriculum resource and own resources to deliver the lesson. We recruit Brown's scale of artefact appropriation as a framework for determining how teachers mobilise the content of the textbook, as depicted in Figure 1 below.

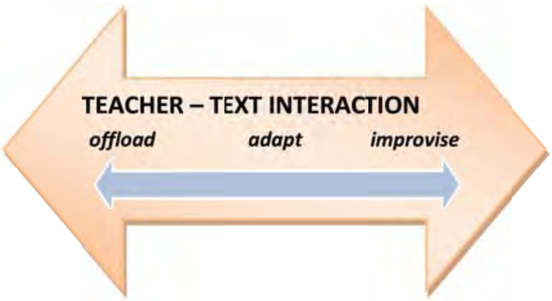

Figure 1. A framework for teachers' appropriation of textbook content

Using this framework, the twenty lessons in the study were analysed for how teachers appropriated the content of the prescribed textbook which happened to be the same for the three schools.

In determining how teachers offloaded or improvised or adapted the content in the lesson, we also noted if and how teachers were making insertions, or omitting aspects of the topic stipulated in the textbook. We define *insertions* as the aspects of the topic which do not feature in the prescribed textbook, but which teacher brings into the lesson. For example, two of the teachers demonstrate the vertical line test for distinguishing between functions and non-functions in their lessons. However, the vertical line test does not feature in the prescribed textbook that teachers use, and therefore we regard it as an insertion to the content. In analysing the lessons, we realised that there were two kinds of insertions: those which served to enhance the object of learning, such as the vertical line test, which we call robust insertions and code ins+ in the analysis; and those which were unproductive and sometimes led to erroneous mediation of the object of learning, which we are calling distractive and code ins-in the analysis.

In a similar manner, we define *omissions* as those aspects of the textbook content which teachers omit from the lessons. For example, in one lesson, teacher assigned questions from the following textbook exercise for learners as homework.

- Work in groups.

1. Draw the graphs of the following on separate axes. Use the table method. Include both positive and negative values for the independent variable.
   a) $g(x) = x^2$ b) $f(x) = 2^x$ c) $h(x) = \frac{8}{x}$
2. Look up the meaning of the word "symmetry" in a dictionary (or refer to Chapter 9 for a definition).
   a) Which of the curves you have drawn has symmetry?
   b) Identify the line(s) of symmetry in each case.
3. a) Investigate what happens to the function values for each function when the x values become:
   (i) bigger and positive (for example use $x = 10$; 100; 1 000)
   (ii) smaller and negative (for example use $x = -10$; -100; -1 000)
   (iii) closer to zero, either positive or negative (for example use $x = 0{,}1$; $0{,}01$; $0{,}001$; $-0{,}1$; $-0{,}01$; $-0{,}001$).
   What effects do these values have on each graph?

Figure 2. An extract from a prescribed textbook

Learners were assigned questions 1 and 2 above for homework, which required them to draw the graphs of the three functions as mentioned and to identify their lines of symmetry where applicable. However, question 3 which required learners to investigate the end-behaviour of the functions drawn in question 1, and question 4 on maximum and minimum values (not shown on Figure 2), were not assigned as homework, or for class work in the lesson after discussing the answers to questions 1 and 2. Instead, teacher chose to assign questions from a different exercise which featured neither the investigation of the end-behaviour nor maximum and minimum values of functions.

This is an example of an omission which we consider as critical to the mediation of the object of learning, as this omission detracts from the object of learning. In this particular example, the end-behaviour of functions is a key feature for the hyperbola and exponential functions, while the maximum/minimum value feature is critical to quadratic functions; their omissions hence detract from the object of learning. The critical omissions are coded om- in the analysis, while the *impotent* omissions, which do not disrupt the object of learning, are coded om^+.

RESULTS AND DISCUSSION

In Table 1 below, we provide a summary of the analysis of the twenty lessons in the study, with respect to the appropriation of content. The table indicates the teacher; how the content has been appropriated; the insertions and omissions made in the lessons; and, comments about the mediation of the object of learning.

Table 1. A summary of teachers' appropriation of the content of the textbook

Teacher	Appropriation of content	Insertions and omissions	Comments
A1	Offloads in 2 lessons improvises in 1 lesson	2 robust insertions; 2 impotent omissions	
A2	Adapts in 1 lesson; offloads in 1 lesson; improvises in 1 lesson	2 distractive insertions 1 critical omission	Mediation erroneous in 3 lessons
A3	Offloads in 1 lesson; improvises in 2 lessons	3 robust insertions 1 critical omission	
A4	Improvises in 3 lessons		Mediation partly erroneous
B1	Improvises in 2 lessons	3 robust insertions	
B2	Improvises in 3 lessons		Mediation breaks down in 3 lessons
C1	Adapts in 1 lesson; improvises in 2 lessons	1 robust insertion 1 critical omission	

The results from Table 1 show that in 4 out of 20 lessons (20%) teachers offload the content from the textbook; adapt the content in 2 lessons (10%); and improvise content in 14 lessons (70%). Thus, there is preference for external resources other than the prescribed textbook by teachers participating in this study with respect to the content in lessons. With respect to insertions and omissions, we present the results from Table 1 in Figure 3 below.

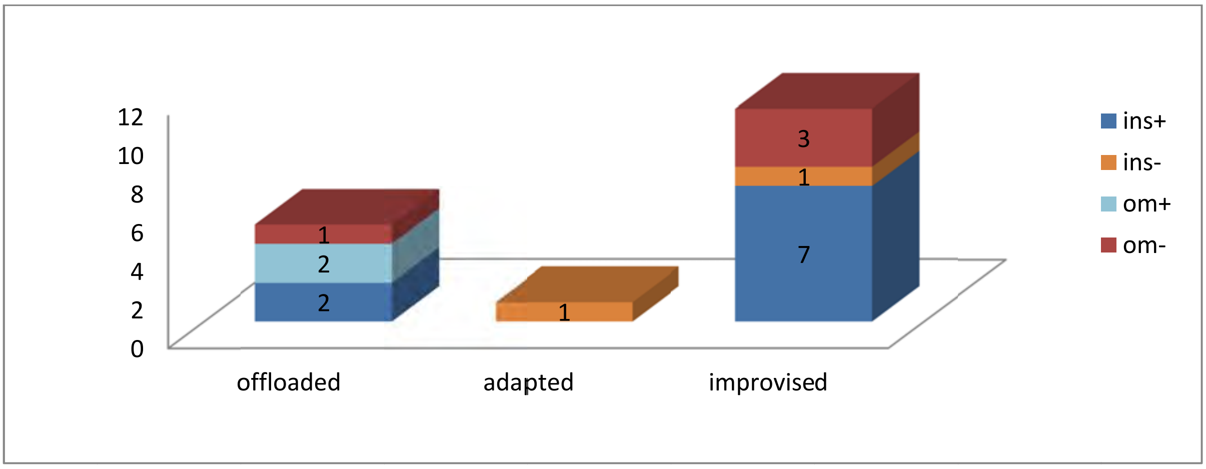

Figure 3. Distribution of insertions and omissions

We observe from Figure 4 above that *critical omissions (om$^-$)* exist in both offloaded and improvised lessons, and similarly for *robust insertions (ins$^+$).* In other words, whether teachers predominantly offload to the textbook in the process of teaching, or improvise own content, they still overlook some critical aspects of the object of learning from the textbook.

At the same time, these results suggest that teachers also perceive content from external resources which can enhance the textbook content, judging by the relatively large number of *robust insertions (ins$^+$)* in the lessons.

CONCLUSION

We have demonstrated in this paper that the insertions and omissions play an important and decisive role on the kind of mediation that takes place in the classroom. While *impotent omissions* do not harm the object of learning, *critical omissions* on the other hand detract from the object of learning and therefore affect the end result of mediation. With respect to insertions, *distractive insertions* have been shown to have potential for being harmful to the object of learning, as they may lead to erroneous mediation; thus, if insertions to the content have to be made, they should be *robust insertions* which serve to enhance the object of learning. The study is ongoing, but the message about the importance of insertions and omissions in the appropriation of the content is clear.

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A COMPARATIVE ANALYSIS OF NATIONAL CURRICULA RELATING TO FRACTIONS IN ENGLAND AND TAIWAN

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The importance of a working knowledge of fractions in mathematics learning, coupled with the difficulties that pupils have with learning fractions, have prompted researchers to focus on this area of mathematics teaching. Research has reported that Taiwanese pupils show greater fluency in operations with fractions than their British peers.

In view of the impact of the mathematics curriculum on a pupil's opportunity to learn mathematics, this paper explores and presents findings on what can be learned from an analysis of respective curricula in England and Taiwan. Specifically, fractions-related content as presented in the national curricula in England from 1999 to 2013 is compared with that of the "Grade 1-9 curriculum" in Taiwan as published in 2003 and implemented from 2005 to the present.

The analysis of the 1999 and 2007 national curricula in England, and of the "Grade 1-9 curriculum" in Taiwan with respect to fractions shows that Taiwanese pupils begin to learn fractions addition at age 8, and learn the other three operations on fractions by age 12 (the end of primary education), while all four operations with fractions are not introduced to pupils in England until age 12. Taiwan's curriculum also emphasises proficiency in computing with fractions by the end of primary education. The new 2013 curriculum in England relating to fractions shows some important changes. For example, pupils will start to learn fractions calculations from age 7. The implications of the results from this study on mathematics textbook development and research are discussed.

Keywords: fractions, mathematics curriculum, England, Taiwan, comparative analysis

INTRODUCTION

The importance of a working knowledge of fractions in mathematics learning, coupled with the difficulties pupils have with fractions, have prompted researchers to focus on this area of mathematics. However, research confirms that fractions continue to be a difficult topic in mathematics teaching and learning (Li, 2014; Saxe, Diakow, & Gearhart, 2013). For example, Hart (1981) found that nearly 30% of 12-13 years old pupils in England added numerators and denominators for 1/10 +3/5, that is, 1/10 + 3/5 = 4/15. Recently, in Li's (2014) study of 561 British and 648 Taiwanese pupils' knowledge of fractions at ages 12 and 13, she reported a similar percentage of the British group using this erroneous strategy to add fractions as Hart found in 1981. Li also found that the Taiwanese pupils showed greater fluency in operations with fractions than their British peers.

Research shows that the mathematics curriculum plays a forceful role in pupils' opportunity to learn mathematics (Fan & Zhu, 2007; Lui & Leung, 2013). Studies of curriculum can benefit from comparative, cross-cultural research since many issues can be revealed more clearly (Kulm & Li, 2009). Moreover, researchers have found that textbooks have a significant impact on teaching and learning (e.g. Wong , Zang & Li, 2013) and can support the goals and vision of the curriculum (Shield & Dole, 2013). However, Fan and Zhu (2007, p. 72) contended that "there existed considerable gaps between national syllabuses/curriculum standards and the textbooks". Fan, Zhu and Miao (2013, p. 636) further commented that "researchers' conceptualization about the relationship between the textbooks and curriculum is particularly noteworthy".

Motivated by (a) Li's (2014) study which pointed out the differences in pupils' achievements between England and Taiwan in the area of fractions, and (b) the need to develop pupils' proficiency of fractions, our study seeks to provide insights into the similarities and differences in the national curricula between England and Taiwan. The intention of this study is not to identify the superiority of one curriculum over another. Rather, the analysis seeks to cast light on how the intended curricula vary and what can be learned from this comparison, in particular for mathematics textbook development and research.

BACKGROUND

England

Education is compulsory for children aged between 5 and 17 (and up to 18, from 2015) in England. There are four Key Stages in its education system; primary education includes Key Stages 1 and 2 (from Reception to Year 6, ages 4 to 11), and secondary education includes Key Stages 3 and 4 (from Year 7 to Year 11, ages 11 to 16). At the time of writing, the national mathematics curriculum is given by Department for Education and Employment (DfEE) in 1999 (DfEE, 1999). The more recent 2007 National Curriculum (QCA , 2007) for secondary was published by Qualifications and Curriculum Authority (QCA) in 2007 and implemented in 2008, which "only contains very broad statements for domains" (DfE, 2011, p.65). The new National Curriculum for mathematics in England for Key Stages 1, 2 and 3 was published by Department for Education in 2013 (DfE, 2013a, 2013b) and will be implemented from September 2014.

Taiwan

Nine years of compulsory education, including six-years at primary school (Grades 1 to 6, ages 6 to 12) and three years at junior high school (Grades 7 to 9, ages 12 to 15), is imposed throughout the country. At the time of writing, the national mathematics curriculum is given in the "Grades 1 – 9 curriculum" which was initiated by the Ministry of Education (MOE) in Taiwan in 1998. After several revisions, it was finally published in 2003 and implemented in 2005. The nine years of compulsory education policy in Taiwan will end in 31st July 2014 and be replaced by the new policy, 12 years of compulsory education (covering three years at senior high school, Grades 10 to 12, ages 15 to 18) from 1st August 2014. The *Framework for the new mathematics curriculum* will be published in 2015 and implemented from August 2018.

METHOD

Content analysis of the statutory curricular documents relating to fractions in England and Taiwan from 1999 to 2013 was used for this study. The analysis does not include wider non-statutory guidance and other related resources. We use "ages" to present our analysis as age-based measurement is recommended to be more appropriate for international comparative studies than other measurements such as those based on school years (Wiley & Wolfe, 1992).

The 1999 and 2007 National Curriculum in England (hereafter, called England (1999, 2007)) and the "Grades 1-9 curriculum" in Taiwan (hereafter, called Taiwan (2003)) are the current curricula that are implemented in these two different educational systems in recent years. We also include the new 2013 National Curriculum in England (hereafter, called England (2013)) in this study as it will be implemented from September 2014. However, the new 2013 National Curriculum for mathematics was only published for Key Stages 1, 2 and 3 (ages 5-14) at the time of writing. Hence our analysis of these curricula – Taiwan (2003), England (1999, 2007) and England (2013) focuses on the content relating to fractions for pupils aged between 5 and 14.

Proficiency of fractions acquires both conceptual understanding and procedural fluency. The curriculum analysis undertaken here focused on the content that relates to (i) the concept of fractions (*part-whole, quotient, measure, ratio, and operator constructs*, see Kieren (1988)), (ii) equivalence between fractions to compare and order fractions, (iii) the relationship between fractions, decimals and percentages and (iv) the four operations with fractions.

RESULTS

We found that the curricula on fractions share some similarities. As far as the four aforementioned areas of fractions were concerned, all of these curricula specify detailed objectives for teaching fractions and cover the similar breadth of content and expectations in children's learning of fractions. The differences between the curricula are mainly found in the way the content is broken down and sequenced.

Table 1 provides an overview of when the content that relates to fractions is introduced across the curricula. Taiwan (2003) appears more intensive in the area of fractions than the England (1999, 2007) and England (2013). The content that relates to the four areas of fractions as shown in Table 1 is organised for pupils aged 7-12 in Taiwan (2003). England (1999, 2007) cover them for pupils aged 7-14, while England (2013) starts to introduce fractions to pupils from age 5. The England's curricula tend to use a spiral-shaped approach, whereby each subject area is repeated or developed further during each key stage.

However, most of the content or subject areas in the Taiwan's curriculum are presented only once at each grade level. For example, in the Taiwan (2003) curriculum, the introduction to fractions at each new Grade is not necessarily restricted to unit fractions. In contrast, in England (1999, 2007) and England (2013), the introduction to fractions at each new school year always begins with unit fractions, and followed by simple fractions in their next school year. Table 1 also shows that the content that relates to the relationship between fractions, decimals, ratios and percentages in Taiwan (2003) is organised over the course of one year for pupils aged 10-11, while it is organised over the course of five years for pupils at ages between 9 and 14 both in England (1999, 2007) and England (2013).

Table 1: Sequence of fractions across curricula for pupils aged between 5 and 14

	Taiwan (2003)	England (1999, 2007)	England (2013)
The concept of fractions	Ages 7-11	Ages 7-11	Ages 5-11
Equivalence between fractions to compare and order fractions	Ages 8-10	Ages 7-10	Ages 6-10
Relationship between fractions, ratios, decimals and percentages	Ages 10-11	Ages 9-14	Ages 9-14
The four operations with fractions	Ages 8-12	Ages 11-14	Ages 7-14

The concepts and skills of fractions are divided into two steps in the England (1999, 2007) curriculum. The former is introduced at primary schools and the latter at secondary schools. It is not the case in Taiwan (2003), since the concepts and skills of fractions are integrated for primary pupils at ages 7-12. England (2013) makes a significant change in the sequence regarding fraction calculations with the four operations for pupils in England. For example, under England’s new 2013 National Curriculum, pupils will begin to add and subtract fractions with the same denominator (within one whole) from age 7. This is a huge change as all the four operations with fractions are not introduced to pupils until age 12 under England (1999, 2007). Also, this means that pupils in England start to learn fraction calculations when they are one year younger than their peers in Taiwan.

It is also worth noting that while proficiency in computing with fractions are specifically emphasised in all the curricula, they show different aspects of using calculators. Taiwan (2003) notes that the use of calculators is only allowed when teachers consider that pupils have developed computational fluency in using written methods and a good understanding of number operations. In contrast, England’s curriculum (1999, 2007) suggests that pupils use a calculator where appropriate, for example, when recognising the equivalence of fractions, ratios and percentages. However, England’s new 2013 National Curriculum states that calculators should only be introduced near the end of Year 6 (ages 10-11) when pupils’ mental and written arithmetic are secure.

DISCUSSION AND IMPLICATIONS

While the findings from the curriculum analysis undertaken in this study are subject to the limitations of the methodology used, some valuable insights into the differences in the national curriculum between the two countries are worthwhile. Due to the restrictions of word length, two major findings are discussed briefly below.

First, the introduction of the four operations with fractions varies significantly between England (1999, 2007) and Taiwan (2003). Pupils in Taiwan begin to learn fraction addition from age 8, and learn the other three operations on fractions by age 12, while all four operations with fractions are not introduced to pupils in England until age 12 according to England (1999, 2007). This difference between Taiwan (2003) and England (1999, 2007) may therefore partially explain Li's (2014) finding that at ages 12 and 13, Taiwanese pupils are able to add fractions far more successfully than their British peers. A series of international comparisons over the last 30 years has led to significant changes of policy and practice in education in England (e.g. DfE, 2011). It is evident that the new 2013 curriculum on fractions places more emphasis on pupils' development of computational fluency than its previous version and introduces fractions operations to pupils from much earlier, at age 7 instead of age 12.

Secondly, the sequence of the content that relates to fractions varies substantially between these two countries. As shown in Table 1, the four areas of fractions in Taiwan's curriculum are introduced to pupils at ages 7-12, with the intention of equipping them with necessary fraction knowledge before they enter junior high school at age 12 (MOE, 2003). In contrast, 12 year olds in England have not yet been introduced to all of these four areas (Table 1), and fractions continues to be a difficult topic for many secondary pupils in England (Hodgen, Küchemann, Brown, & Coe, 2009).

The findings above have important implications for mathematics textbook development and research. Firstly, mathematics textbook development relating to fractions following England's new 2013 curriculum needs to consider carefully how best to introduce fractions concepts and operations to pupils as young as ages 5 and 7 respectively. Pupils' cognitive developments at ages 5, 7 and 12 are very different. Moreover, many teachers may not be equipped to teach fractions to pupils at a younger age to maximise their understanding and learning outcomes. Mathematics textbooks may therefore not only play an important role in presenting fractions in a way that interests and enables pupils to learn well, but also have instrumental function in supporting teachers' teaching strategies.

Secondly, researchers interested in textbook comparison and pupil attainments may examine the impact of topic sequence on children's learning of fractions across textbooks in different jurisdictions. For example, to what extent, an "intensive" or "extensive" content of textbooks on fractions in England and Taiwan helps pupils develop their understanding of fractions.

Finally, there has been a lack of research focusing on the process of textbook development (Fan et al., 2013). The changes in England's new curriculum in fractions provide a great opportunity for textbook researchers to centre on how fractions-related content of textbooks is produced. We hope that this paper stimulates further research to examine how national curricula are interpreted in textbooks and curriculum materials relating to fractions. More international comparative research is also needed to investigate how textbooks and curriculum materials could support the teaching and learning of fractions.

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IMPROVEMENT IN TEACHERS' INTERPRETATION OF MATHEMATICS TEXTBOOKS

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This study was designed to examine how teachers used case discussion to improve their understanding of the textbook to be used in their teaching. Five 5th-grade teachers and a facilitator in the same school worked to enhance their expertise of interpreting the textbooks in a meaningful way. Pre- and post- interviews were conducted to document the impact of the use of case discussion. Results show that the teachers improved their interpretation of the textbook tasks and the structure of the textbook, from unawareness, awareness move toward understanding and applying to other situations.

Key words: teacher interpretation, case discussion, textbook use, textbook tasks

INTRODUCTION

Textbooks are not only an important resource guiding teacher's mathematics instruction in the classroom but also an essential potential factor affecting students' mathematics learning. Besides this, textbooks also play an essential role in the curriculum. The textbook as the implemented curriculum is viewed as a mediator between the intended curriculum and attainted curriculum in school mathematics (Schmidt et al. 1997; Valverde et al. 2002), since textbooks help teachers to identify the content to be taught, the instructional strategies appropriate for a particular age level, and possible assignments to be made for reinforcing classroom activities (Thompson, et al. 2012). Given their importance, textbooks are increasingly being recognized as an important topic for research.

The literature on mathematics textbooks reveals that textbook research has received increasing attention over the last three decades, but research is currently devoted to the textbook analysis and comparison. For instance, Fan, Zhu, and Miao (2013) reveal that 63 % of the studies focus on textbook analysis and comparison and 25% on textbook use including how textbooks are used by teachers or students. However, it is important to examine textbooks not only in terms of their content and structure, but also their use in real classrooms. For instance, an in-depth analysis of lessons and studying textbooks in-depth is a way of supporting Japanese teachers learning and ability to teach (Shimizu, 2002).

It is recognized that teachers play a crucial role in the use of textbooks in classrooms. Teachers have great variation in the use of textbooks (Remilard, 2005). For instance, McNaught et al. (2010) conducted a three-year project on teachers' use of mathematics textbooks. The result suggests that teachers tend to assign fewer problems to students than the textbook recommended. Furthermore, the research on textbook use displays that the differences in teachers' implementation of textbooks under the context of teaching the same content of one textbook among teachers result into the differences of the effectiveness of teaching and the outcome of students' learning (Freeman & Porter 1989). This suggests that the use of textbooks may not be effective without additional professional development. Thus, there is a need to support teachers to develop their expertise in how to best use of the textbooks, particularly, teachers' understanding the texts of the textbook prior to teaching.

Teachers' interpretation of the texts of textbooks is an aspect of teachers' knowledge for teaching mathematics. Its importance is also shown in the work of Ball and her colleagues (Hill, et al. 2008). The knowledge of curriculum is a subdomain of mathematics knowledge for teaching. A number of researchers have studied how the use of textbooks affects teachers' learning and mathematics instruction (Remillard, 2005). However, there is little research on developing expertise in using the textbooks to teach mathematics. Thus, this study was designed to help teachers in better understanding of the textbooks prior to teach mathematics through research-based cases.

Why do we use research-based cases to facilitate teachers' interpretation of textbooks? Research has suggested that the support for teacher learning is more effective when it is linked closely to teachers' classroom context (Cohen & Hill, 1998). Cases are constructed on the basis of the narratives of real teaching. Taking this advantage, cases are claimed to be an effective tool to develop teacher's mathematical knowledge and pedagogical content knowledge (Steele, 2008). Furthermore, analysis of textbooks requires teachers to consider subject matter and pedagogy together, in relation to one another. Therefore, if teachers are enhanced their expertise in mathematics teaching through the use of research-based cases, then it is more likely to improve their interpretation of textbooks. Thus, the use of research-based cases for this study is considered to be a pedagogical tool for supporting teachers learning, the interpretation of textbooks.

METHOD

Participants

To facilitate teachers' better interpretation of the textbook, six inservice teachers in a same elementary school including a facilitator (F) and five teachers (T1~T5) with 5 to 12 years of teaching experience set up a case discussion group. The facilitator has received 108 hours courses related to cases when she studied in graduate institute. The courses helped her not only to understand the contents covered in the cases but also to enhance her competence in facilitating case discussion. The facilitator invited five teachers based on intimately close friendship and willingness of improving mathematics instruction. They were teaching in grade 3 to 5 at the time of the study.

Case discussion

The research-based cases are featured as real teaching and constructed by teachers and a researcher based on research (Lin, 2002). Parts of contents in each case consist of: learners' prior knowledge, instructional objectives, and activities engaged in the lesson including the problems given by the case teacher, students' various solutions, the dialogues between teacher and students, and Discussion Questions. The problems in Discussion Questions are for stimulating critical reflection on the case with respect to key issues related to students' mathematical thinking, ways of representing, and formulating mathematical concepts. The discussion group met once every two weeks lasting for 2 hours on Monday afternoon after school. Each case was discussed in two meetings. Four cases of fractions in middle grade were selected to be used for the purpose of study. Each case was read in advance individually at home before case discussion. The facilitator used the questions of Discussion Questions in the case as prompts to initiate teachers' discussions.

Data collection and analysis

Data for this study included pre- and post- interview and the transcription of the audio-taped for the case discussion. Each teacher was interviewed individually lasting for an hour at the beginning and the end of the study. In order to document the effect of the case discussion, the questions conducted in the two interviews were identical. The teachers were asked to respond to 10 questions focusing on the goals of lessons, problems, representations, anticipated students' possible solutions and misconceptions. They were asked to make an distinction between the contents of the textbook, explain which they preferred for students, indicate any change they might make, suggest what they thought were the main concepts, indicate mathematics understandings that students would gain, and then choose which activity they would prefer to use in their classrooms. A question in the interview was that "*What could be the factors impacting students' difficulties from the two problems in the textbook?: "Problem 1*: *A box has 18 pieces of chocolates. Joseph ate 4/9 box. How many chocolates did Joseph eat? Problem 2*: *David has 9 marbles. 3/9 of the marbles are blue. How many blue marbles does David have?"*. Each interview was tape-recorded. There were eight transcriptions of case discussions and ten interviews in all.

Table 1: Four levels of teachers' interpretation of textbook on the aspect of problems

Levels		Description
1	Unawareness	Teachers were not aware that problem 2 is easier than problem 1 in that each part divided into from a whole in problem 2 is a single entity instead of 2
2	Awareness	Teachers were aware that Problem 2 is easier than problem 1 because the size of each part is the same as the denominator in problem 2, while the number of each part is not the same as the denominator in problem 1.
3	understanding	Teachers realized that 2 is the size of each part in Problem 1 and 1 is the size of in each part in problem 2 that is a main factor impacting students' difficulty of learning fractions.
4	Application	Teachers were able to apply the concept of "the size of in each part" to distinguish other problems involving in the textbook.

After repeatedly reviewing the data of the interviews, the emerged four levels of teachers' interpretation of textbook were unawareness, awareness, realization, and application. The four aspects of texts of textbook consist of problems, representations, students' cognition, structures of texts including objectives, learners' prior knowledge, sequence of activities, and main ideas of instruction. The codes were further verified by two experienced teachers working independently with a full set of data before coming together to agree on the best integrated categories. The four levels of teachers' interpretation of textbook are depicted in Table 1 from the above the question of interview to illustrate as an example.

RESULTS

This part presents the findings on the improvement of teachers' interpretation by levels on texts of the textbook. The texts consist of the aspects of problems, representations, students' cognition, and the structures of the text. Due to the limited page, the improvement of teachers' interpretation on the aspects of representations and students' cognition are not presented in the paper.

Improvement in teachers' interpretation of textbook tasks

The data of teachers' improving their interpretation on the contexts of the problem is summarized in Table 2. When the teachers were asked to review the activities of ordering fractions in the textbook, data in Table 2 suggests that at the very beginning of the study all of them were not aware that the difference of the size of each equal part that if it is equal to a singular entity could be a factor contributing to students' difficulty in ordering two fractions. Three teachers neither concerned about the problem involving in discrete or continuous quantity. They were not aware that the discrete model plays an important role in equivalent fractions and it causes students' more difficulty than continuous model.

Table 2: Improvement of teachers' interpretation on problems

Levels / Contexts of problems	Pre-interview				Post-interview			
	1	2	3	4	1	2	3	4
Continuous vs. discrete quantity	T2, T3, T4	T1,T5						T1, T2, T3, T4,T5
Size of each equal part	T1, T2, T3,T4,T5						T3	T1, T2, T4,T5
Units of the context		T1,T3 T2, T4,T5						T1, T2, T3, T4,T5
Numerals in the problems	T3,	T1,T2T4,T5					T5	T1, T2, T3, T4
Semantic structure	T1, T2, T4, T5	T3,					T5	T1, T2, T3, T4

Note: T1 at Level 4 stands for T1 promoted to the highest level 4 from level 1 go through level 2, 3 and to 4.

Through discussing a case related to ordering fractions, the teachers improving their views on the factors of affecting students' difficulty related to problem itself. The factors consist of either discrete or continuous quantity, the size and the number of parts, number of objects as one whole unit, and partition division and measurement division. For instance, T4 learned that if the number of objects is not the same as the number of parts, then it is much harder than the counterparts (shown on line 3~6).

3 T4: Problem 1 is more difficult than problem 2, because problem 1 is kind of the division of whole number.

4 F: How many sizes are in each part in the problem 1 and 2, respectively? What is one part? Which one did the case-students perform better in these two problems?

5 All: Better on Problem 2.

6 T4: I am surprised with the case-students' they thought beyond what I anticipated. ……

In the post-interview, T2 stated that she learned from case #1 involving in the relationship between the size and number of parts into which a whole is partitioned is very important in determine the order of fractions. This relationship should be distinguished from what students have learned about the size of the parts is the same, but fewer parts is smaller, such as (2/5 < 3/5). T1 and T3 also mentioned that they restructured the pairs of fractions to be ordered relying on students' prior cognition. Likewise, T5 attended to the numerals involving in the problems as small as possible to use a set model to develop students' meaningful learning of fraction equivalence, since a smaller number makes student's partition easier than the large numbers.

Improvement in teachers' interpretation on the structures of the texts

Teachers' attentions to the structure of textbook when they read the textbook were composed of objectives, learners' prior knowledge, sequence of activities, and main concepts to teach. The improvement of teachers' interpretation on each component was evidenced by data shown in post-interview compared to pre-interview seen in Table 3. The data of Table 3 suggests that teachers were not aware that they are authorized to criticize the activities in the textbook.

At the beginning of the study, all responses to "What is the most important concept in the lesson for the students to understand? Why do you think so?" were centred repeatedly on describing the objectives of the lesson, such as "to understand the fraction equivalence". T3 reported in pre-interview that she was not aware of students at risk of learning from the tasks in the textbook tended to rush to teach procedure without via repartitioning or regrouping underlying the fraction equivalence until she participated in the discussion of case #4. At the end of the study, all responses have shown that they are able to decompose original objectives into detailed objectives. For instance, T2 stated that "*the activity on page 69 is to learn to find equivalent fraction by raising a fraction, while the activity on page 70 is to learn to find equivalent fraction by reducing a fraction*".

All teachers specifically cited the case discussion as major influence on their better understanding the importance of students' prior knowledge. In the post-interview, T5 stated that students' various solutions presented in the cases were based on what they have learned. Whether a student's solution is acceptable was determined by students' prior knowledge. In addition, T4 complemented that prior knowledge was the basis for her re-modifying the activities and restructuring the sequence of activities in the textbook.

Table 3: Improvement of teachers' interpretation of structures pre-and post- interview

Levels Contexts of problems	Pre-interview				Post-interview			
	1	2	3	4	1	2	3	4
Objectives	T1, T2, T3,	T4, T5						T1, T2, T3, T4,T5
Prior knowledge		T1, T2, T3,T4,T5					T3	T1, T2, T4,T5
Sequence of activities	T1,T3 T2, T4,T5							T1, T2, T3, T4,T5
Main concepts to teach	T1, T2, T3,T4, T5						T1, T2,T5	T3, T4

Note: T1 at Level 4 stands for T1 promoted to the highest level 4 from level 1 go through level 2, 3 and to 4.

In terms of sequence of the learning activities, in the pre-interview, most teachers responded that they followed the scope and sequence of the textbook closely. T2 mentioned that she learned about the tasks the case-teacher designed for exploring mathematical ideas and developing students' ability to give multiple solutions from the case #2, which is to compare the case-teacher's version with two other versions of the textbook on the sequences of activities. In the post-interview, T2 revealed her ability in modifying a task and restructuring four pairs of fractions to encourage students with multiple methods in meaningful way to compare two fractions instead of the use of algorism, from unit fractions (1/5 *vs.* 1/7), same numerator (5/16 *vs.* 5/19), to general fractions (4/9 *vs.* 8/12) and (4/15 *vs.* 8/26).

DISCUSSION AND CONCLUSION

The results show that in the initiation teachers highly relied on textbook and read it with a superficial and a narrow way. Teachers' interpretation tends to the approach of "teaching textbook". Through the case discussion, they became a critical reader with the approach of "using textbook to teach mathematics". The discussion on the learners' prior knowledge and instructional objectives provided in each case supported teachers understanding the need of students' prior knowledge for constructing a reasonable sequence of tasks. The discussion on the problems given in each case increased the teachers' understanding of the importance of the size of numbers, the number of each equal part, the contexts involving in problems influencing students' difficulty. Students' solutions and the dialogues between teacher and students listed in the text of the cases supported teachers to have a deeper understanding of main mathematics concepts involving in the cases. The results indicate that research-based case discussion switched teachers' interpretation toward level 3 and level 4 from level 1 and 2 as suggested by Sugiyama's ideas of the use of curriculum materials (2008).

The results imply that the effect of the use of cases on mathematics teaching is much different from the textbook. The difference of the research-based cases from the textbook is saliently describing students' prior knowledge, comparison of the case-teacher's version with the textbook, and the case-teachers' orchestrating the discourse of classroom. These critical key ideas are valued for the teachers in the preparation of teaching. The most essential element is that the set of "Discussion Questions" readily draws users' attention to the key aspects and create salient focus for the case discussion. Those questions readily initiated the teachers' reflection on students' learning and cause their cognitive conflicts of mathematical teaching, and hence triggered teachers' change. The discussion of research-based cases seemed to be a catalyst for the teachers to have high-level interpretation of the textbooks.

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DEVELOPMENT OF CURRICULUM UNITS FOR A BASIC COURSE FOR CALCULUS

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This study was to design, develop, and investigate instructional units for University freshmen to learn before they study calculus. Because the concepts, skills, and theories of function are fundamental for the calculus course but the below average students were not familiar with the basic knowledge and ability in function when they studied in the high schools and it will affect their learning calculus, the investigator in this study has analyzed the calculus course to find out the relevant functional concepts, skills, and theories and taken some actual research studies as references to deliberately design, compile, and write an instructional unit on function. Then the teaching material was tried out in the classroom. During trying out the unit, the investigator found that students were also unfamiliar with the concepts and operations of numbers and sets. Therefore, the investigator thought that it is indispensable to integrate the content of numbers and sets into the course. Finally the curriculum units including numbers, sets, and functions have been completed to be a basic course for calculus. After preparing the curriculum units, the teaching materials were sent to experts to ask for reviewing and giving feedback for revising the content. Then the curriculum will be tried out again in the beginning of the calculus course to test the degree of appropriateness and find where should be revised again. During the next year, a formal instruction will be carried out. Finally, it is to complete a set of curriculum units on number systems, sets, and functions for freshmen to take as basic content for calculus course.

Keywords: number system, set theory, functions, calculus, Higher Education

INTRODUCTION

This study is to design and develop curriculum units for freshmen to take as basic contents before learning calculus. In the beginning, the investigator made effort only to develop a curriculum unit on function because the concepts, procedures, and theories of functions are essential for studying calculus. During trying out the unit, the investigator found that students were also unfamiliar with the concepts and operations of numbers and sets. Therefore, the investigator thought that it is indispensable to integrate the content of numbers and sets into the course. Finally the curriculum units including numbers, sets, and functions (including algebraic functions, absolute value functions, exponential functions, logarithmic functions, trigonometric functions, and inverse trigonometric functions) have been completed to be a basic course for calculus.

Numbers, sets, and functions together with the related concepts are fundamental content for learning calculus. However, many college students did not get familiar with those contents when they studied in high schools. It will affect students' learning calculus. Although the high school curriculum and instruction provide sufficient content in numbers, sets, and functions for students to learn calculus in college, the below average students still have problems in the content. Therefore, this study had been planned to design and develop a pre-calculus course including the content of numbers, sets, and functions to provide the related concepts, procedures, and generalizations for freshmen to make clear and to help them to learn limit, differentiation, and integration thereafter.

During the past decade, the relevant content of function was integrated into the instruction of calculus when it was necessary in my course. Perhaps integration could give students only piecewise knowledge of function instead of continuous and integral knowledge and procedures. If we can review the content of function before teaching calculus to renew students' memorization and let them get inspired to obtain correct concepts, knowledge, and procedures on function, it will establish a good base for them to learn calculus.

According to my research studies during the past years, I found that many students in junior high school, senior high school, or junior college were unclear about functional concepts even they had learned those concepts in the junior high school. When they were asked about the meaning of function, they could only give examples without any explanation or only with incomplete interpretation. Few students could give the essential meaning or definition of function. In addition, rare students could give any non-example of function. Most students could only do calculations instead of understanding abstract concept. The content of function in the junior high school curriculum contains only linear and quadratic function. Although there are examples of function with discrete variable and examples of rational function in that curriculum content, many first year students in the junior college misunderstood that only linear or quadratic functions are then functions. Being linear or quadratic was used as criterion to distinguish whether the given examples in any representation is a function. They even could not accept that an absolute function or rational function is a function because the independent variable x cannot be put inside the sign of absolute value or in the denominator. They used to give incorrect reasons.

The initial purpose of this study was to design and develop a curriculum unit on function for college freshmen to take as basic content before learning calculus and to explore the following questions:

1) What content of function should be included in the curriculum unit in terms of experts' opinions?
2) When the curriculum unit is tried out in the college with low level students, how appropriate is the content and what content should be revised or added?
3) Are there any misconceptions that freshmen have when they study the curriculum unit?
4) Is there other content that should be included for pre-calculus course?

LITERATURE REVIEW

Experience of developing a curriculum unit

In 1984, I developed a lesson on the topic of stem-and-leaf plot to do a pilot study at the University of Georgia to see how appropriate the content is, what learning difficulties that students have, and what content should be revised. Then I went back to Taiwan to design and develop a descriptive statistics curriculum unit that was to be integrated into the mathematics curriculum for Grade 11 students in the business colleges. Twenty-eight experts in Taiwan were asked to comment on the curriculum materials and give advice for improving the content. The detailed results were reported in Lue (1985). From this study, I understood that not only design and compilation of curriculum is important for the development work but also the process of trying out the curriculum materials are indispensable. Because only trying out the curriculum can find out the students' learning difficulties and the defects of the teaching materials to improve the content and propose useful teaching guide. On the other hand, I recognized that the arrangement of spiral curriculum is helpful for students' learning. Spiral arrangement of the curriculum content used to be advocated by educators. Fennema, Carpenter, & Franke (1992) also advocated cognitively guided instruction. The gist is to ask the teacher to recognize the cognition status of their students to give instruction according to their capabilities. Of course, it is an important principle in developing a curriculum material and teaching.

Students' understanding about the concepts of numbers

Lue (2008) pointed out that "comparing students' performances on five kinds of definition domains, the order of the difficulty was about in the form FNZRQ in the pilot study and in the form FZN'QR in the main study from the easiest to the most difficult (here F stands for the finite set including -1, 0, 1, and 2, N stands for the set of natural numbers, N' stands for the set including natural numbers and other three integers 0, -1, and -2, Z stands for the set of integers, Q stands for the set of rational numbers, and R stands for the set of real numbers).…It is easy to see that students' performances were affected by the corresponding forms of function and its definition types. Moreover, students could deal with constant function and finite domain best". Lue (2004, 2005) also said "Students were only familiar with simple routine algorithm. But they could not deal with more complicated data, such as radicals. It is easier for students to get extrema for bounded monotonic functions but more difficult for them to find extrema for unbounded functions." In addition, he mentioned "Although two variables have linear relationship, students were unable to point out the function relation between the two variables. To get the function expression from a given algebraic expression was an uneasy task for them. It was more difficult to ask students to get domains of function, especially the range. Students used to draw a continuous graph for even a function with discrete domains. Few students could draw the graph or find the extrema for the irrational function h( )＝ ." Liu (2006) also found that students used to think that a function should be continuous and tend to regard that a point-wise graph cannot represent a function.

Students' learning difficulties about functions

Lovell (1971) found that middle school students have miscellaneous misconceptions and puzzles in learning functions. Only a few elder students could deal with composite functions. Markovits, Eylon, & Bruckheimer (1986 & 1988) found that students could not understand functional concepts, such as domain, range, pre-image, and image. Students were also puzzled with constant function, piecewise function, and point-wise function. Students used to cite linear functions as examples for function and use algebraic expression or graph as the representation of a function. Moreover, it is more difficult to transform graph to algebraic expression than vice versa. Tall & Vinner (1981) and Vinner & Dreyfus (1989) found that it is a difficult task for students to reflect from their brain although the definition of function is short and brief. To deal with problems, students used to reflect concept image instead of definition. Concept images were formed from their experiences and might distort correct cognition, such as regarding functions being a formulated expression.

In Taiwan, many researchers found that students had a lot of misconceptions or alternative conceptions in functional concepts and even could not distinguish example or non-example from representations of functions. My experiences tell me that students were easier to do simple procedure tasks, such as computing function value (image), than to deal with concept questions. In 2001, I interviewed two junior middle school students: one is a Grade 9 girl student whose academic performance was average and the other is a Grade 8 boy student whose academic performance was above average. Both could use algebraic expression f(x) to represent a function but could not explain the meaning of function. The boy could also use algebraic expression y= f(x) to interpret the variations of variables. I hereby concluded that example recognition is easier and earlier than meaning recognition. The former is concrete and may be perceived by sight. The latter is abstract and should be understood by insight and expressed by verbal statement. Both form the process of the formation of concept. Meaning understanding is the primary goal of concept learning.

RESEARCH METHOD

The researcher paid attention to the content related to function in the calculus curriculum and the presentations of the curriculum materials of function in the high school textbooks and then designed a curriculum unit on function including functional concepts (such as function, variables, definition domain, corresponding domain, range, function value, and extrema) and their properties (such as one to one, onto, increasing, decreasing, and invertible), types of functions (such as varieties of polynomial function, varieties of absolute value function, varieties of rational function, and varieties of irrational function), the operations of functions (such as addition, subtraction, multiplication, division, and composition), and the representations of function (such as verbal statement, algebraic expression, table, graph, arrow diagram, and machine analogue). The designed content was tried out in five classes called A, B, C, D, and E in this report in a private university of technology and science at Taipei. The students' academic abilities were below average in Taiwan. The students in Class E took classes during the night and worked during the day. Their academic performances were generally worse than students in other classes. The only teacher used 12 periods within 6 weeks to teach the students in each class the curriculum content. Lecture method was the major way for instruction. The related history of mathematics was provided and reported in the class to soften the curriculum materials.

To understand the students' learning achievements, a set of test items were designed. The test items contain two parts: the first part includes distinguishing whether a representation can represent a function and stating the reason and the second part includes the examinations of functional concepts and the properties of functions. The test items had been evaluated by four experts and they thought that the items were appropriate to test the students' understanding about function.

After the class of 12 periods, the test was administrated. There were 51, 49, 53, 54, and 45 students in the five classes who took the test. The test time was 70 minutes. After the test, the students' performances were graded and analyzed by using EXCEL to denote their performances: giving the score 2 for correct answer, the score 1 for half correctness, the notation 'x' for incorrectness, '0' for inappropriate answer, and '?' for puzzled answer. Moreover, about 10 students in each class were interviewed to see their thinking and understanding.

DATA ANALYSIS AND RESULTS

The researcher had discussed with four other mathematics teachers about the outline and content of the curriculum unit. The four experts had no special opinion about the unit. They thought that the topic of function is basic and essential for studying function and calculus.

After testing, the standard of grading was given to the graders. Each examination paper was reviewed by another grader to assure precise grading.

There are 45 questions in the test and the total score is 90. The rate of correctness of students in the five classes was 30.90%. Their performances were poor. The rate of correctness of students in the best class was about one third. If we disregard the parts about stating reason or computation, the rate of correctness was 40.05% in the five classes. However, the rate of correctness in the parts about stating reason or computation was only 10.63%.

The first three items asked students to distinguish whether the given representations in the three common forms: algebraic expression, table, and graph can represent functions and asked students to explain. The rate of correctness was 34.08%. If we disregard the explanation parts, the rate of correctness was 57.28% but the rate of correctness of explanation was only 10.88%. During interviews, more than half of the interviewee said that their answers for whether the representation is a function were due to guess and got the correct answers. Moreover, few students could state the definition of function. A few students could cite an example of a function and explain it. Their explanations used to be incomplete.

The fourth item including five sub-items was to examine students' functional concepts. The first sub-item was about function values. The given function is f(x) = $\frac{3x+1}{x-1}$. The rate of correctness for f(2) is 80.16%. The rate of correctness for f(-1) is 69.05%. However, the rate of correctness for finding the function value of 1/2 is 21.83%. To find the function value of 1 was a difficult question for students. Considering the results of interviews, I conclude that the students' computation accuracy depended on number systems. Moreover, many students were unclear about the meaning of "function value." They mistook that it is to find pre-image. More than half of the students did not have exact idea about the denominator being 0.

The second sub-item was about transforming an equation $x^2y=1$ into an algebraic expression of function and ask them to point out the independent variable and dependent variable and to give the reason. The rate of correctness for the 4 questions was 17.90%. The rate of correctness is 27.45% if we ignore the part of give explanations. The rate of correctness for transforming the equation $x^2y=1$ into the function expression $y=1/x^2$ was only 19.05%. About 30% of the students could point out that the independent variable is x (30.95%) and dependent variable is y (32.34%). However, only 7.14% of the students could give reasons.

The third and fourth sub-items asked students to find extrema of a rational function $f(x)=\frac{1}{x}$ ($3\leqq x\leqq 7$) and an irrational function $g(x)=\sqrt{x+2}$ ($2\leqq x\leqq 6$). The rate of correctness was 47.67%. Most students were unfamiliar with fractional numbers and radicals together with their properties and computations. It made the researcher think that adding the content of number systems is indispensable.

The fifth sub-item provided questions about functional concepts including definition domain, corresponding domain, range, one to one, inverse function, and composite function and asked students to find range, inverse function, and composite function and to give explanations. The rate of correctness of all answers in the five classes was 11.61%. The rate of correctness in the parts of finding range, inverse function, and composite function was only 1.09%. The rate of correctness of other parts was 15.81%. The students' performances were very poor in this sub-item. It made the researcher and the teachers consider the step of instruction.

DISCUSSION, CONCLUSION AND SUGGESTIONS

The experts had no particular opinion about the designed content on function for the course. However, the content seemed to be difficult for students even though they had been taught the same content in high schools. For the learning of students of this level, the step of instruction should be slower. More interpretations in the classes are required.

From the test, we find that most students did not have clear recognition about function. They could not distinguish whether a representation can represent a function. Some students used guessing to answer questions. Few students could give explanations for their answer. The accuracy of the students' computations was affected by different number systems and the degree of complication. Most students were also unclear about several functional concepts respectively. Most students were not able to transform an equation to algebraic expression of a function.

Considering students' understandings and computations in numbers, the researcher thought that a unit on number systems is indispensable for pre-calculus course. The properties of numbers together with their computation procedures should be emphasized in the curriculum and instruction. In addition, the concepts of both set and function are unified concepts in mathematics. It is recommended that both units on number system and set be added to the pre-calculus course. The research group has hereby completed teaching materials on three topics, including number system, set, and function containing not only elementary content but also transcendental functions, such as Gaussian function, exponential functions, logarithmic functions, trigonometric functions, and inverse trigonometric functions.

Nevertheless, the teacher found that students in this course were not diligent in learning, even when they went to class. The students cannot have good academic achievement if they lack diligence. How to motivate students' learning is the most important issue to teach the students of low level. Even so, the teacher did not find that students were more interested in listening to the mathematical history. No questions related to history were given in the test to understand students' knowledge.

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ASSESSING A NEW INDONESIAN SECONDARY MATHEMATICS TEXTBOOK: HOW DOES IT PROMOTE AUTHENTIC LEARNING?

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Indonesia has implemented its new national curriculum since 2013, and accordingly the Ministry of Education and Culture has published the new textbooks for all subjects, including mathematics. This study aims to examine how the new mathematics textbooks reflect one of the features of the new curriculum, that is, authentic learning. The study focuses on to what extent authentic tasks are presented in the textbooks. For this purpose, we established a framework for analysing the mathematics tasks presented in the textbooks. The year 7 mathematics textbook was selected and the analysis was carried out through two layers. First, all the mathematics tasks were classified into two categories: authentic tasks and non-authentic tasks. Second, the authentic tasks were further categorized into two different levels of authenticity, which are real authentic and semi-authentic. Furthermore, the analysis also compares the authenticity of mathematics tasks between topics of mathematics. The results show that only about 22 percent of the tasks were authentic tasks which comprise 19 percent semi-authentic tasks and 3 percent authentic tasks. The findings of the study suggest that there is room for improvement of the textbook.

Keywords: authentic learning, Indonesia, National Curriculum

INTRODUCTION

In terms of student population, Indonesia has the fourth largest system in the world (Suryadarma & Jones, 2013). Like many Asian countries, Indonesia also adopts a centralized education system in which the Ministry of Education and Culture decides and administers most of educational policies at the national level. Provincial governments, however, decide and administer some aspects of the educational policies such as teacher recruitment, and teacher professional development. Education system in the country is divided into formal and non-formal education. Formal education consists of three levels: primary, secondary, and tertiary education.

International studies such as TIMSS and PISA have shown that the Indonesian educational system does not work well in acquiring a good quality of education at the primary and secondary levels (Suryadarma & Jones, 2013). These indicators have been a driving force for the government to undertake the latest curriculum reform in 2013. In the first year of the implementation, the new curriculum has been implemented at years 1, 4, 7 and 10, and it will be implemented at all levels in 2015. One of the features of the new curriculum is authentic learning (Kemendiknas, 2013). To implement this new curriculum, the Ministry of Education and Culture is publishing textbooks for all subjects, including mathematics. At the time of writing, mathematics textbooks for Years 1, 4, 7, and 10 are the only ones available.

Since the textbooks are used in public schools in Indonesia as the main teaching and learning resources in implementing the new curriculum, and textbooks have great influence on classroom teaching learning, it is both timely and important to assess the new textbooks particularly mathematics textbooks. This study is intended to examine how the new national mathematics textbooks reflect one of the features of the new national curriculum, that is, authentic learning. For this purpose, a year 7 textbook was analysed in the study. Focusing on tasks presented in the textbook, we intend to address the following research question: to what extent the authentic tasks are presented in the year 7 textbook?

LITERATURE REVIEW

Textbook research

It has been widely acknowledged that textbooks play an important role in teaching and learning. Howson (2013), for instance, in the sense of coherent mathematical courses, argued textbooks have played and will continue to play a vital role in mathematics education. Moreover, as a supporting material on classroom teaching and learning, mathematics textbooks potentially shape the ways of teaching and learning. Fan (2000) claimed that textbook can affect not only what to teach, but also how to teach, as "textbook could convey different pedagogical messages to teachers and provide them with encouraging or discouraging curricular environment, promoting different teaching strategies" (Fan, 2000).

Because of their importance as instruments for teaching and learning in mathematics, mathematics textbooks have received increasing attention in the international research community of mathematics education over the few decades (Fan, Zhu, & Miao, 2013). One of the themes of textbook research is textbook analysis. Even though textbook analysis research does not fully provide comprehensive information or knowledge on how a textbook influences teaching and learning, according to Fan et al. (2013), research on textbook analysis can still meaningfully tell us about what the textbooks look like.

Problems or tasks have been widely acknowledged as one of the most important aspects in teaching and learning mathematics. For instance, Kilpatrick, Swafford and Findell (2001) argued that, in term of instruction, the quality of instruction depends on the tasks. Barr (1998) conducted a study on conditions influencing content taught to nine fourth grade mathematics classrooms in five US schools. The study revealed that the nature of lesson and problems in the textbooks determined what was taught. She argued that the number of problems include in the books directly influenced the number of problems assigned by teachers in their classroom (Barr, 1998). Furthermore, Doyle (1988) claimed tasks serve as a context for students' thinking not only during, but also after instruction. Tasks which are most likely chosen from textbooks influence to a large extent how student think about mathematics and come to understand its meaning (Henningsen & Stein, 1997).

Type of tasks

According to Palm (2008), an authentic task refers to "a task in which the situation described in the task including a question or assignment is a situation from real life outside mathematics itself that has occurred or that might very well happen". The task asks students to address concept, problem, or issue that is similar to one they have encountered or are likely to encounter in life beyond the classroom (Newmann, Secada, & Wehlage, 1995). Moreover, authentic tasks use comprehensive representations of the context, real-life performance of adult that are often non-routine yet meaningful and engaging for students (NCTM, 1995). In addition, an authentic task is based on situation which, while sometimes fictional, represents the kinds of problems encountered in real life (OECD, 2001). Authentic tasks can also have different meaning. For example, a teacher can use real-life data such as actual recent unemployment data and ask questions about the increase or decrease of unemployment at different period of time (Franskestein, 1989). In addition, authentic tasks can also refer to “applied mathematics” used in in the workplace, for instance, in engineering (Franskenstein, 1989).

CONCEPTUAL FRAMEWORK

Before we go further, it should be pointed out that in this paper the terms ‘problem’ and ‘task’ are used interchangeably. Moreover, we basically use Zhu and Fan’s definition of a problem, which is “as a situation that requires a decision and or answer, no matter if the solution is readily available or not to the potential problem solver” (Zhu & Fan, 2006). Hence in this study examples and exercises are called tasks or problems.

Following the definition of an authentic task mentioned earlier, we established a conceptual framework for this study. We first categorized all the tasks into authentic and non-authentic tasks, and then further classified the authentic tasks into semi-authentic and real-authentic. Table 1 portrays the conceptual framework of this study.

Table 1: Conceptual Framework of Authentic Tasks

Classification of Mathematical Tasks		
Non-Authentic	Authentic	
Tasks do not connect or relate to real life contexts	Represent all kinds of problems encountered in the real life; students have encountered or are likely to encounter in real-life	
	Semi-authentic	Real-authentic
	Authors or developers make up the real life context in which mathematical problems represented	Original data Students collect data or do activity in the real life

METHODOLOGY

Textbook selection

As previously mentioned, the Indonesian government has published new textbooks in order to implement the new curriculum. An ad hoc team was formed by the Ministry of Education and Culture which is intended to write the textbooks. Furthermore, the ministry then published the textbooks and freely distributed them to the students. However, in the first year of the implementation, the government only published textbooks for year 1, 4, 7, and 10. It means only textbook for year 7 is available for middle school when this study was conducted.

The selected textbook is intended for teaching and learning mathematics at year 7, which is the first level of middle school in the Indonesian education system. Students at this school level are usually about 12 and 13 years old. The main reason for the study to focus on this particular school level is that this level is a crucial stage in the development of students' ability. Zhu and Fan (2006), for instance, argued that this level is key stage in the development of students' ability in problem solving.

Procedure

According to the conceptual framework in Table 1, we examined and coded all the problems presented in the selected textbook. There are 681 problems, which consist of 272 examples and 409 exercises. Furthermore, reliability of coding was checked with an external coder who was invited to code all the problems. The coding result by the independent coder was compared with that obtained by the researchers. Intra-class correlation coefficients (ICC) on absolute agreement between the researchers and the coder are presented in Table 2.

Table 2: Intra-Class Correlation Coefficient between the Researchers and the Coder

Type of Tasks	Intra-Class Correlation
Authentic	.789
Semi-authentic	.771
Real-authentic	.702

The intra-class correlation coefficients of all types of authentic tasks are higher than 0.7. and hence we believe that the result of this study reasonably reliable (also see Devellis, 2003).

RESULTS

The results reported below are first about the ratio of non-authentic and authentic tasks, and then the ratio of examples and exercises of the authentic tasks. Finally, a detailed tabulation of ratios of semi-authentic and real-authentic tasks for all topics is also reported in this paper.

The ratios of non-authentic, semi-authentic and real-authentic tasks are presented in Figure 1. The results show that the authentic tasks presented in the selected textbook are around 22 percent, which consist of 19 percent semi authentic tasks and 3 percent real authentic tasks.

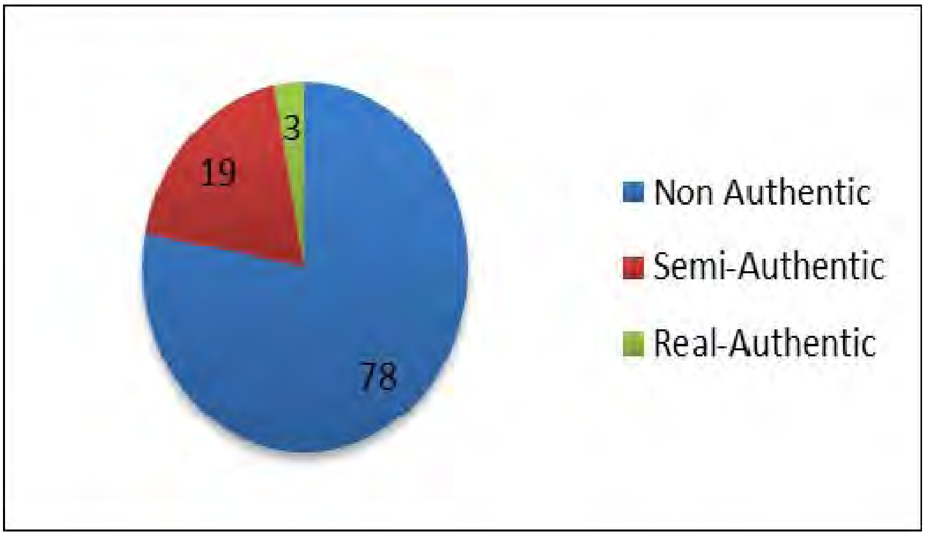

Figure 1. Ratio of Non-Authentic, Semi-Authentic, and Real-Authentic Tasks

The ratios of exercise and examples of the real-authentic and semi-authentic tasks are portrayed in Figure 2.

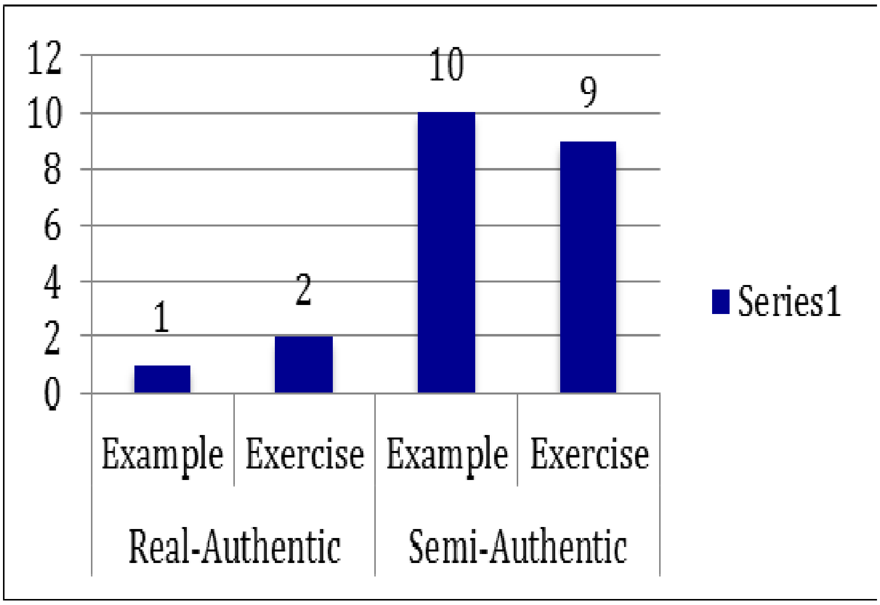

Figure 2. Ratio of exercises and examples

From Figure 2, we can see that the authentic tasks are equally presented in exercises and examples which is 11 percent for each the category. The results indicate that the percent of authentic tasks especially that of real-authentic tasks are very low in the textbook. Fan (1990) once pointed out that, in general, the ratio of application problems to non-application problem in a textbook should be around 40 to 50 percent. It appears that the number of authentic tasks especially real authentic tasks should be increased in order to improve the textbook.

In order to reveal more detailed distribution, the ratios of semi-authentic and real-authentic tasks among the topics are presented in Table 3.

Table 3: Ratios of semi-authentic and real-authentic among the subtopics

Topics	Percentage						
	Semi-Authentic			Real-Authentic			Tot.
	Exp.	Exc.	Tot.	Exp.	Exc.	Tot.	
Set	7	1.4	8.4	1.4	2.8	4.2	12.6
Number	5.6	4.7	10.3	1.4	0.9	2.3	12.6
Line and Angle	9	3	12	6	6	12	24
Quadrangle and Triangle	6	9.4	15.4	0	1.6	1.6	17
Ratio and Scale	7.1	20.4	27.5	3.1	4.1	7.2	34.7
Linear Equations and Inequalities (1 Var.)	10	8.9	18.9	0	1	1	19.9
Social Arithmetic	29.5	48.2	77.7	0	3.5	3.5	81.2
Transformation	23	3.8	26.8	0	3.8	3.8	30.6
Statistics	24	8	32	4	12	16	48
Probability	3	23.5	26.5	0	3	3	29.5

DISCUSSION

The results show that the semi-authentic and real-authentic tasks were found in all topics of the book. However, there is a large variety in terms of the ratios of authentic tasks among the different topics. The topic of Social Arithmetic, for instance, has 81.2 percent of the tasks being authentic. On the other hand, in the topic of Set, only 12.6 percent of the tasks are authentic. We think that this happens because it is probably not so easy to find an appropriate context related to the tasks in the topic. In contrast, in the topic of Social Arithmetic, which mostly focuses on human activities, has many related contexts. Hence, it seems important, though also challenging, to increase the number of authentic tasks for topics that have small proportion of semi- authentic and real-authentic tasks such as Set, Number, Quadrangle, and Triangle.

Concerning about the ratios of semi-authentic and real-authentic tasks among the topics, the results show that the proportion of semi-authentic tasks are higher than real-authentic tasks in almost all topics. Nonetheless, it is interesting to note that topic of Line and Angle has an equal proportion between semi-authentic and real authentic tasks which is 12 percent for each classification. Furthermore, some topics such as Linear Equation and Inequalities, Social Arithmetic, Transformation and Probability have big differences between semi-authentic and real-authentic tasks proportion. For these topics, it needs to increase the number of real-authentic task in order to have a good proportion of semi-authentic and real-authentic tasks.

Regarding the ratios of examples and exercises of the authentic tasks, we found that both are equally presented in the textbook, which is 11percent for each category. Furthermore, it is somehow surprising to see that there is no example of real-authentic tasks presented in the five topics including probability, transformation, and social arithmetic, etc. We believe that this finding indicates these topics should receive more thoughtful attention for improvement of the textbook.

CONCLUSION

To conclude, the new Indonesian mathematics textbook is comprised of non-authentic, semi-authentic, and real-authentic tasks. However, the proportion of authentic tasks particularly real-authentic tasks remains very small. In terms of distribution, a large variety was found about the ratios of semi-authentic and real-authentic tasks presented in the textbook either among the topics or between examples and exercises. Overall, it appears clear that there is room for improvement of the textbook in order to support authentic learning.

To end this paper, we wish to point out that the tasks, as they are written in the textbook, are static in nature in that they may not communicate with the students as they are intended to do (Bayazit, 2013). Therefore, it needs further empirical research in the classroom on investigating how the textbook can promote authentic learning during teaching and learning process, and how this book could influence student’s learning. In addition, this study only examines the authentic tasks in terms of quantity, and further study is needed to investigate the tasks in terms of quality.

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SCIENTIFIC MATHEMATICS AND SCHOOL MATHEMATICS: KNOWLEDGE, CONCEPTIONS AND BELIEFS OF TEACHERS AND MATHEMATICIANS DURING THE DEVELOPMENT OF AN E-TEXTBOOK

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The aim of this work is to investigate the process of sharing and negotiation of knowledge (Shulman, 1986), conceptions and beliefs (Thompson, 1992; Ponte, 1992), on scientific and school mathematics, among the participants of team involved in the development of a set of e-textbooks for the elementary school (MatDigital Project, Brazilian Mathematical Society). The set of e-textbooks includes four volumes in digital interactive format, integrating a range of multimedia resources. The Project is run by a team with diverse academic and professional backgrounds, which gathers elementary school teachers and university lecturers. The research methodology is based on observant participation (Wacquant, 2002), and includes analysis of written records and semi-structured interviews.

Keywords: e-textbook, primary school, MatDigital Project, Brazil

INTRODUCTION: THE DIALOGUE BETWEEN SCHOOL MATHEMATICS AND SCIENTIFIC MATHEMATICS

The interplay between school mathematics and scientific mathematics on teachers' education and practice has been extensively discussed on the mathematics education research literature. This concern is not new or geographically situated. In the early 20th Century, the German mathematician Felix Klein (Klein, 2010) denounced a rupture between school and university mathematics – which he identifies as a *double discontinuity* on prospective teachers education: the mathematics they have contact with during undergraduate courses have little connection with, on the one hand, the one they have previously learned as school students, and, on the other hand, the one they will deal with as school teachers. Moreover, Klein regards the school as having a key role on the development of science: rather than merely spreading knowledge that is produced at the university, school is responsible for independently assessing education needs and establishing categories that will determine forthcoming production of knowledge (Schubring. 2014).

Such concerns have overlaps with more recent research. Since the 1980's, mathematics teachers' education and knowledge has gained prominence on research literature. For instance, Ball (1988) maps out assumptions that tacitly underpin the scheme of teachers' undergraduate courses in the USA (and that make these courses virtually innocuous to classroom practice). In particular, the author concludes that the mastery on advanced mathematical topics is assumed to be sufficient to fully equip teachers with the knowledge needed for practice. Literature has been focusing on the content knowledge needed for teaching practice, its construction, and how and to which extend it is related with (so-called) advanced mathematical topics (e.g. Even & Ball, 2009).

The use of textbooks is an important component of practice, since it largely influences the aspects of the content that will be given less or more stress and the way they are conveyed at the classroom. Therefore, the investigation on teachers' choices concerning textbooks can help to understand their conceptions in school mathematics on scientific mathematics and the ways their mathematical knowledge is linked with practice.

In this paper we report results from a research on the process of sharing and negotiation of knowledge and conceptions, in school mathematics and on scientific mathematics, among the participants of team involved in the development of a set of elementary school e-textbooks (*MatDigital* Project, Brazilian Mathematical Society). The team gathers elementary school teachers and university lecturers, with diverse academic and professional backgrounds. Thus, we focus on negotiation of conceptions between these two groups and, as a result of this process, which aspects of school and scientific mathematics are privileged.

We understand the notions of school mathematics and scientific mathematics as they are formulated by Moreira & David (2003). Thus, scientific mathematics comprises all the academic production on the field, which has its own rigour standards and warrants for truth, as accepted by the academic community at large. School mathematics concerns not only the insertion of pedagogical strategies in teaching, but the whole context of the discipline in elementary education, with its inherent production processes and knowledge validation criteria, as well as the choices of what teach and what not to teach in elementary school.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

A main reference is the work of Shulman (1986), which identifies *pedagogical content knowledge* (PCK) as a kind of knowledge which "goes beyond knowledge of subject matter *per se* to the dimension of subject matter knowledge for teaching […] the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (Shulman, 1986, p.9). Due to its nature, this dimension of knowledge cannot be exhausted in pre-service teachers' education and is continuously constructed from and with classroom practice. To Schulman, teachers must, therefore, take on a protagonist role on the construction of their own knowledge, by forming a clinical view on pedagogical practices. Taking this perspective into account, the links between knowledge involved in pre-service education and knowledge activated in practice can help to understand the constitution of school mathematics (Moreira & David, 2003, p.59).

Shulman's perspective is opposite to hierarchical conceptions that, as underlined by Chervell (1990), considers scientific mathematics as the privileged source of knowledge and constrain school mathematics to a simple vulgarization of scientific knowledge, which is "didactized" in order to be understood in school. Such conceptions disregard classroom as a space for production of knowledge (which is a key aspect for our research).

Besides the dimensions of knowledge proposed by Shulman, several other variables that influence teachers' practice have pointed out on research literature. Thompson (1992) highlights that personal beliefs and meanings attributed to mathematics and mathematics teaching, which may differ from the consensual understanding of scientific mathematics, have a major influence on practice. For Ponte (1993, p. 2, our translation):

> Whilst beliefs are commonly understood as those things people believe in (often by a totally unjustified way), conceptions are more related with general ideas the underlie thinking and action, been much more on the implicit than explicit.

RESEARCH SETTING

Elementary education and textbook use in Brazil

In Brazil, compulsory education is organised in three sections: fundamental school I (grades 1 to 5, ages 6 to 10), fundamental school II (grades 6 to 9, ages 11 to 14), and middle school (grades 1 to 3, ages 15 to 17). Textbooks used in public schools are distributed for free and are chosen by each school out of a list previously approved by the Ministry of Education, through an assessment process that mainly based on evaluations by experts.

The *MatDigital* Project

MatDigital is a sub-project of the Brazilian branch of ICMI's Klein Project, run by Brazilian Mathematical Society (SBM). The aim of the Project is to develop a set of e-textbooks for elementary education (fundamental school II, grades 6 to 9). With inspiration on the ideas of Felix Klein, the guiding principle of the Klein Project is to establish links between a comprehensive view of academic mathematics, the contents and approaches of school mathematics, and the curricula of undergraduate pre-service teachers' courses (Barton, 2008). Aiming to fulfil the Klein Project's this orientation, the methodological design of *MatDigital* project was based on the collaborative work of a team of 60 members, gathering elementary school teachers and university lecturers, from different parts of the country. The team members was organized into subgroups (of 4 or 5 members), coordinated by a central editorial board. Each chapter of the e-textbooks was assigned to a subgroup. Each one of the subgroups as well as the central editorial board included school teachers and university lecturers.

During the development of the Project, communication between the subgroups and with the editorial board was extensively made through an online discussion forum (*Moodle* platform). Moreover, subgroups were instructed to produce a weekly written report of their work, and share them through the platform.

Research questions

The goal of this research is to investigate how the negotiation process, within the team of *MatDigital* Project, to reach a consensual result for the content of the e-textbooks may (or may not) be affected by the participants' knowledge, conceptions and beliefs towards mathematics and school mathematics. In particular, we focus on the relationship between school and university teachers in each subgroup. We aim to identify if this relationship was complementary or hierarchical, and to which extend knowledge, conceptions and beliefs of participants prevails over the others.

More specifically, we aim to answer the questions: (1) What are the main conflicts and consensus about school mathematics that emerge from the discussions on the contents of the e-textbooks? (2) How is the relationship between teachers of elementary education and higher education? (3) How different types of knowledge – the knowledge provided by elementary school practice and the knowledge provided by academic research – are activated in the production of the e-textbook?

METHODOLOGY

Data collection

As the authors are participants of *MatDigital* team, research methodology is based on observant participation (Wacquant, 2002). Data was collected from different written sources: notes taken during subgroups meetings; online discussion forums; and weekly reports of the subgroups.

Moreover, a semi-structured individual interview was conducted with a sample of six volunteer participants, selected in such a way to balance school teachers and university lecturers, and members who were working in different subgroups. Questions focused on the process of production of the e-textbook, on the discussion that took place within the subgroup and between the subgroup and the editorial board, and on choices of topics and approaches to compose the e-textbook. The interviews were tape recorded and fully transcribed.

Data analysis

From the initial exploration of the interview transcripts, most recurrent topics were identified. Four analysis units emerged from this data exploration: (1) why teach mathematics? (2) the relationships between school mathematics and scientific mathematics; (3) knowledge and reflections on the practice; (4) the e-textbook and the collective construction. The discourse concerning each of these units was then compared with data from written sources (notes, forums and reports).

RESULTS

Why teach mathematics?

The participants referred to the importance of the discipline mathematics in elementary school. Both elementary school and university teachers used three kinds of justifications for this importance: applications to students' daily lives; development of specific skills (such as logical reasoning, sense of organization and mental strategy); preparation to following undergraduate studies. We identify these justifications as a *utilitarian* view of mathematics teaching: learning mathematics is only justified to serve *purposes external to the discipline*.

Relationships between school mathematics and scientific mathematics

School teachers point out that they need to have a view of mathematics that differs from the view of academic mathematics. Such different view is mostly associated to pedagogy, that is, to a more didactical approach to mathematical concepts. They also recognize that their undergraduate courses at the university did not achieve the preparation they needed to the challenges they face in the elementary school classroom. Nevertheless, they regard the university as the source where they must seek for improvements to their practices.

Knowledge and reflections on the practice

University lecturers and school teachers define their own ways to contribute on the production of the e-textbooks very differently. University lecturers believe their main contribution concerns providing a proper theoretical grounding and assuring mathematical correctness of the concepts, whilst school teachers claim that their role regards the adequacy to what they call “elementary school reality”, assuring suitable (not too formal) language and approach. Both university lecturers and school teachers associate their respective roles with their objectives and experiences as teachers.

The e-textbook and the collective construction

Participants stress the importance of developing a textbook that incorporates digital tools. They point out that these tools can facilitate learning in several situations, such as visualization in geometry. However they express concerns about the weak preparation of teacher to use digital materials in the classroom, due to low proficiency on handling these tools, and to the changes in classroom dynamics triggered by them.

All the participants agree that, due to challenges involved in the conception and development of a textbook with these characteristics, the collaboration between school and university teachers is imperative, since each one has a specific role in the enterprise.

DISCUSSION

Surprisingly or not, we found more consensus than conflict in the discussion about the e-textbook. School teachers and university lecturers agree on the importance of teaching mathematics in the elementary school (which we have identified as a utilitarian view), on the well-defined roles of each group in the production of the e-textbook, on the importance of developing a textbook with digital tools and on the challenges involved.

In particular, they seem to agree that the role of university lecturers concerns mathematical content correctness, whilst the role of school teachers is more related to pedagogy. The task of criticizing or interfering on mathematical content has not been assigned to school teachers in the discourse of the participants. This indicates a view of the relationship between school mathematics and scientific mathematics: the university is the main source of mathematical (correct) knowledge, which must be didactically adapted in order to be taught in the elementary school; and it is the job of school teachers to perform these adaptations – or the “didatization” of correct mathematical content.

This view is reinforced by the fact that school teachers acknowledge the university as the privileged source to improve the knowledge needed for teaching, even though they admit that their undergraduate courses have poorly contributed to building up their classroom skills. A possible interpretation for this is that teachers associate the knowledge needed for teaching solely with mathematical knowledge, and attribute their own difficulties on teaching to a lack of mathematical knowledge. However, this remains a question for future investigation.

These results also suggest a hierarchical relationship between university lecturers and school teachers. On the one hand, school teachers would comply with university lecturers' orientation when mathematical content is concerned; and on the other hand, university lecturers would follow school teachers' recommendations on pedagogical approach.

In this research we did not focus specifically on the conceptions of the participants about the use of digital technology on the textbook. This was only a context to discuss school mathematics, scientific mathematics, and the process of development of the e-textbook. However, this is an interesting subject for future research.

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FUNCTIONS OF PROOF: A COMPARATIVE ANALYSIS OF FRENCH AND JAPANESE NATIONAL CURRICULA AND TEXTBOOKS

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The aim of this paper is to identify, through a comparative analysis of French and Japanese national curricula and textbooks, different functions attributed to and/or played by proof in geometry, and clarify the relations between the identified functions and the nature of related objects (e.g., theory, statement, diagram). As a result, it is made clear that proofs in two countries play some functions in different ways, and two countries take distinct positions on the choice of a raison d'être of proof (reason why proof is necessary in mathematics) around justification function: justification without perception in France and justification of general case in Japan. The analysis also shows that the functions identified in curricula and textbooks affect and/or are affected by the nature of related objects. For example, justification without perception is closely related to the theoretical geometry as an object to be taught, and provokes the use of 'incorrect' diagrams in French textbooks.

Key words: mathematical proof, geometry, secondary school, France, Japan

INTRODUCTION

Research in mathematics education has identified and valued different roles and functions that proof and proving play in mathematical activities (e.g., Bell, 1976; de Villiers, 1990). Proof is not only for verifying a statement true, but also for explaining why it is true, communicating mathematical ideas, systematising the body of mathematical knowledge, etc. To what extent are these roles and functions reflected in curricula and textbooks?

In my previous work, I identified some differences in the nature of proof according to the country through an analysis of French and Japanese mathematics textbooks: for example, differences in its form (what should be written, in what form), statements to be proven, and properties used (definition, theorems) (Miyakawa, 2012). This analysis also indicates some differences in functions played by proof in mathematics, that is to say, functions of proof reflected in textbooks also differ from country to country. In this paper, I further develop this last point about functions of proof.

The reason of focusing on the functions of proof comes from a conviction that they are principal elements that form the nature of proof. According to the *ecological* perspective proposed by the Anthropological Theory of the Didactic (Chevallard, 1994, 2002), a body of mathematical knowledge to be taught is formed under the influence of *conditions* that allow it to exist and *constraints* that hinder such existence in an institution or an educational system. I consider that this is also the case for proof, and in consequence, different natures of proof in different countries are formed under the influence of a system of conditions and constraints.

In this system, functions attributed to or played by proof occupy a central position, because they are often used as a reason why proof is necessary in mathematics (*raisons d'être* of proof).

In this paper, through a comparative analysis of French and Japanese national curricula and textbooks of lower secondary mathematics, I try to identify different functions attributed to and/or played by proof in geometry, and clarify the relations between the identified functions and the nature of related objects (e.g., theory, statement, diagram). The principle research questions are: *What functions are attributed to or played by the proof in two school mathematics? How do the identified functions affect or are they affected by the nature of related objects?* I expect this research provide some information for textbook design and development, such as another possibility of *raison d'être* of proof.

METHODOLOGY

The study reported in this paper focuses on the domain of geometry where proof is usually introduced in lower secondary school. The analysis consists of two parts according to the research questions. The first part is to identify several functions of proof mentioned in the national curricula and textbooks and shed light on the differences in two countries. An interpretive analysis is carried out from the viewpoint of main functions of proof identified in prior research: verification (conviction or justification), explanation, systematisation, discovery, and communication (de Villiers, 1990). After identifying a function, I further analyse its meaning given in the curricula and textbooks, because a same function might have different meanings. In the textbooks, I analyse especially the chapters of grade 8 textbook both in France and in Japan, where proof is formally introduced. The second part is to analyse the relation between the functions identified in the first part and the nature of objects related to proof, how a function affects the nature of objects such as geometrical theory behind the proof, statement to be proven, diagrams used, etc., or inversely how these objects affects the functions of proof.

Data to be analysed in this study are French and Japanese national curricula and mathematics textbooks. One may easily find some correspondences of data between two countries, due to the similar educational system adopted in each country. The lower secondary level is a single-track educational system, that is, all students go to the same kind of school: four years of *collège* in France and three years of middle school in Japan. And in both countries, teaching contents are determined in the national curriculum written by the Ministry of Education. The textbooks of both countries are based on the national curricula, while the approving processes are different (no approval is required for French textbook).

In this study, I analyse the French national curriculum (MEN, 2008) on the one hand, the guide of Japanese national curriculum which is published by the Ministry of Education (MEXT, 2008) on the other hand. The guide is used for the latter, because the Japanese national curriculum is very simple and there is not enough explanation. As for mathematics textbooks, the ones which are relatively known and shared in each country were chosen for the analysis: for French textbooks, *Triangle* series published by Hatier (Chapiron et a., 2011) and *Sésamath* series by Génération 5 (Sésamath, 2011); for Japanese textbooks, *Atarashii*

Suugaku [*New Mathematics*] series published by Tokyo-shoseki (Fujii et al., 2011), which has the number-one market share position in Japan (I call "*Tokyo-shoseki*").

FUNCTIONS OF PROOF IN FRANCE AND IN JAPAN

In the national curricula and the textbooks I analysed, several functions could be identified. Many of them, such as verification function, convincing function (proof is for convincing someone or one's own self) and communication function, are commonly referred to in both countries. However, looking closely at each function, some significant differences could be identified in some functions.

Justification (or verification) function

In both countries, proof is a means to justify or verify that a statement is true in mathematics. And this function is often referred to as a reason for the necessity of proof in mathematics. However, the way the proof played this function is not same.

In France, proof is a means to justify statements without relying on perception, that is, proof is necessary in mathematics, because what we see and get on a diagram is not precise (*justification without perception*). This is clearly stated in *Triangle* textbook as follows.

> "One cannot prove that a geometrical statement is true by uniquely doing affirmations on a drawing or measurements. […] In order to prove that geometrical statements are true, one has to carry out mathematical proofs" (*Triangle 4e*, 2011, p. 147, my translation).

This function can be also identified in students' activities proposed in *Sésamath 4e* textbook (grade 8). In this textbook, there is a section named "Tool for reasoning" where mathematical proof is introduced. This section consists of some subsections. One of them is named "Attention to illusion" which provides some activities based on optical illusions such as Titchener illusion and Kanizsa triangle, followed by an activity named "Why prove?" (*Sésamath 4e*, 2011, pp. 218-219). The textbook is therefore designed so that unreliability of perception provokes the necessity of proof.

I consider that this function is a principal reason for the existence of proof (*raison d'être*) in French geometry. One may find an account in the French national curriculum. One of objectives of teaching geometry is "Passing from the perceptive identification (recognition by sight) of figures and configurations to their characterisation by properties (passing from drawing to figure)" (MEN, 2008, p. 10). This objective implies a transition from a perceptive geometry to a theoretical geometry where perception is not allowed for justification. As long as a theoretical geometry is an object of study for students, a proof is required to be taught. Otherwise, there is no way to show that a statement is true.

On the other hand in Japan, justification without perception is not mentioned either in the national curriculum or the textbooks. Instead, the guide of national curriculum refers to a *raison d'être* of proof in the section named "Necessity, meaning, and method of proof" as "proof is a means to show that a proposition is true without exception" (MEXT, 2008, p. 96). This citation means that proof is a means to justify a statement about not a particular case but any cases satisfying some conditions. In other words, proof is necessary in mathematics, because without proof there is no way to justify general case (*justification of general case*). In the textbook, one may also find a comment about this function with an example of the sum of interior angles in a triangle: "One cannot check out all triangles by means of experiments or measurements, but one can show that the sum of interior angles of any triangle has 180 degrees by means of a proof like the one above" (*Tokyo-shoseki*, 2012, p. 98).

In Japan, justification function is considered in relation to the nature of statement to be proven, while in France, it is considered in relation to the nature of theory behind proof.

Systematisation function

In my previous work, I reported that proof creates interrelations between geometrical objects in different ways in France and in Japan (Miyakawa, 2012, pp. 230-231). This result shows that proof plays systematisation function in both countries, but the way the proof plays this function is not same.

In Japan, proof is a means not only to create interrelations between geometrical objects, but also to *construct a quasi-axiomatic system of geometry*. The guide of Japanese national curriculum explains it as follows.

> "Verifying by deduction a conjecture obtained by induction or analogy deepens students' understanding on the contents and helps correlating and systematising the knowledge" (MEXT, 2008, p. 29, my translation).

This citation refers to two functions of proof: understanding on the one hand and correlating and systematising on the other. In the latter, one may identify two sub-functions related to the systematisation function. I consider that "systematising the knowledge" implies the construction of a quasi-axiomatic system of geometry. This is not explicit in other parts of the national curriculum, but it can be easily found in the way geometry is structured and presented in textbook. The chapters devoted to the geometry in *Tokyo-shoseki* grade 8 textbook are Chapter 4 "Parallelism and congruence" and Chapter 5 "Triangles and quadrilaterals", where proof is introduced and several statements are intensively proven. In these chapters, the authors first introduce proof and some elementary properties such as properties of parallel lines and congruent figures, and then prove one by one properties of triangles and quadrilaterals. The property (theorem, definition, etc.) that can be used for proving is either the one introduced and admitted at the beginning (e.g., the ones related to parallel line and congruent triangles) or the one proven in these chapters (not the ones introduced earlier in grade 7). While a large part of properties found in these chapters is well known to students since elementary school, it is not allowed to use them for proof until they are proven. In this way, an axiomatic system of geometry, similar to the one given in *Euclid's elements* (but simplified), is constructed step by step. This is carried out without using the term "axiom" or "postulate" (this is a reason why I put prefix "quasi-").

In France, proof is a means to create interrelations between geometrical objects as I reported in my previous work. Some particular geometrical properties (parallelism, perpendicularity, and midpoint) are intensively proven using different properties. However, proof is hardly a means to construct an axiomatic system of geometry. When proving, students are allowed to use any properties introduced earlier in lower secondary school, even the ones introduced in grade 6 where little justification is required (mathematical proof is formally introduced in grade 8). Therefore, proof in France plays rather *organisation* than systematisation, that is, it is a means to carry out “local organisation” (Freudenthal, 1971) in geometry.

RELATION BETWEEN FUNCTIONS AND RELATED OBJECTS

As we have seen, proofs in two countries play some functions in different ways. In particular, one may see distinct positions on the choice of a *raison d’être* of proof around justification function: justification without perception and justification of general case. The nature of these functions is closely related to several aspects of proof. In Japanese textbooks, statement to be proven is always about the general case satisfying a certain conditions and the justification of a particular case (e.g., Fig. 1) is not called “proof” but just “explanation”. And geometrical objects given in proving tasks do not have a fixed dimension in Japan (see Fig. 2). On the contrary, in French textbooks, justification required for the statement about a particular object with fixed dimension (Fig. 1) is also called “mathematical proof” (démonstration). These are consequences of the justification function attributed to the proof in each country.

I mentioned above that the justification without perception is related to the theoretical (but not axiomatic) geometry to be taught in French school mathematics. This function and the geometry to be taught affect the nature of diagrams given in proving tasks. In French textbooks, one may find “incorrect” diagrams in which measures of angles or lengths of segment are not precise and/or which are like the ones drawn by hand (Fig. 1). These diagrams are expected to make students notice that the point is not the realistic/physical objects perceived on diagrams but the ideal objects in a theoretical geometry, and to make them feel the necessity of proof.

In contrast, all of given diagrams in Japanese textbooks are “correct” like Fig. 2. This choice would be due to the functions of proof. First, “incorrect” diagram is not necessary, because “justification without perception” is not taken into consideration. The geometry to be taught is considered to be quasi-axiomatic but not necessary theoretical. In fact, ideal feature of the geometrical object is not mentioned in the guide of actual Japanese national curriculum (it did in the guide of 1989). Second, as proof is a means to justify a general case without exception, “Diagram drawn for proving is the one given as a representative of all” (MEXT, 2008, p. 96). “Representative” should exemplify the characteristics of represented object, and is expected to play a heuristic role of finding ideas for proving. Therefore it could not be “incorrect”.

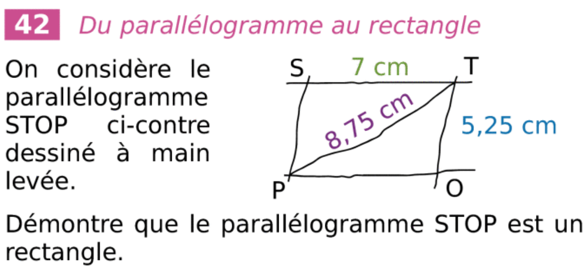

Fig. 1 Prove rectangle STOP
(*Sésamath 4e*, 2011, p.146)

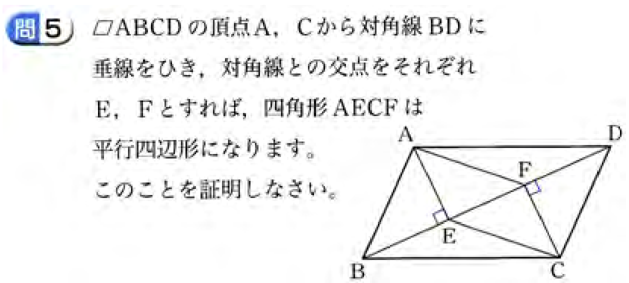

Fig. 2 Prove parallelogram AECF
(*Tokyo-shoseki 2*, 2012, p.137)

As for the systematisation function, it is easy to see that the nature of geometry (or theory) is closely related to this function of proof. In Japanese school mathematics, since the quasi-axiomatic geometry is an object to be taught, the proof plays a function of systematisation. In contrast, the proof plays a function of organisation in French school mathematics, since not the axiomatic geometry but the locally organised geometry is an object to be taught.

DISCUSSION: FOR TEXTBOOK DESIGN AND DEVELOPMENT

In this paper, through the analysis of national curricula and mathematics textbooks, I have shown that proofs in two countries play some functions in different ways, and the nature of these functions is closely related to several aspects of proof. What are implications of these results for textbook design and development? On the one hand, the results propose another possibility of *raison d'être* of proof (why teach proof) in the geometry of each country. On the other hand, the results show that the change of functions of proof in curricula and textbooks requires the change of the nature of other objects related to the proof (e.g., theory, statement).

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HOW TECHNOLOGY USE IS BEING REFLECTED IN JUNIOR SECONDARY MATHEMATICS TEXTBOOKS IN HONG KONG

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The use of digital technologies has been proposed as one of the five basic principles of curriculum design in the curriculum document for the secondary mathematics curriculum (CDI, 1998). Since then, the Hong Kong curriculum has undergone continual reforms of different scales and the use of digital technologies in mathematics teaching has been promoted. Textbook is the medium for informing what should be taught in the curriculum and often recognized as the potentially implemented curriculum. To what extent has the technology been used and reflected in the junior mathematic textbooks under the promotion of top-down curriculum reforms? A popular textbook series was analyzed. The use of technology was categorized into the use of calculators; the use of internet and the use of software, internet and CD-ROM; and other supplementary materials provided by the publishers. The use of technology varies according to the topics in the different strands, namely, algebra, geometry and data handling. Finally, Hong Kong as an example in an Asian context to the use of IT in mathematics teaching is argued to be at a very preliminary stage and a wish list is proposed.

Keywords: technology, secondary school, Hong Kong

INTRODUCTION

The use of digital technologies, including calculators, software and the internet, has been introduced in mathematics education for more than two decades. Technology is currently widely recognized as essential in enhancing the teaching and learning of mathematics (Churchhouse, et al., 1986; Laborde and Sträßer, 2010). In pace with the global trend, the use of digital technologies has been proposed as one of the five basic principles of curriculum design in the curriculum document for the secondary mathematics curriculum (CDI, 1998). Since then, the Hong Kong curriculum has undergone continual reforms of different scales, e.g. "Learning to learn" (EDB, 2001), "The future is now: From vision to realisation" (EDB, 2009), and the use of digital technologies in mathematics teaching has been promoted. On the one hand, the implementation of the use of IT in mathematics teaching needs a lot of teachers' initiative and development of professional knowledge. On the other hand, the implementation also relies much on the availability of the resources and development of teaching materials. Textbooks playing a significant role in curriculum implementation are often recognized as the potentially implemented curriculum (Johansson, 2005; Mullis, et al., 2009, Park and Leung, 2006), therefore, providing a window for understanding how technology may have influenced the mathematics teaching. Using Hong Kong as an example of promoting the use of IT in mathematics teaching in top-down curriculum forces in Asian context, this study aims to investigate how the technology has been used and reflected in the junior mathematic textbooks in Hong Kong. A popular textbook series was analyzed. The research question is:

How has the technology been used and reflected in the current junior mathematics textbooks (Grade 7 to Grade 9) in Hong Kong?

CURRICULUM IN HONG KONG

The current mathematics curriculum in Hong Kong for junior secondary level is based on the curriculum guide prepared by the Curriculum Development Council (2002). With respect to information technology, the short-term targets with respect to IT are: "use diversified learning activities and tools (including project learning and using IT) to arouse students' interest in learning mathematics and to foster high-order thinking skills" (CDC, 2002, p. 5). The objectives of the curriculum include those for strands or learning dimensions of mathematics knowledge and generic skills (collaboration, communication, creativity, critical thinking, information technology (IT), numeracy, problem solving, self management and study). The mathematics contents are arranged in learning units under the three learning dimensions (numbers and algebra; measures, shape and space; and data handling). Information technology is listed as one of the generic skills and the curriculum framework provides exemplars of implementation in mathematics education that are listed below (CDC, 2002; p.22):

1. To use scientific calculators/graphing calculators for various computational and exploratory activities (e.g. input data and create statistical graphs; draw straight lines and explore their relationship with slope).
2. To use suitable software to explore the relations of numbers (e.g. number patterns), algebraic formula (e.g. formulae of area and volume) and graphical representations (e.g. pie charts and straight lines).
3. To use suitable software to construct/explore appropriate statistical diagrams/graphs (e.g. bar charts, pie charts, line charts) to represent given data; to find simple statistical measures (e.g. mean, mode) and to explore the meaning of experimental probability (e.g. tossing coin simulation).
4. To use geometry software packages to explore properties of 2-D rectilinear geometric figures dynamically (e.g. the relationship among the angles or sides of a parallelogram); to explore and visualize geometric properties of 2-D and 3-D figures intuitively (e.g. transformation and symmetry).
5. To use the information obtained through Internet/Intranet in self-directed learning and when doing projects (e.g. statistical projects, projects on the development of mathematics in China, stories and achievements of mathematicians).
6. To judge the appropriateness of using IT in solving mathematical problems (e.g. to calculate $2\sin 30^{\circ}$ mentally).

METHOD

Selection of textbooks: One of the most popular textbook series in Hong Kong was selected for the study. Each grade consisted of two volumes with 5 to 7 chapters per volume and there were about 40 to 60 pages for each chapter. Each volume was supplemented with a CD-ROM. The textbooks including the CD-ROM were read from top to bottom to categorize all examples and activities/exercises that used IT. The categorization was based on Fan's framework (2011) and the categories are listed below:

- Calculator: Using the calculators to find value, to calculate and to explore.
- Internet E-tutor: The E-tutor in the publisher's website providing e-guidance for the selected questions in the revision exercises.
- Internet: Additional resources and information for projects.
- Software, Internet and CD-ROM: One type of activities uses in the Internet for exploration and these are guided by the activity sheets and files in the CD-ROM. Another set of activities can be carried out in the computer offline and use software such as Excel, GeoGebra or Animation for exploring the mathematical concepts. The activities are guided by the activity sheets and the files in the CD-ROM.
- Other supplementary materials provided by the publishers.

RESULTS

Calculator

In general, scientific calculators could be used for any parts of the curriculum. There was an approved list of calculators that could be used for public examinations held by the Hong Kong Examinations and Assessment Authority. Matching the objectives of the curriculum, students were expected to judge the appropriateness of using calculators for solving mathematical problems. Therefore, there was no specific indication whether students should use calculators or not for a certain problem or exercise in the textbook. In the textbooks, the key sequence in using calculator occurs as a side feature near the examples or the content. Those key sequences are included for the new mathematical terms such as irrational number, trigonometric ratios, etc., e.g., 'SHIFT' 'π' '÷' 9 '×' 2 '+' 6.

Internet: E-tutor

At the end of each chapter, there was a revision exercise with support provided by the E-tutor on the internet for selected problems of varying difficulty. Students might login their accounts to use the e-tutor in the publisher's website. The e-tutor provided hints, outlines of the method and a list of the knowledge for solving the problem. These exercises were traditional exercises that can be completed by pencil and paper.

Internet: Websites for additional resources and information

Projects were another type of activities that students might use the internet. Project was not a popular option for the mathematical work suggested in the textbooks; there was only one project for each grade level. These were: (1) A statistically study on the population in Hong Kong (Grade 7, Data Handling), (2) Pythagoras theorem (Grade 8, Measures, Shapes and Space), and (3) Taxation in Hong Kong (Grade 9, Numbers and Algebra). For example, in the project of Taxation in Hong Kong, students were expected to access the Inland Revenue Department's website to study the tax system and comment on the taxation system of Hong Kong. Some guided questions were provided, e.g., "What were the two main sources of tax collection in 2008-2009?" "Present your findings with suitable statistical diagrams, which may include: distribution of various taxes collected in the last three financial years, etc."

Software, Internet and CD-ROM

An icon of IT activity in the textbook was used whenever there was an IT activity relating to the section and the worksheets were found at the end of the chapter with a softcopy in the publishers' CD-ROM, and the link of the software in the internet would be given if appropriate. The learning activities depended on the mathematical topics and the softwares might be Excel, Sketchpad or GeoGebra; with supplementary resource materials provided in the CD-ROM (see examples in Table 1 and Figure1). The activities were mostly for exploration of mathematical concepts. Besides IT activities for exploring mathematical concepts, there was a section with the heading "constructing statistical diagrams with computer software" in Grade 7, which provided a step-by-step instruction with illustration how to create statistical charts with the Chart Wizard function in Excel.

Other supplementary materials provided by the publishers

The CD-ROM provided by the publishers was a major IT resource package for the textbooks. The CD-ROM was designed to be either viewed by resource type or by chapters. When viewed by chapter, only the relevant resources for the chapter were shown. Apart from the IT activities mentioned in the earlier sections, other resources provided by the publishers include:

- Drilling programs were self-evaluated drilling exercises matching the content in the chapters could be run on the computer. The questions were prototypes for the purpose of practice and drilling (e.g., "If 16% of a number is 36, find the number.") The interaction between the student and the computer was only limited to the checking of answers and recording the numbers of attempted questions and correct answers. A help-function displaying the formulae was included.
- 5-minute lectures were Powerpoint files for selected topics in the chapters.
- Software demonstration included files with screen video and verbal instruction of using the software such as Excel, Sketchpad and GeoGebra. E.g., calculating the mean, mode and median of a group data using Excel, construction of parallelogram with Sketchpad or GeoGebra.
- Glossary was a dictionary for mathematical vocabulary in the textbooks, providing definition and audio.
- 'Graph and grid paper' contained files of different scales of graph and grid paper including polar coordinates and isometric grid which could be printed for paper-and-pencil use.

DISCUSSION AND CONCLUSION

The implementation of IT in mathematics teaching is indispensable for the agenda in the curriculum but there are many issues, for example, the technology itself, the impact of the technology on the teaching content, the pedagogical design, the availability of resources, teachers' professional development, equity and policy (Trouche, et al., 2013). In this paper, Hong Kong is used as an example of how the use of IT was represented in the textbook as a result of the top-down curriculum reforms in an Asian context. The use of IT depends much on the mathematical content stipulated in the curriculum. In addition to the use of scientific calculators, the use of IT in the textbooks can be categorized into three major categories. The first type is the provision of self-learning platform such as drilling programme with self-evaluation provided by the publishers. This type can be described as only a change of working platform because the content was basically the same as the traditional pencil-and-paper exercises. These exercises mostly appeared in the Drilling program in the publishers' CD-ROM and the E-tutor on the internet giving quite a complete coverage of the topics but having little implication on the pedagogical design. They could be optional supplementary exercises for students. The second type was IT activities designed to make use of specific software platforms such as Excel and Geogebra. Such activities often provide opportunities of exploration within a limited context. The third type is to let students to use the internet for mathematical activities with a project nature. The latter two types represent a genuine integration of the use IT in the teaching with a change of pedagogical direction and the nature of the learning activity. Textbooks play a very important role as a medium of what should be taught in the curriculum (Park and Leung, 2006) but the results showed that the availability of such activities was very limited as reflected in the textbooks. Moving onto the era of mobile devices and tablets, there are indeed much room for further work. To end, I would like to propose a wish list:

- More insight into the design of the mathematical tasks and activities;
- More insight to the support for the teachers;
- More insight to the empowerment of the students' capacity of learning.

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Table 1. IT activities in the textbooks for Grades 7 to 9

Grade 7	Grade 8	Grade 9
• Sum of all the interior angles of a triangle (Geogebra) • Rotational symmetry of plane figures (Geogebra) • Reflection and rotational transformation (Geogebra) • Order of Transformations (Geogebra) • Estimation of π (Excel) • Constructing Statistical Diagrams (Excel) • Project: A statistical study on the population in Hong Kong	• Identity of the difference of two squares (CD-ROM animation) • Investigating the graphs of linear equations in two unknowns (Excel) • The value of √2 (Excel) • Tessellation (link to activity on internet) • Proofs of Pythagoras' theorem(CD-ROM animation) • Properties of sine ratios and cosine ratios (Geogebra) • Project: Pythagoras theorem	• Simple interest and compound interest (Excel) • Experimental probability (Excel) • Project: Taxation in Hong Kong

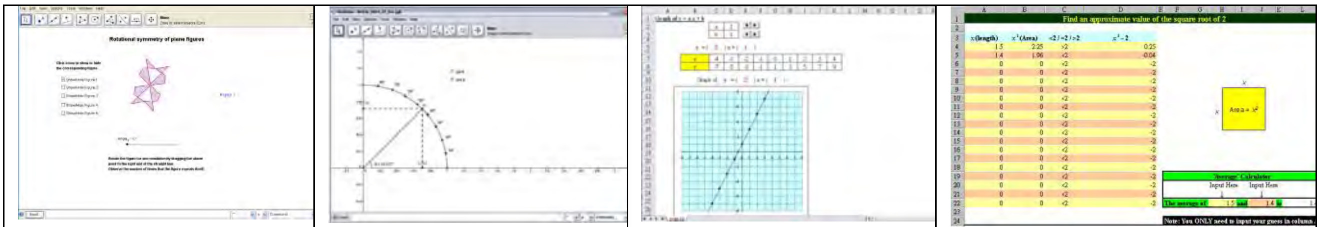

Figure 1. Examples of activities and e-worksheets: Transformation (left most), Trigonometry Linear Graph, Estimation of $\sqrt{2}$ (right most)

THE USE OF TECHNOLOGY IN TEXTBOOKS: A GRADE-7 EXAMPLE FROM HONG KONG

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Information technology for interactive learning is one of the four key tasks recommending in the curriculum reform in Hong Kong (Curriculum Development Council, 2000). According to Fan (2011), the use of technology in textbooks can be reflected in the categories of calculator, computer, Internet, software. A set of Grade 7 mathematics textbooks from a popular series in Hong Kong was analyzed. The exercises or activities using the technological tools in the textbooks were identified and the tools were categorized. Their mathematical contents were classified according to the three mathematics strands (Number & Algebra; Measures, Shape & Space and Data Handling) in the curriculum. Findings show that the use of technological tools in the textbooks included Internet, software and a CD-ROM produced by the publisher. The publisher provided an "E-tutor"-website for a direct guidance for selected questions in the revision exercises in the textbooks and a drilling program in the CD-ROM. There was one project for carrying out a statistical study of the Hong Kong population. The steps for generating of statistical graphs with Excel were demonstrated with illustrations. IT exploratory using Excel and GeoGebra were found.

Keywords: secondary school, technology, Excel, GeoGebra, Hong Kong

INTRODUCTION

Information technology has been receiving much attention in the mathematics curriculum. Textbooks as a major vehicle for the teaching in the classrooms play a major role for facilitating the use of IT. The aim of this paper, on the one hand, is to illustrate a possible way to categorize the use of IT in the textbooks; on the other hand, is to answer the research question: What are the technological tools used in the Grade 7 mathematics textbooks in Hong Kong?

METHODS

One of the most popular textbook series used in Hong Kong was selected for the study. The Grade 7 textbooks consisted of two volumes, each accompanied by a CD-ROM. The textbooks including the CD-ROM were read from top to bottom to code all examples and activities/exercises that used IT. The coding was based on Fan's framework (2011) three major ways of using technology in mathematics textbooks:

- Use of scientific calculators, e.g., to find value, to calculate, and to explore
- Use of Internet as a resource, e.g., for project work
- Use of specific software such as excel and GeoGebra for mathematics activities, e.g, to construct graphs, to explore geometric figures.

Based on the above ideas, the use of technology in the textbook was further categorized by considering the content of learning and how the IT tool might facilitate the learning as the usage was represented in the textbooks. Finally, there are the following categories:

- Calculators: In general, calculators can be used in all parts of the textbooks. For some new mathematics terms, there was some guidance in terms of key sequences for using the calculators.
- Using the internet as a resource of a project
- Using the internet as a resource of e-guidance: The publisher provided a E-tutor website to provide e-guidance for the selected questions in the revision exercises
- Using software in the publisher's CD-ROM
- Using software (Excel or Geogebra) for constructing graphs or exploratory activities guided by worksheets.
- Using the self-evaluated drilling program in the CD-ROM

EXAMPLES OF DIFFERENT USE OF TECHNOLOGY TOOLS IN THE TEXTBOOKS

Table 1 summarizes the technological tools used in the Grade 7 mathematics textbooks. Some examples are given below.

An example of using GeoGebra: Sum of interior angles of a triangle

The activity was to explore the sum of interior angles of a triangle using GeoGebra guided by an e-worksheet (Figure 1) in the CD-ROM with the following instructions.

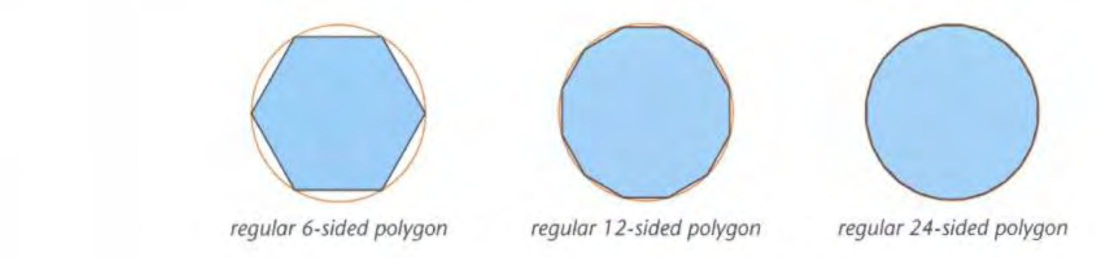
- Select "Step 1" and rotate ΔADE by dragging A until AE coincides with EC.
- Select "Step 2" and move the polygon DBFG horizontally by dragging B until BD coincides with AD.
- What type of angle is formed when the three interior angles (i.e. $\angle ECF$, $\angle EAD$ and $\angle DBF$) are joined together? Circle your answer below.
- (acute angle / right angle / obtuse angle / straight angle / reflex angle / round angle)
- Will you get the same answer of question 1 for other angle? Repeat the above steps with different angels and check the result.
- (Hint: You can get the different triangles by dragging the vertices of ΔABC)
- Write down your conclusion about the sum of all the interior angles of a triangle)

Example of using Excel: Estimation of π

The activity is to estimate the value of π using the software Excel. The activity is guided by the following questions:

1. If the radius of a circle is 1 unit, express the circumference of the circle in terms of π.

2. The following diagram show an inscribed regular 6-sided polygon, inscribed regular 12-sided polygon and an inscribed regular 24-sided polygon. (The radius of the circle is 1 unit).



(a) When the number of sides of the inscribed regular polygon increases, what is the relationship between the perimeter of the polygon and the circumference?

(b) By using the symbol "≈", express the relationship between the perimeter (P) of the regular polygon and π.

3. Open the Excel file provided in the CD-ROM (Figure 1). Input "6" into cell A2. Excel automatically calculates the perimeter of the regular 6-sided polygon as shown in the file. According to the result in question 2(b) and the datum in cell B2, write down an estimation of π

4. In cell A3 to A11, input the number of sides of other regular polygons to obtain the estimation of π. When the number of sides of the regular polygon increases, does the estimation of π become more and more accurate? Yes/No.

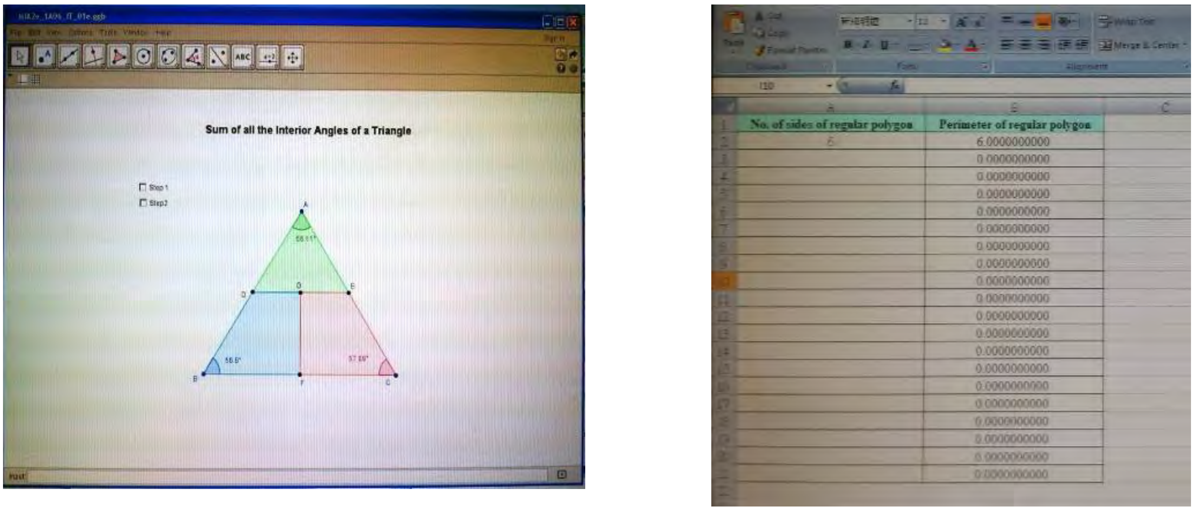


Figure 1. The GeoGebra file for the activity "Sum of interior angles of a triangle" (left); the Excel file for the activity "Estimation of π" (right)

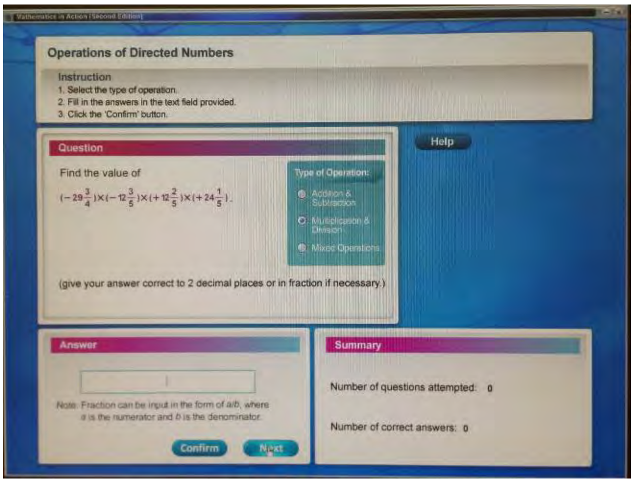
Some examples of the exercises in the self-evaluated drilling program in the CD-ROM

(a) $(+17.3)+(-26.3)+(26.8)+(-0.5)$

(b) A product is sold at a discount of 19% for $103. Find the market price.

(c) $\dfrac{48a^9b^5}{16a^4b}$

(d) Simplify the expression $\left(2x-9x^4-7x^3\right)+\left(9x^4+3x^3\right)$ and arrange the terms in descending powers of the variables.



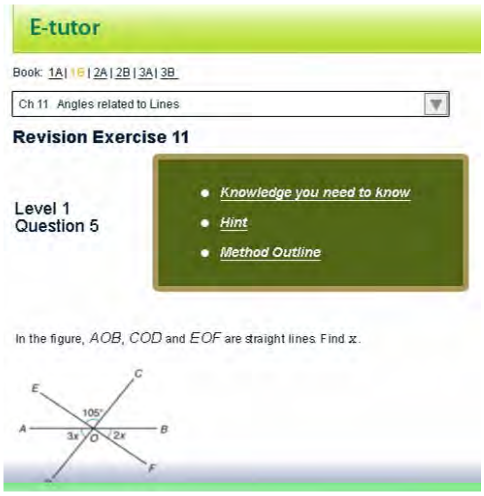


Figure 2. The screen display of an exercise item in the drilling program in the CD-ROM (Left); an example in the E-tutor on publisher's website.

E-tutor in the publisher's website

At the end of each chapter, there was a revision exercise with support was provided by the E-tutor on the internet for selected problems. Students might login the E-tutor website for hints and outline of method (Figure 2).

Using internet for mathematical projects

There was one project in the Grade 7 textbook. The students were asked to carry out a statistical study on the population in Hong Kong by visiting the website of Hong Kong Census and Statistics Department for authentic information to describe the major changes of the population in the last 25 years. In the project, the students were expected to use Excel to produce different types of graphs in the presentation and the steps for generating of statistical graphs with Excel were demonstrated with illustrations in a separate section in the textbook.

SUMMARY

Findings show that the use of technological tools in the textbooks included Internet, software and a CD-ROM produced by the publisher. The publisher provided an "E-tutor"-website for a direct guidance for selected questions in the revision exercises in the textbooks and a drilling program in the CD-ROM. There was one project for carrying out a statistical study of the Hong Kong population. The steps for generating of statistical graphs with Excel were demonstrated with illustrations. IT exploratory using Excel and GeoGebra were found.

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Table 1: Summary of the use of IT tools in the Grade 7 mathematics textbooks

Topics	Technological tools	Exercise/Activity
Number & Algebra Strand		
Basic Mathematics		
Directed Numbers and the Number lines	E-tutor	Revision Exercise
	Student CD-ROM	Drilling program
Introduction to Algebra	E-tutor	Revision Exercise
Algebraic Equations in One Unknown	E-tutor	Revision Exercise
Percentages (I)	E-tutor	Revision Exercise
	Student CD-ROM	Drilling program
Manipulation of Simple Polynomials	E-tutor	Revision Exercise
	Student CD-ROM	Drilling Program
Measures, Shape and Space Strand		
Introduction to Geometry	E-tutor	Revision Exercise
	Software & Learning CD-ROM	Sum of all the interior angles of a triangle (Geogebra)
Symmetry and Transformation	E-tutor	Revision Exercise
	Software & Learning CD-ROM	Rotational symmetry of plane figures (Geogebra) Reflection and rotational transformation (Geogebra)
Areas and Volume (I)	E-tutor	Revision Exercise
Congruency and Similarity	E-tutor	Revision Exercise
	Student CD-ROM	Drilling program – Congruency and similarity

Introduction to Coordinates	E-tutor	Revision Exercise
	Software & Learning CD-ROM	Order of transformations (Geogebra)
Angles related to lines	E-tutor	Revision
	Learning CD-ROM	Drilling Program
Data handling Strand		
Introduction to Various Stages of Statistics	E-tutor	Revision Exercise
Simple Statistical Diagrams and Graphs (I)	E-tutor	Revision Exercise
	Software	Demonstrate how to use the "chart wizard" function of Excel
Project	Internet-website	A Statistical Study on the Population in Hong Kong
Combined strands: Number & Algebra/Measures, Shape and Space		
Estimation in Numbers and Measurement	E-tutor	Revision Exercise
	Software & Learning CD-ROM	Estimation of π (Excel)

PRE-SERVICE AND IN-SERVICE TEACHERS' PREFERENCE WHEN SELECTING MATHEMATICS TEXTBOOKS

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The paper presents results of a research survey carried out within the frame of a larger research project, in which the author make inquiry into the process creation, choice and use of textbooks of mathematics. In this paper, the author argues that teachers' active participation in production of new culture (as Giroux (1983) claims to be the case at schools) is possible in mathematics only if teachers realize they are working with cultural values in their classrooms. That is why the author constructed a questionnaire to name the criteria to which in-service and pre-service teachers pay attention when selecting a textbook. Are cultural values (e.g. gender correctness) of any interest? The questionnaire was administered to 28 pre-service and in-service teachers. The analysis of the questionnaire data shows, amongst other things, that teachers prefer teaching from textbooks of their own choice and that their priorities in selecting textbooks are much more connected to mathematics than to cultural contents.

Key words: selecting textbooks, pre-service teachers, in-service teachers

INTRODUCTION

Recent years have witnessed a rapid growth of interest in mathematics textbook research. From the different fields and areas of interest – one of the possible categorizations of textbook research by E.B. Johnssen is quoted in Sikorová (2010) and distinguishes between process-oriented (creation, production, approval process, selection, distribution), use-oriented (textbook in a mathematics classroom) and product oriented textbook research (content analyses) – it is important for this paper to look at textbooks of mathematics not just as texts with some content (not just the product-oriented perspective), but also as products of a given society, presenting its values, beliefs, opinions on what it means to be doing mathematics, to be doing it at school, to be working with a textbook, to be learning (process-oriented research – selection of a textbook by school and teachers). However, these research approaches can never be treated separately and the process of production and selection will of course have impact on the use of textbook in the classroom. As Rezat and Straesser (2012) point out what happens in a lesson of mathematics can never be well described by the traditional concept of the didactical triangle 'teacher-pupil-mathematics' as there are a number of other factors that structure a mathematics lesson (artifacts – i.e. also textbooks, culture, community). Rezat and Straesser redefine the classical notion of the didactical triangle to the more complex and plastic model of a socio-didactical tetrahedron, which depicts the situation in a mathematics lesson much more accurately. Artefacts used within a lesson (i.e. textbooks) are one of the factors affecting the course of a mathematics lesson and mathematics education in general. Thus the selection of a textbook has a clear influence on lesson planning and teaching; i.e. is important also for use-oriented research.

It is even more important to focus on textbook analysis in countries where education systems are decentralized like in the Czech Republic (Beneš, 2009), where there is no strictly defined national curriculum. The education reform from the beginning of the 21st century replaced national curricula by Framework Education Programme for Pre-School Education (FEP PE), Framework Education Programme for Elementary Education (FEP EE) and Framework Education Programme for Secondary Education (FEP SE). The FEP EE "delimits all that is shared and necessary within the compulsory elementary education of pupils, specifies the level of key competencies which should be attained by the pupils at the end of elementary education; defines the educational content − the expected outcomes and subject matter; integrates cross-curricular subjects with distinctly formative functions as a binding part of elementary education; supports a complex approach to the implementation of educational content, including the possibility of interconnecting it appropriately, and expects that various educational approaches, different teaching forms and methods will be selected and all supportive measures utilised in accordance with the pupils' individual needs." (FEP, p. 6)

The philosophy behind is "to apply more variable organisation and individualisation of education in accordance with the pupils' needs and potential and to utilise internal differentiation of instruction and to create a wider offer of obligatory optional subjects for the development of the pupils' interests and individual capabilities, and to support the educational autonomy of schools and professional responsibility of the teachers for the outcomes of the educational process". (FEP, p. 7)

Every school is expected to work with the Framework Education Programme creatively and design its own School Education Programme respecting the principles of the FEP. FEP defines the binding scope of education for its individual stages but the school defines how and when the goals will be achieved. At each level, there is compulsory time allotment as well as available time allotment (14 lessons of in 1st to 5th grade and 24 lessons in 6th to 9th grade of available time allotment), which allows each school to construct its programme in such a way that it suits the needs of the school, its pedagogical staff, local background etc. In the ideal state teachers from a particular school collaborate and work very creatively, using their potential and skills, to construct the best possible Framework Education Programme for the given school and the specific group of children. However, what often happens is they take a textbook offered at the market (whatever the reasons for selecting a specific textbook are) and use its table of contents as the matrix defining the subject matter that will be taught in a particular grade. Needless to say that this gives a lot of power to those who write and/or publish textbooks, as they indirectly define what is in School Education Programmes. Of course there is a process in which the Ministry of Education of the Czech Republic gives each set of textbooks official approbation and only those can be purchased by schools from finances they can in their budget. However, a lot of power in textbook production lies outside the ministry.

The research carried out by the author of the paper (e.g. Moraová, 2014a) shows that many mathematics textbooks used in the Czech Republic were first published in the beginnings of the 1990’s. Although they have been subject to several revisions to meet the needs of official school documents (curricula, standards, Framework Education Programme), or modifications caused by changes in the outside world (e.g. when hellers stopped circulating or postage changed), e.g. their “literary” content, the setting of the word problems, as well as their methodological approaches, have changed very little. Thus not only is everyday life presented in the textbooks at the best the life lived in the 1990s’ but also the manner of presentation of the subject matter, the methodology will not be really up-to-date. Textbooks of mathematics in the Czech Republic have become rather old-fashioned and culturally distant.

As suggested above, the selection of a particular set of textbooks for a school will have much influence on what mathematics and how it is taught at school and should be paid attention.

METHODS AND MATERIALS

The research question for this paper is: What criteria when selecting a textbook do teacher trainees find important and who (for in-service teachers) selects the textbook at their school?

To answer this question the author of this paper designed a questionnaire (using the principles described by Chráska, 2007) which had two parts: in the first part she asked about the age, sex and year of study of the respondents. This was followed by a question about their teaching experience. If the respondent had none they were asked to proceed to Part B. If they were in-service teachers, they were asked to say what textbook they teach from, who selected it, whether they were happy with it. There were also two open questions where the respondents were asked to describe the pros and cons of that particular textbook.

The criteria used in Part B to operationalize the definition of “preferences when selecting a textbook” were the following:

- Extra materials available (workbooks, printable worksheets, methodological materials, video recordings, online support).
- Reliable and useful teacher’s book
- Correspondence of the textbook with School Education Programme Reasonable price
- Amount of details, of information related to the subject matter
- Motivating elements
- Pace
- Age appropriateness
- Manner of presentation of new subject matter (exposition, definitions, story-motivated, situation based, real-life)
- Cross-curricular references
- Attractive design
- Gender correctness

The last three properties are those that are not connected to mathematics content and may be seen as non-mathematics aspects. The order of these properties was random in the questionnaire, mathematical and non-mathematical criteria were mixed. The respondents were asked to prioritize them (choose the 5 most important and number them 1-5).

The questionnaire was administered to students of Department of Mathematics and Mathematics Education at Charles University in Prague, Faculty of Education. Out of the 50 questionnaires, 28 were responded to and returned to the author of this paper. 12 of them were female students and 6 male students of bachelor programme, i.e. mathematics, not teaching of mathematics (courses of didactics of mathematics are s studied in the follow-up masters study programme), 3 were master degree students, all women, and 7 (6 women and 1 man) were in-service teachers taking a course within the frame of life-long learning to extend their qualification to teaching mathematics; 6 of them were already teaching mathematics, one only other subjects.

FINDINGS

The answers were analysed and interpreted separately for the group of female bachelor students, male bachelor students, master students and in-service teachers (their responses were analysed together as there was only one man).

As far as Part A of the questionnaire was concerned, out of the six in-service teachers, two report to have selected the textbook on their own (one claims to be using more than one textbook and combining the best of each), two use a set of textbooks selected by their predecessor and two use a set of textbook selected by the Department of Mathematics (teachers in Czech elementary schools are united in departments where they should meet for methodological discussions, to solve problems, to discuss plans etc.). In some schools, decisions about what textbook will be used is made after discussion in the department, in some cases each teacher uses a textbook of their own choice and not all teachers of one subject at one school use the same textbook. In small schools, there might be just one teacher of mathematics – in case of this survey these teachers were using a textbook selected by their predecessor. Textbooks are provided to school children for free and thus the school cannot afford to be changing the textbooks used too often as they are reused several years. The respondents who had selected the textbook on their own were satisfied with it and liked it. The two teachers who teach from a textbook selected by their predecessors report they are very unhappy with it, one of the teachers who use the textbook selected by the department says she likes it, the other says she can cope with it. The comments from the open questions cast some light on what teachers like and dislike. Three criticize there are too few examples and problems, and two criticize that explanations are too complicated, one does not like that the layout of the textbook is confusing and one criticizes that the textbook requires very little initiative from the pupil (this is the only comment in which methodology and pupils' activity are considered). Those, who are happy with their textbook value its clear layout, extra activities, usability for self-study (2). Of course, the low number of the filled in questionnaires allows no generalization. However, these questionnaires confirm that teachers should have influence on the textbook they teach from (selecting it on their own or after discussion in the department) as they are then happier when using it. The few questionnaires also suggest that teachers focus most on amount of details, layout/structure and form of mathematics content. Non-mathematical content is not relevant for them.

The relative frequency of priority 1, cumulative frequency for position 1 and 2, and cumulative frequency for positions 1-5 were calculated. In the group of female bachelor students, the highest frequency of position 1 was for availability of extra materials (33%), followed by correspondence with framework education programme (25%) and age appropriateness (15%). If positions number 1 and 2 were considered, then priority number one was availability of extra materials (67%), followed by the amount of details, motivation elements and correspondence to school education programme (all 33%). The cumulative frequency for all top five positions was 75% for manner of presentation and 67% for correspondence to framework education programme and availability of extra materials.

In the group of male bachelor students, the highest frequency of position 1 was the amount of details (50%), followed by price, cross-curricular links, manner of presentation and age appropriateness (17%). If positions number 1 and 2 were considered, then priority number one is the amount of details (83%), followed by availability of extra materials, price, teacher’s book, cross-curricular references, manner of presentation, pace, age appropriateness (all 17%). The cumulative frequency for all top five positions was 83% for details and 67% for price and existence of teacher’s book.

In the group of master students (all of them were female), the highest frequency of position 1 was availability of extra materials (67%), followed by correspondence with school education programme (33%). If positions number 1 and 2 were considered, then priority number one was amount of details (83%), followed by availability of extra materials, price, teacher’s book, cross-curricular references, manner of presentation, pace, age appropriateness, (all 17%). The cumulative frequency for all top five positions was 100% for manner of presentation and 67% correspondence to school education programme, age appropriateness, motivation elements and availability of extra materials.

In the group of in-service teachers (6 female, 1 male), everybody chose a different top priority (availability of extra materials, reasonable price, design, teacher’s book, cross-curricular links, correspondence with school education programme, manner of presentation). If positions number 1 and 2 were considered, then priority number one was age appropriateness (43%), followed by cross-curricular links and correspondence to school education programme 29%). The cumulative frequency for all top five positions was 100% for motivation elements, 71% for manner of presentation and 57% for reasonable price (which shows that in-service teachers know the problems and reality of school budgets and will consider economy of the chosen textbook as well but also that they consider a particular group of pupils – age appropriateness).

It can be concluded that non-mathematical properties of the textbook are in general regarded as less important. Gender correctness was somehow prioritized only 3 times (1x position 2, 1x position 4 and 1x position 5), design 6 times (2x position 1, 2x position 4 and 2x position 5). The only more considered non-mathematical aspect was cross-curricular links with a cumulative frequency of 32% and 3 first positions.

Another criterion considered not very important was pace, which was selected ten times (2x position 2, 1x position 3, 1x position 4 and 6x position 5). On the other hand, the two most important criteria are correspondence with school curricular programme and manner of presentation with the cumulative score of 67%. These are followed by motivation elements in the textbook (60%).

Comparing priorities of female and male students, it is interesting to note that female students find the amount of details much less important than male students (only 50% of female students have it in top 5 positions while 83% of male students). Another interesting difference among the groups is the in-service teachers' attention to age appropriateness and price which cannot be observed in other groups and will be caused by their everyday experience.

CONCLUSION

The survey reported in this research shows that there are significant differences in what different groups of teacher trainees regard important when selecting a textbook. Much more attention is paid to the mathematical than to the non-mathematical content and e.g. gender correctness (after all the debate and product oriented research of the past decades) is not seen as important. This confirms the conclusions of analysis of problems posed by teacher trainees (Moraová, 2014b), which are very stereotypical and do not show much subversion as far as culture is concerned.

This only shows that textbook analyses and work with textbooks should become part of teacher training. This would raise teacher trainees' awareness of what to consider when making the decision or partaking on the decision which set of textbooks to use at their school.

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IMPACT OF CHANGES IN TEACHING STRATEGIES ON HOW TEACHERS WORK WITH A TEXTBOOK

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The paper presents one of the findings of a longitudinal research project focusing on improving culture of problem solving by pupils through the use of various heuristic solving strategies. The paper classifies how teachers work with textbooks in mathematics lessons. It shows changes in approaches of teachers involved in the experiment to the use of mathematics textbooks in the classroom that are the consequence of their experimental teaching. The findings are based on questionnaires and in-depth interviews with the participating teachers. The survey shows there is a shift towards creativity in these teachers' approaches to the use of textbooks. A significant increase in their autonomy was observed. The findings of the survey are very important for teachers´ school practice as well as for pre- and in-service teacher training.

Keywords: mathematical problem solving, heuristic strategies, textbook use

INTRODUCTION

Textbooks are one of the basic teachers' instruments in planning and conducting their lessons. Research of Baldwin and Baldwin (1992) from Canada showed that teachers were using textbooks 70 to 90% of the teaching time. More recently Askew et al. (2010) use TIMSS 2007 data to show that 65% of 5th grade math teachers and 60% of 9th grade teachers claim to be working with the textbook most of the teaching time. Textbooks have been in the center of attention of teachers, educators and researchers for a long time (see e.g. Triantafillou, Spiliotopoulou, Potari, 2013). Considerable attention is paid to textbook research at various international events (Veilande, 2014); it is the main topic of the work of Nordic Network of Research on Mathematics Textbooks.

Rezat and Sträßer (2014) divide the field of research on mathematics textbooks into three areas: (1) research that focuses on the mathematics textbook itself, (2) research on the use of mathematics textbooks and (3) research on the impact of mathematics textbooks. Research reported in this paper is from area (2). It focuses on the relationship between development of the use of heuristic strategies and a teacher's approach to the use of textbooks.

The paper presents one of the findings of a longitudinal research project *Development of culture of problem solving in mathematics in Czech schools* focusing on improving culture of problem solving by pupils through the use of various heuristic solving strategies (Břehovský et al., 2013). It combines the following aspects: It classifies how teachers work with textbooks in mathematics lessons (ignoring the case when the textbook is used for pupils' self-study) and follows the changes of approaches in the use of mathematics textbooks in the classroom by teachers participating in the research.

THEORETICAL BACKGROUND

Textbooks

It is a generally accepted fact that mathematics lessons should develop pupils' creativity and independence in their search for suitable solving strategies. However, very little or no attention has been paid to whether or how this is supported by textbooks. Karp (2013) states that:

> … despite of the fact that the importance of problem solving has become universally recognized, problems in textbooks are often seen as mere exercises, whose aim is to develop various skills, and which are consequently more or less typical and traditional, and therefore of little interest for discussion.

Heuristic strategies

Many authors focus on the use of heuristic strategies when solving problems, see e.g. (Fan & Zhu, 2007), (Eisner, 1982), (Sandford, 1985), (Kaufmann, 1985) or (Stacey, 1991). However, no attention is paid to the influence of textbooks while using heuristic strategies.

The teaching/learning process can be characterized as a sequence of situations (natural or didactical) that result in modifications in the students' behaviour that are typical for getting new knowledge (Brousseau,1997). The term “problem solving” is a vague notion, a kind of umbrella under which different theoretical approaches are taking place (Nesher, Hershkowitz, Novotná, 2003). Mathematicians agree that problem solving occurs in cases where there is no clear algorithm to be performed. Acknowledging that solving a genuine problem is not just a matter of following a given algorithm, first Polya (1945, 1973) and then Schoenfeld (1985) suggested general strategies for solving word problems, asking questions such as: What is the unknown? What are the data? What are the conditions? Do you know a related problem that was solved previously? Prepare a plan for the solution. Examine the solution obtained.

In the research in the project *Development of culture of problem solving in mathematics in Czech schools*, the following 12 heuristic solving strategies are dealt with (Eisenmann, Přibyl, 2014): Guess – Check – Revise, Systematic experimentation, Use of false assumption, Graphical representation – Solution drawing, Introduction of auxiliary element, Working backwards, Generalization and specification, Specification and generalization, Problem reformulation, Decomposition into simpler cases, Omitting a condition and Analogy.

Effective use of the heuristic strategies is discussed e.g. by (Charles, Lester and O'Daffer, 1992).

OUR RESEARCH

Research questions

The research presented in this paper tries to answer the following research questions: Do the selected textbooks use heuristic strategies in problem solving and, if so, to which extent? How do teachers use textbooks in mathematics lessons and how was a teacher's approach to textbooks influenced by long-term experimental inclusion of heuristic strategies into their lessons?

Research methodology

The original as well as the changed teachers' attitude after their participation in the experiment were investigated using a questionnaire and in-depth structured interviews. All the respondents were teachers participating in the research experiment (11). Throughout this period the teachers used problems designed for the experiment in their lessons and encouraged their pupils to solve them using a heuristic strategy.

Data collection

The experiment was conducted with the total of 342 pupils aged 12-19 and 11 lower and upper secondary school teachers in the Czech Republic. The classes were selected with the intention of having a variety of classrooms as far as geographical position, specialization and pupils' intellectual levels are concerned. The teachers did not undergo any special training on how to conduct the lessons. They were only informed about organization of the experiment and were provided with sufficient number of tasks and problems that could be solved more easily using various heuristic strategies.

The questionnaire survey was sent out and collected by email; the interviews were conducted by the researchers. The following part presents the main findings of the survey.

DISCUSSION AND RESULTS

Implementation of heuristic strategies and textbook problems supporting their use

The researchers focused on several mathematics textbooks for lower secondary school that are widely used in the Czech Republic. Most of these textbooks do not work with heuristic strategies. The textbooks are based on problems and tasks whose aim is to practice and drill selected parts of mathematics through school algorithm strategies. Very little attention is paid to development of pupils' creativity.

We see the following as the causes of this situation:

- Textbooks are usually designed for teachers' practice, therefore they are guided by the prevailing teachers' demands; heuristic solving strategies are not commonly used by Czech teachers of mathematics.
- Non-algorithmic character of heuristic strategies often requires a lot of additional explanations, which exceeds the required scope of the textbook (it would be too thick);
- There is the potential danger that misuse of heuristic strategies will make them just another algorithmic procedure.

Use of textbooks when teaching mathematics

All the teachers involved in the questionnaires and interviews were using textbooks and also collections of problems. Based on their responses we classified their use of textbooks in lesson planning and in the classroom. This classification ignores the case when the textbook is used only for pupils' self-study. The classification distinguishes these cases:

The teacher uses the textbook only when planning the lesson but not in the lesson.
The teacher supplements its content with his/her own material or modifies the content.
The teacher works exclusively from the textbook.

The most common is the case when the teacher uses the textbook together with a collection of problems (2). It is often the case that the teachers modify the problems to meet the needs of the particular group of pupils, albeit by changing the context, simplifying it, reformulating the question etc. They use different collections, pose their own problems, look for problems on the internet etc. The textbook is a guide for making a cascade of new topics of increasing difficulty. The following comments justify this approach:

Teacher 1:

> Like my colleagues, I use Czech textbooks as a collection of problems. I also use collections of problems. I don’t think current Czech textbooks are good to be used with pupils – they are very academic, austere, unsuitable for self-study for average and below average pupils. There aren’t enough problems, not enough types of solutions, there aren’t many applications. They offer the teacher no extra service – there are no methodological teacher’s books with solutions of problems, suitable methodology, extra materials etc. Teachers and pupils would appreciate if there was this service offered in the textbook.

Teacher 2:

> I’m afraid textbooks available on our market don’t give much space for use of various solving strategies. There are not many problems supporting reasoning. I know that even the simplest reasoning is hard to explain in writing. Textbooks most often contain problems asking for precise mathematical reasoning. It’s up to the teacher to explain different solving strategies. But this requires a lot of experience and ability to improvise, which is very hard to do as a fresh graduate – beginning teachers are more likely to insist their pupils use prescribed procedures. They are more textbook-bound. … The aim of textbook authors is to have the texts mathematically precise. This makes the solving procedure more difficult for pupils than they would be in everyday life. That’s comprehensible – pupils must master certain procedures and algorithms. For example in case of the rule of three, most problems could be solved by reasoning. But the rule of three must be explained as e.g. in chemistry reasoning would not do and pupils must know the mechanical procedure.

Teacher 3:

> The textbooks I have contain quite a lot of real-life and application problems but they are very artificial. My pupils are quite good and refuse to calculate nonsense. Some of the strategies are nice and really usable in real-life, e.g. experimenting. But there aren’t many problems of this type.

Most teachers do it in such a way that pupils work with the textbook mainly when practicing.

Teachers expressed very different views on what they expect from the textbook. While some of them ask for a large number of routine problems, short assignments, instructive problems, sample solutions etc., others call for real-life problems (preferably not “artificial”) and problems that can be solved using several procedures (including heuristic strategies) that leave space for pupils’ individual discovery. However, there are not many teachers who support pupils' independent use of heuristic strategies. Teacher 2 describes his approach as follows:

Teacher 2:

> I teach in a way that textbooks are used for assignment of problems and it's up to my pupils to look for the solution. If it's something new, I don't follow explanations as they're presented in the textbook but I explain it my way. Sometimes I use several procedures and strategies. Sometimes (very rarely) I look at the presented solving procedure if it's something I can't explain easily. … I often explain several strategies – somebody visualizes the situation, other pupils need an illustrative drawing, somebody needs analogy. My aim is that everybody should find a procedure they understand. I welcome all pupils' procedures; they explain them at the blackboard. However, there's a group of pupils who don't like this variety of several procedures. They only want one which they learn. More possible strategies make it more difficult for them.

Changes in the approach to textbook use in case of the teachers involved in the experiment

The success in changing students' relationship to solving problems requires not only deep teachers' involvement in realisation of the designed activities but also their active involvement in the project design. This change of their role goes hand in hand with the change of their pedagogical approaches and beliefs. We detected changes in their approaches to the use of textbooks independently of their approaches before the experiment. Our findings are based on in-depth interviews with the participating teachers. The survey showed a shift towards creativity in the way teachers use their textbooks. This change was not instantaneous, it was a longer process which had considerable impact also on pupils' attitudes to mathematics and problem solving.

We observed a significant increase in the teachers' autonomy. Let us illustrate by comments of two teachers:

Teacher 2:

> I used to insist on accurate recording of a problem, solution usually using equations, in physics for example first a general solution in which the unknown from a formula was expressed, followed by substitution. Today I see that if I insist on theoretical procedure, my pupils cannot see the general sense in the problem. They focus on formulas and learned algorithms and do not reason. I now appreciate if they solve the problem anyhow. I emphasize simple reasoning. … The goal of my lessons should be to teach the children to think not to reiterate known algorithms.

Teacher 3:

> Soon I started to select deliberately those problems (in collections of problems) that enabled my pupils to practice a selected strategy. I also started to pose problems based on a good model problem from a textbook. Thus I created sets of related problems different in parameters or difficulty, sometimes even context. Sometimes I managed to engage those pupils who had finished earlier than the rest of the class in posing new problems. … I'm now more than before annoyed by problems that ask absolutely stupid questions (for example how many chickens and goats run somewhere if we see 22 legs etc.).

CONCLUDING REMARKS

As textbook problems are very often designed to support application of algorithmic strategies, if teachers want to use them for development of heuristic strategies, their work with the textbook must be very active. Based on experiments conducted within the frame of this research, the paper shows changes in the participating teachers' approaches to the use of mathematics textbooks in mathematics classrooms which are a consequence of their experimental teaching. The survey shows there is a shift towards creativity in teachers' approaches to the use of textbook.

The findings of the survey are very important for pre- and in-service teacher training.

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MATHEMATICS TEXTBOOK ANALYSIS; SUPPORTING THE IMPLEMENTATION OF A NEW MATHEMATICS CURRICULUM

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Following the 2005 National Council for Curriculum and Assessment (NCCA) review in Ireland a new mathematics curriculum was developed. This new curriculum was part of a strategic plan intended to improve mathematics teaching and learning in Ireland and it led to the development of new mathematics textbooks and the updating of already well-established mathematics textbooks. At the time there was significant national debate centered on the new curriculum. This national debate kept the spotlight firmly on the national roll out of the curriculum in late 2010 and no doubt mounted pressure on the relevant Government bodies. Hence in 2011, for the first time in Irish education history, the NCCA made the decision to intervene in the development and redevelopment of the new mathematics textbooks. One of their interventions was to commission the author, as part of the National Centre of Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL), to conduct a review of the available mathematics textbooks in light of the Project Maths curriculum documents. This paper presents a brief outline of this mathematics textbook review, discussing the methodology and the main findings while also intending to highlight the role of textbook analysis in supporting the implementation of a new curriculum.

Keywords: mathematics curriculum, NCE-MSTL, Ireland

INTRODUCTION

There are many conversations about the role, purpose and impact of the textbook in general and specifically in relation to the mathematics textbook. One of the common themes from such discussions is that it is accepted worldwide that mathematics textbooks have a major influence on classroom practice (Valverde et al., 2002). Textbooks are important vehicles for the promotion of specific types of mathematics curricula and, as noted Schmidt et al. (1997), the mathematics textbook is one of the key factors in implementing mathematics curricula. Textbooks are organised in a purposeful way, and consequently their content and structure are important for the promotion of a specific vision of mathematics curriculum, which in turn impact directly on students' learning (Robitaille & Travers, 1992). Further to this TIMSS (2005) conceives and develops a powerful link between curriculum and textbooks, suggesting the textbook can be considered as a 'surrogate curriculum'. Given the central role of textbooks in curriculum development and change, O'Keeffe & O'Donoghue (2011) highlight the need for care to be taken at policy level to ensure that any new mathematics curricula is supported by new mathematics textbooks which are aligned with the reform vision and hence 'fit for purpose'.

Irish context

From an Irish perspective, mathematics teaching and classrooms have historically been influenced by commercially produced school textbooks that have promoted a view of mathematics concerned mainly with skills and instrumental learning (NCCA), 2005). The NCCA (2005) also suggests that over reliance on mathematics textbooks may well be a contributor factor to the low uptake of higher level mathematics in state examinations. Two curriculum mapping exercises have been carried out to determine the true position of Ireland's mathematical education system. The first being a `test-curriculum rating' which involves measuring Irish students' expected `curriculum familiarity' with the concepts, contexts and formats of PISA, based on an analysis of the Junior Certificate examination[1] papers at Higher, Ordinary and Foundation levels (Cosgrove, Oldham, & Close, 2005) and the second of which mapped the 2003 Junior Certificate and 1974 Intermediate Certificate[2] against the PISA three-dimensional framework (Close and Oldham, 2005). The initial curriculum mapping exercise highlighted the discontinuity between the senior primary school curriculum and the lower secondary school textbooks. This discontinuity has been noted by Smyth, McCoy, & Darmody (2004) to be affecting the students' transition from primary to secondary education.

Irish junior cycle mathematics curriculum

The 2005 NCCA curriculum review and consultation discussion paper led to the decision to create and implement a new second level mathematics curriculum which came to be known as *Project Maths* which was implemented nationally in September 2010. Prior to this Project Maths was piloted in twenty four schools in Ireland during the period 2008-2010, with mathematics teachers in these schools exploring a new range and availability of classroom resources and teaching methodologies. Project Maths is a five strand curriculum; Statistics and Probability, Geometry and Trigonometry, Number, Algebra and Functions and its key focus is to change what students learn, how they learn and how they are assessed (Project Maths a). As each strand was piloted in the twenty four secondary schools the cooperating teachers were required to evaluate the resource materials and provide feedback, the intention being to improve the quality of the teaching and the provision of resources. No new mathematics textbooks were introduced by the Project Maths team with the new curriculum. However, as each strand was rolled out into schools nationwide some textbook materials emerged, some of which were supplement material to be used in conjunction with previous textbooks and one of which was a new textbook series. The Project Maths development team made their views on the issue of mathematics textbook known by stating that they acknowledged the role of the mathematics textbook in the classroom but urged teachers to be flexible in their approach to selecting and using textbooks highlighting that 'no single textbook' can meet the needs of all pupils within a class and that while mathematics textbooks

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[1] Irish second level students enter second level education at aged 12-13 and complete a three year programme of study (the Junior Cycle) which terminates with state examinations called the Junior Certificate Examination. Examinations are offered at three levels, Higher Level, Ordinary Level and Foundation level.

[2] Prior to the development of the Junior Certificate Programme of Study in 1989, the previous programme was entitled the Intermediate Certificate.

contain many good ideas and suggestion for teaching they are somewhat limited by the 'linear presentation of ideas' (Project Maths b).

Background to the report

Following very public debates and conversations about the new mathematics curriculum and some criticisms of the lack of support and resources for teachers the NCCA commissioned the National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL) to conduct a review of school mathematics textbooks published commercially for Project Maths. The report (O'Keeffe & O'Donoghue, 2011) was published in 2011 at a time when a significant number of new textbooks were made available for Project Maths but anecdotal evidence was suggesting that they are not a good match for the new mathematics curriculum. The intention of the O'Keeffe & O'Donoghue (2011) report was to offer an objective evaluation of a selection of new textbooks available for Project Maths.

This paper aims to provide an outline of the methodology employed in the aforementioned report, highlight the key findings of the report and finally discuss the implications of such findings on the process of curriculum development and introduction.

METHODOLOGY AND THEORETICAL FRAMEWORK

The O'Keeffe & O'Donghue (2011) report referred to the TIMSS (1995) for its theoretical underpinnings. Curriculum is a central variable in TIMSS and is used to compare national systems of education, the conceptual framework for which is based on the now well-known tripartite model of intended, implemented and attained curriculum (Robitaille et al., 1997). The TIMSS model was formulated to deal with evolving curricula based on a devised common framework to compare systems of education through analyses of curricula, related documents and artefacts, known as *curriculum frameworks*. Each framework is characterised by the same three elements that are further sub-divided; subject matter content, performance expectations and perspectives or context (Robitaille et al., 1997).

According to TIMSS these frameworks can be applied to the curriculum or any piece of the curriculum that is seen as promoting the intended, implemented or attained curriculum and includes artefacts such as textbooks, curriculum guides, standards documents etc.. While the original framework had three dimensions (Structure, Performance expectations and Perspectives) the mathematics framework was adapted and refined for use in the O'Keeffe & O'Donoghue (2011) report. The evolved model is identified as the TIMSS+ instrument (Figure 1).

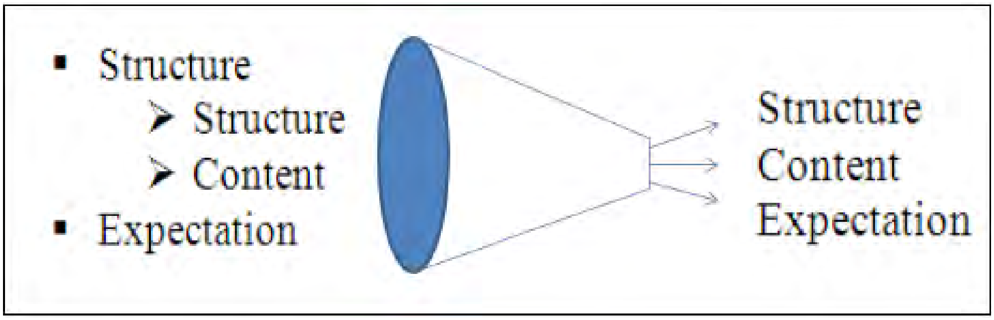

Figure 1: Development of the TIMSS+ Instrument

Hence the study outlined by O'Keeffe & O'Donoghue (2011) is aligned with TIMSS theory and methodology which in turn makes available the TIMSS superstructure as needed. Each textbook available for analysis at the time was treated as a representation of the intended Project Maths curriculum, and hence could be treated as individual stand-alone representations or compared to the other textbooks.

At the time of the study there was a number of supplement textbook materials and one new textbook series available to support Project Maths. However, there are no complete textbooks for Project Maths (Strands 1-5) covering the entire curriculum from year 1 to year 5 at Higher and Ordinary levels. In order to allow for comparative analysis the authors had to create *specially constructed curricula (SCC)*, whereby strands of the curriculum were identified as subsections of the curriculum and each were treated independently. Once the SCC were established (See Tables 1 and 2), the associated disaggregated textbooks and disaggregated Project Maths Syllabus documents were identified and the TIMSS+ instrument was systematically applied to each (See O'Keeffe & O'Donoghue (2011) for further detail).

FINDINGS AND IMPLICATIONS

The key findings from each disaggregated textbook are presented in the main body of the report according to each SCC under the three key framework sections; Structure, Content and Expectation. The summary of the key findings suggests that no one textbook met the needs of the new mathematics curriculum but there were some genuine attempts. While there were structural differences, as expected, across the 10 disaggregated textbooks the mismatches with regard content and expectation are of primary concern. It is important to note that all material is compulsory in Project Maths and there are no 'optional' topics of study hence content omissions such as 'Domain and Range', 'Linear Functions' and 'Proportionality Problems' are of consequence. Similarly key expectation omissions within strands or across textbooks such as a focus on 'Developing Algorithms', 'Performing more Complex Procedures', 'Formulating & Clarifying Problems' and 'Across and Inter Subject Connections' can have an impact on implementation of the new mathematics curriculum. Further to this a note in the expectations data indicates that a greater emphasis could be placed on 'inquiry based learning', 'problem solving in context' and the 'use of graphics to assist with problem solving'. A further key finding was the noteworthy omissions of the integration of ICT throughout all textbooks and disparities between approaches to teaching for understanding and problem solving.

Project Maths identifies itself as a new mathematics curriculum with a new focus aimed at improving the teaching and learning of mathematics and hence improving students' comprehension and understanding (Jeffes et al., 2012) and in light of this the key findings summarised above can impact on the successful implementation of such a curriculum. The previous mathematics curriculum is well acknowledged, in an Irish context, as providing predictable exams, whereby large chunks of the curriculum could be omitted entirely and students could rote learn some key information (Oldham, 2001). Project Maths intends to counteract this with a change in teaching methodologies and a focus on teaching for understanding and application (Lynch, 2011). In order for such changes to take place the teacher must be fully informed on expectations and good practices (a separate issue from the one discussed in this paper) and the textbook, given its unquestioned link to the curriculum, must echo the expectations of the curriculum (Ball & Cohen, 1996, Remillard, 2005). The above findings indicate a mismatch between curriculum and textbook expectations which could obstruct a complete 'change' in approaches to teaching and learning (Ball & Cohen, 1996).

Further to this, numerous reports by the NCAA identify that Irish mathematics teachers focus entirely on routine procedures and place little or no value on the concepts of understanding, communicating, validating and justifying mathematics. This is reflected in the previous Irish textbooks (O'Keeffe, 2011), the authors of which are all mathematics teachers, by the dominance of the expectation to perform routine procedures and minimised or omitted focus on predicting, verifying, justifying, critiquing and discussing mathematics. There has been much discussion about the difficulties teachers face when asked to change practices, strategies and approaches to teaching (Ball & Cohen, 1996). Such difficulties are increased in light of the findings of Ní Ríordáin & Hannigan (2009), which identified that nearly 50% of the teachers teaching mathematics in Ireland at the time were out-of-field teachers. It is feared that a new phase of mathematics textbooks which do not reflect the intentions of Project Maths could impact on teachers' willingness to adopt new teaching strategies and approaches or even their abilities to change given a suspected lack of pedagogical training in some cases. Similarly key content omissions echo the era of the old mathematics curriculum and may unknowingly encourage minor omissions of content as discussed by Remillard (2005).

CONCLUSION

This paper set out to identify the role that textbook analysis can play in supporting the introduction of a new curriculum. It set out the key areas of a textbook report aimed at identifying the correlation between a new mathematics curriculum and its supporting textbooks. This report identified a number of key content and expectation omissions which were central to the ethos of the new mathematics curriculum and hence provided an opportunity for oversights to be addressed and hence textbooks be improved. By engaging in the process of textbook analysis the publishers and authors gained an external review of their material and the NCCA gained an insight into how the focus and expectations of the new mathematics curriculum was developed in the supporting mathematics textbooks. Further to this, the report raised awareness among teachers about the differences between textbooks, the options available to them (as they would now have to purchase new textbooks for the new curriculum) and also some of the key features and expectations of Project Maths were discussed from a different perspective. With regard to the international context, this paper reinforces a method of textbook analysis than enables direct comparison between mathematics curricula and textbooks while explicitly identifying the impact that such can have on improving the supporting materials and ensuring greater cohesion between the message of the curriculum and the message of the textbook.

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CHANGE COMES SLOWLY: USING TEXTBOOK TASKS TO MEASURE CURRICULUM IMPLEMENTATION IN IRELAND.

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Textbooks are an important resource in Irish mathematics classrooms, which can have both a positive and negative impact on teaching and learning. The mathematics curriculum at post-primary level in Ireland was reviewed in 2005. The Project Maths initiative was introduced to reform the curriculum, bringing about changes to what students learn in mathematics, how they learn it and how they are assessed. Publishers have produced new texts in response to the expectations of the revised curriculum and the changed needs of the classroom. This paper presents a framework to consider how tasks found in mathematics textbooks are meeting the objectives of this new curriculum. Sections of textbooks currently being used in Irish classrooms at second level have been analysed using this framework and the results indicate that, while all textbooks incorporate a significant number of these objectives to some extent, key aspects are being neglected.

Keywords: curriculum reform, mathematical tasks, Ireland

INTRODUCTION

Secondary education in Ireland is divided into two parts – a junior cycle and a senior cycle. The former spans three years, building on the education received at primary level while preparing students for the sitting of the Junior Certificate state examination. The senior cycle, spanning two years, follows the junior cycle and culminates with the sitting of the high stakes Leaving Certificate examination where students generally sit exams in seven subjects. Mathematics is compulsory throughout junior cycle and because of university level matriculation requirements, most students continue with Mathematics at senior cycle. The subject is examined at three levels –of the students sitting the Leaving Certificate in 2013, 26%, 63% and 11% took Higher, Ordinary and Foundation level respectively.

Project Maths is an initiative, led by the National Council for Curriculum and Assessment (NCCA), to bring about positive change in the teaching and learning of mathematics at second level in Ireland. It emphasises the development of student problem solving-skills (NCCA, 2012a, p.1). One of the key aims of Project Maths is to encourage students to think about their strategies, to explore and evaluate possible approaches, and so build up a body of knowledge and skills that they can apply in both familiar and unfamiliar situations. The new syllabuses contain five strands in total: 1) Statistics and Probability, 2) Geometry and Trigonometry, 3) Number, 4) Algebra and 5) Functions.

Change was not limited to syllabus content; Project Maths also advocates different learning and teaching practices. Looking at the early experiences of teachers within this reformed curriculum, it has been noted that 'for many teachers there has been a change in their role, teaching practices and methods as they have moved away from teacher led and didactic approaches to more student-centred and active methodologies' (NCCA, 2012a, p.20).

Historically, there has not been a lot of research carried out on the nature of post primary mathematics textbooks in Ireland (Conway and Sloane, 2005, p. 31). However, there is some evidence that textbooks play an important role in Irish classrooms. O'Keeffe and O'Donoghue (2009) found that over 75 per cent of Irish second level mathematics teachers use a textbook on a daily basis. It has also been reported that a lot of the time in the classroom appears to be related to the textbook and very often it is the only resource which students have access to during the lesson aside from the teacher, with most of the problems assigned for classwork and homework coming from the textbook (Project Maths, 2012).

Fan and Kaeley (2000) conducted a study investigating the influence of textbooks on teaching strategies. Their findings show that textbooks can impact not only on the content of teachers' lessons but also how teachers actually teach. They concluded that it would be difficult to reform teachers' teaching methods without corresponding reform of the textbooks being used due to the important role that textbooks play in affecting teaching strategies. Indeed, Valverde, Bianchi, Wolfe, Schmidt and Houang (2002) have suggested that textbooks act as 'mediators between the intentions of the designers of curriculum policy and the teachers that provide instruction in classrooms' (p. 2).

The analysis presented here in this paper will give some insight as to whether the Project Maths curriculum is being implemented as intended through textbook tasks in three different textbook series. It had been noted, before the implementation of Project Maths, that mathematics textbooks in the Irish system promoted 'retention and practice, with little focus on active learning' (O'Keeffe and O'Donoghue, 2009, p.290). Similarly classroom inspections in Ireland, before the introduction of Project Maths, have shown that teaching was highly dependent on the class textbook which had a tendency to reinforce this drill and practice style (NCCA, 2006). The work reported on here will re-examine this phenomenon.

More recent research in Ireland has raised questions as to the suitability of textbooks. In 2011, O'Keeffe and O'Donoghue (2012) conducted a large study of the textbooks published in response to Project Maths that were available at the time (ten in all). The study found that all textbooks analysed fell short of the standard needed to support the Project Maths (intended curriculum) effectively, as outlined in the Project Maths Syllabus documents for junior cycle and senior cycle. Worryingly, while the conclusion of the analysis was that the textbooks display a genuine attempt to match Project Maths expectations, no one textbook series was found to address problem solving satisfactorily. However, it is not possible to identify how well specific curricular goals are met based on the results presented and this makes it difficult to determine how improvements should be made. Davis (2013) examined the prevalence of reasoning-and-proving in the topic of complex numbers in six Irish textbooks and one teaching and learning plan produced for teachers during the introduction of Project Maths. The results from Davis' study suggest that there is a 'misalignment between the six textbook units and the Leaving Certificate syllabus' (p. 54). He found that the textbooks underemphasize the importance of pattern identification, conjecture development, argument construction and the use of technology in reasoning-and-proving. If these textbooks are not revised it will fall to the teacher to supplement this shortfall with suitable activities.

PROJECT MATHS SYLLABUS FRAMEWORK

The development of synthesis and problem solving skills is outlined in every content strand of the syllabus through a list (NCCA, 2012b, p. 20) of seven objectives: 1) explore patterns and formulate conjectures, 2) explain findings, 3) justify conclusions, 4) communicate mathematics verbally and in written form, 5) apply their knowledge and skills to solve problems in familiar and unfamiliar contexts, 6) analyse information presented verbally and translate it into mathematical form, 7) devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

To garner as much information as possible, it seems appropriate to subdivide some of these objectives, especially those which cover multiple situations. In particular with reference to contexts, it is more useful to make the distinction between familiar and unfamiliar. An important separation also has to be made between the very different purposes of devise, select and use. Another framework (Lithner, 2008) classifies tasks in terms of their requirement for imitative (IR) or creative (CR) reasoning. IR is dependent on the use of algorithms while CR makes use of more original thinking. Using Lithner's framework, use and select would be classified as IR but devise would be categorized as CR. An amended set of objectives is outlined below.

1-4 and 6 as given above
5(a) Apply their knowledge and skills to solve problems in familiar contexts.
5(b) Apply their knowledge and skills to solve problems in unfamiliar contexts.
7(a) Devise appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.
7(b) Select appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.
7(c) Use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Figure 1: Amended objectives in the Project Maths (Leaving Certificate) syllabus.

Using the amended list in this way allows one to classify the activity in which the student is engaged when completing tasks. It is possible for a task to address more than one of these objectives and classifying the tasks in this way gives a clear picture of whether tasks are meeting the Project Maths curricular goals in relation to synthesis and problem solving.

METHODOLOGY

The study reported here analyses tasks on the topic of Patterns, Sequences and Series from three Leaving Certificate textbooks series (at both Higher and Ordinary level) on the Irish market: referred to as Text A, Text B and Text C. There are no textbooks for Foundation level available currently. A total of 1838 tasks were analysed, each task was examined very carefully and the Project Maths objectives it addressed were identified using figure 1, taking into account that more than one objective could be in evidence.

After I had coded the textbook tasks, at least one of my two PhD supervisors also looked at each task independently. We subsequently compared our classifications and discussed any differences,, coming to agreement on how the coding should be applied. Having clarified our coding, we reviewed and revised the existing classifications of previous tasks, in order to ensure consistency throughout the analysis.

RESULTS

As it is possible for tasks to address more than one objective using this framework, I will consider the incidence of the different objectives as well as expressing them as a percentage of the total number of tasks. Results are presented in table 1**.**

Table 1: Results of analysis using Project Maths objectives

Project Maths Objectives	Text A (Higher)	Text A (Ordinary)	Text B (Higher)	Text B (Ordinary)	Text C (Higher)	Text C (Ordinary)
No Objective	18(6%)	62 (18%)	49 (25%)	54 (23%)	67 (18%)	135(37%)
1	54 (17%)	57 (16%)	70 (36%)	70 (30%)	5 (1%)	45 (12%)
2	21 (6%)	15 (4%)	2 (1%)	14 (6%)	0	7 (2%)
3	7 (2%)	1 (<1%)	1 (<1%)	3 (1%)	5 (1%)	16 (4%)
4	7 (2%)	8 (2%)	1 (<1%)	0	0	1 (<1%)
5(a)	0	0	16 (8%)	0	0	0
5(b)	36 (11%)	18 (5%)	25 (13%)	1 (<1%)	18 (5%)	12 (3%)
6	17 (5%)	7 (2%)	17 (9%)	0	11 (3%)	14 (4%)
7(a)	21 (6%)	13 (4%)	20(10%)	7 (3%)	3 (1%)	10 (3%)
7(b)	10 (3%)	4 (2%)	3 (2%)	0	0	2 (<1%)
7(c)	233(67%)	225(64%)	133(68%)	176(76%)	284(77%)	170(46%)

Objective 7(c) is encountered very often across all three textbook series. Across the three textbook series, the objectives 7(a) and 7(b) are far less frequent, with 7(b) appearing least often. The second most commonly addressed objective in Textbooks A and B was objective 1. This is not surprising given the content of the chapters (on patterns, sequences and series) analysed. Objective 3 does not feature strongly in the material analysed with the highest incidence at Ordinary Level in textbook C. Objective 4 also appears to have been neglected in the three textbook series.

DISCUSSION AND CONCLUSION

The greatest difference between textbook levels can be observed in textbook series C where nine objectives are encountered at Ordinary Level with just six at Higher Level. Objective 7(c) appears to be dominant at Higher Level and strong at Ordinary Level. There is a greater variety of Project Maths objectives addressed at Ordinary Level when compared to Higher Level, although the incidence for some objectives is very low.

The most common Project Maths objective encountered in all three textbook series is 7(c); its greatest frequency is in textbook series C. Only one of the textbooks¸ the Higher Level Text B, made use of all the Project Maths objectives listed. There is also a strong incidence across all three textbook series of exercises not matching any of the Project Maths objectives (over 36% in one textbook). Objectives 3 and 4, in particular, are rarely addressed. It would appear that the textbooks have yet to fully embrace the goals of Project Maths and there is still a tendency for exercises to emphasise the practice of skills and algorithms rather than asking students to devise new techniques or apply their knowledge in unfamiliar contexts. This supports Davis' (2013) view that there is a misalignment between these textbooks and the Project Maths syllabus. However, the analysis reported here looks at all of the Project Maths Objectives while Davis' study focused exclusively on the area of reasoning-and-proving. In Davis' study, he found that there was little opportunity for students to engage in reasoning-and-proving and very little evidence of tasks requiring the explanation of findings or the justification of conclusions.

Fan and Kaeley (2000) have noted the important role that textbooks play in affecting teaching strategies. It would appear that the Project Maths curriculum will not be realised effectively until further development takes place in these textbooks in light of the deficiencies outlined above. The NCCA (2012a) has reported that teachers are beginning to employ different methodologies to better promote the development of mathematical skills, yet it appears that the textbook tasks analysed here do not support this initiative sufficiently. Houang and Schmidt (1998) have cautioned that textbooks can have varying interpretations of curricular intentions. The findings of my analysis would support the views expressed by O'Keeffe and O'Donoghue (2011) that the textbooks in their study did not meet the required standard to support the revised curriculum. There has been some attempt to meet the Project Maths objectives in the textbooks considered here but my results indicate that more attention needs to be given to the design of tasks that meet the goals of Project Maths. If not, the more traditional mathematics classroom where the emphasis is on the development of procedural skills rather than applying mathematics in real-life contexts or considering properties of mathematical concepts and how they interconnect will persist (NCCA, 2006, p.7).

The Project Maths curriculum designers (NCCA, 2012b, p.12) want students to build up a body of knowledge and appropriate skills that they can apply in both familiar and unfamiliar situations. However, my findings suggest that the current textbooks being used in Irish classrooms do not facilitate this.

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MATHEMATICS TEXTBOOK DEVELOPMENT AND LEARNING UNDER DIFFICULT CIRCUMSTANCES IN SCHOOLS IN NIGERIA

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The nine-year Basic Education Curriculum in Nigeria has paved way for recent innovations in the educational sector. This has included but not limited to revision and new inclusions of curriculum resources to make sure learners benefit and develop their potentials maximally from the free, universal, basic and compulsory education system for all children of school going age. Textbook development a crucial aspect of the process, has witnessed proliferation in the country. This study examined how students learning under difficult circumstances are considered in textbook development especially in the learning of mathematics in schools in Nigeria. A survey research involving stratified random sampling technique comprising of 3600 students was carried out in the six (6) geo-political zones of the country. Each stratum was made up of 50 students from 4 different schools in the most volatile villages in 3 states of each geo-political zone in the country. Quantitative and qualitative analysis of results revealed that the needs and interests of students learning under difficult circumstances were inadequately considered in mathematics textbook development. The paper concludes by encouraging a shift in paradigm for textbook developers from the ideal classroom environment to incorporate extreme learning situations. Pedagogical implications as regards students learning of mathematics were put forth. The paper also suggested that contemporary national issues should serve as context for solving mathematics problems while developing textbooks in general.

Keywords: textbook development, difficult circumstances, Nigeria

INTRODUCTION

The system of education in Nigeria involves four stages known as 6-3-3-4.This system comprises of a six (6) years of primary education, three (3) years of junior secondary education, three (3) years of senior secondary education, and four (4) years of tertiary education. Any graduate from the second stage of education (i.e junior secondary level) is expected to live a useful living in the society by being self-reliant. Unfortunately, the reverse has been the case. Hence, Government at all levels decided to provide free and compulsory Universal Basic Education (UBE) since 1999. This Basic Education programme consists of the 6 years of primary education and 3 years of junior secondary education. This 9-year basic education curriculum exposes children of school going age to free, universal and compulsory basic education from primary 1 to junior secondary 3. The Nigerian Educational Research and Developmental Council (NERDC) is that arm of government in Nigeria responsible for curriculum development and provision of material resources to schools. HKEAA (2007) asserts that textbooks, amongst other things, provide learning activities and task-oriented problems at the students' level of understanding to engage them in exploratory work and encourage high level thinking.

Textbooks are mainly used as teaching and learning resources for instruction in schools. Textbooks are innovative measures in making mathematics simple, real and consolidate learning of knowledge. Since mathematics is a compulsory subject in schools, students are expected to have access to this material for consolidating knowledge. NERDC in maintaining quality assurance carries out quarterly assessment of textbooks, where recommendations are forwarded to the states and other stakeholders through federal ministry of education. The revision and production of curricular materials used in the basic education programme gave rise to series of materials for teachers, students alike in the school system. Due to the importance of learning mathematics in nation building, mathematics is treated as a core and compulsory subject in schools.

Ale (2007), in his report after re-appointment as the director of the National Mathematical Centre, mentioned that, in enhancing the teaching of mathematics in Nigeria, the Centre provided teaching modules, textbooks, mathematical games and kits for the universal basic education programme through his policy direction and research simulation.

LEARNING UNDER DIFFICULT CIRCUMSTANCES

Learning mathematics in schools over the years has been characterised with poor performance. But close looks at the problems students undergo suffice to say that these students are actually learning mathematics under difficult circumstances. As a result of the economic situation in many homes some students engage in the following:

- Working while schooling (ie working before and/or after school hours)
- Missing school deliberately to work and meet up with payment of bills
- Become absent from school on market days that fall during school days
- Illegal business leading to mayhem in the community
- Frequent communal clashes over sharing of farmlands

Such students that belong to the category mentioned above among others will not only have poor performance in schools but have difficulties in learning generally. It is unfortunate that the school system does not have intervening measures of dealing with these students. One wonders if such students have access to textbooks. These categories of students are challenged by the environment which learning takes place especially in mathematics. In choosing textbooks, schools need to take into account the mathematical abilities, needs and interests of their students, as well as the quality of the books recommended for use. It therefore becomes necessary to question whether these students interest were taken into consideration prior to the development of these textbooks. Since, it is necessary that students practice what they learn or read in textbooks. Sutcliffe (2010) asserts that many teachers of mathematics are currently doing a truly outstanding job, sometimes under extremely difficult circumstances, of preparing their students for the world of work or advanced studies beyond school. Twenty-first century skills are beyond computer literacy, students' need not only need to be educated but the education should be used to liberate them as well as to show good examples in the society. Textbook related background information might not be meaningful to these students due to the differences in their learning experiences and those in other learning environment. Again, Ogunkunle and Gbamanja (2007) suggested the teaching of mathematics in alternative learning environments so that mathematics could be more meaningful and transferable to other contexts.

The research questions guiding this study include:

- Which mathematics textbook is mostly used for junior secondary schools in the states?
- How often is mathematics textbook used by students learning under difficult circumstances in junior secondary schools?
- How does mathematics textbook development affect students' learning under difficult circumstances in schools in Nigeria?

The null hypothesis tested for this study was:

- There is no significant difference in the perception of rural and urban students learning under difficult circumstances about textbook development in schools in Nigeria

METHODOLOGY

A descriptive survey research using multi-stage sampling technique involving 3600 students was carried out in the six (6) geo-political zones of the country. Stratified sampling comprised of 50 students from 4 different schools in the most volatile villages in 3 states of the 6 geo-political zones in the country. The students were able to respond to statements on the mathematics textbooks used in schools for the 9-year basic education curriculum in Nigeria. A questionnaire titled Mathematics Textbook Development and Students' Learning under Difficult Circumstances was developed to elicit data for the study. Face and content validity was carried out by prominent mathematics educators, while Cronbach Alpha coefficient (0.72) was computed to test the internal consistency of items.

RESULTS

Which mathematics textbook is mostly used in junior secondary schools? Table 1 reveals that most students do not have access to textbooks.

Table 1: Mathematics textbooks used frequently in schools

s/no	Textbooks	State1 (%)	State2 (%)	State3 (%)	Total (%)
1	Effective Mathematics	120(10)	200(17)	300(25)	620(17)
2	Essential Mathematics	80(7)	150(13)	40(3)	270(8)
3	MAN Mathematics	98(8)	72(6)	103(9)	273(8)
4	Melrose Functional Mathematics	-	-	10(1)	10(0)
5	Nelson Functional Mathematics	-	-	-	-
6	New General Mathematics	250(21)	320(27)	400(33)	970(27)
7	STAN Mathematics	94(8)	206(17)	452(38)	752(21)

As evident in Table 1, about 27% of the entire respondents have New General Mathematics textbook, which is used in most West African countries. Science Teachers' Association of Nigeria (STAN) Mathematics textbook is next popular textbook amongst the students.

How often is mathematics textbook used by students learning under difficult circumstances in junior secondary schools?

Table 2 shows that students learning under difficult circumstances hardly use their textbooks in studying mathematics. They rarely do their homework (31%), but 36% of the students study their textbooks for examination purposes. This could be attributed to the manner in which the textbooks are developed.

Table 2: Frequency of mathematics textbook used by students

s/no	Name of Textbooks	For Home work	For Exams	Once in a while	Not at all
1	Effective Mathematics	5%	8%	-	-
2	Essential Mathematics	16%	22%	-	-
3	MAN Mathematics	32%	11%	27%	22%
4	Melrose Functional Mathematics	-	-	-	-
5	Nelson Functional Mathematics	-	-	-	-
6	New General Mathematics	31%	36%	15%	78%
7	STAN Mathematics	12%	31%	24%	55%

How does mathematics textbook development affect students' learning under difficult circumstances in schools in Nigeria? Ho: There is no significant difference in the perception of rural and urban students learning under difficult circumstances about textbook development in schools in Nigeria.

Table 3 shows that there is no significance difference in the perception of rural and urban schools regarding their perception on textbook development for schools. The null hypothesis was therefore retained.

Table 3: Z-test showing perception of rural and urban students about textbook development

	n	mean	Std Dev	df	Z-Cal	Z-critical	Remark
Urban	2000	26.94	5.925	3598	0.92	1.96	NS*
Rural	1600	25.44	6.150				

*NS= Not Significant at .05 sig level

CONCLUSION

The study revealed that most students do not have access to mathematics textbooks. Less than 30% of the students involved in the study have access to New General Mathematics textbook, which is used in most West African countries. Students learning under difficult circumstances hardly use their textbooks in studying mathematics. There is no significance difference in the perception of rural and urban schools regarding their perception on textbook development for schools.

The outcome of this study suggests that textbook development has not incorporated all categories of students. Thus students studying under difficult circumstances have not seen why they should be part of such textbooks which they have limited or no time to study. Hence textbook developers need “a shift in paradigm” from “the ideal classroom environment” towards incorporating “extreme learning situations”, especially for those students having difficulty in accessing good education. It is also recommended that contemporary national issues should serve as context for solving mathematics problems while developing textbooks in general.

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CONCEPT OF PROBABILITY: DISCURSIVE ANALYSIS OF JAPANESE SECONDARY SCHOOL TEXTBOOKS

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I report here a part of findings on my curriculum research about probability. The purpose of this paper is to clarify some differences between lower secondary school natures of the concept of probability and upper ones in Japan. For this, Japanese secondary school textbooks are analysed from the discursive framework in the commognitive theory. The results of analysis show two differences, which are not simple like the simple distinction of 'frequentistic vs. classical,' but more complex. The first difference exists between general natures of the concept of probability in the lower secondary textbook and in the upper one: the lower level nature is a hybridity of frequentistic and classical nature, while the upper level nature is a purity of classical nature. The second difference exists between their classical natures in the lower secondary textbook and in the upper one: a lower secondary school classical probability is different from an upper one in terms of their roles and realizations. These findings suggest that the number of natures of the concept of probability 'to be taught' around the world is more than what we have considered so far.

Key words: probability, mathematical concepts, commognitive theory, discourse analysis, Japan

INTRODUCTION

In Japanese school mathematics curriculum, the word of *probability* is introduced at lower secondary schools and developed at upper secondary schools. The probability in the curriculum are usually understood by Japanese mathematics educators as following: on one hand probability in lower secondary school is founded on the frequentistic perspective which regards probability as convergent values of relative frequency, but on the other hand probability in upper secondary school is based on the classical viewpoint which considers probability as ratios about equally likely events, and these philosophical terms are not taught for students. However, taking a closer look at the curriculum, we can observe ways of thinking on the classical probability such as 'equally likely' in lower secondary school probabilistic contents that are commonly assessed as the frequentistic. Similarly, several upper secondary school teachers, who should teach the classical probability, deal with ways of thinking on the frequentistic probability through experiments. From these facts, I think that *didactical* taxonomy, not philosophical, on natures of the concept of probability needs to describe, explain and improve curriculum, teaching and learning of probability. And, this taxonomy may be constructed and elaborated by curricula analysis on different probabilistic educations in various places (e.g., cultural sphere, country, educational level, and school) and times (e.g., present time, the modernization era, the reformation era, and future). In this paper, I grapple with this task in a case of Japanese secondary education at the present time.

THEORETICAL FRAMEWORK

In order to clarify different natures of the concept of probability in Japanese school mathematics curriculum, I use the commognitive theory on discourses (e.g., Sfard, 2008). Based on this theory, curricula are regarded as discourses in language (Newton, 2012), which are distinguishable by their vocabularies, visual mediators, endorsed narratives, and routines. *Vocabularies* are word uses in discourses. For example, in probabilistic discourses these are *probability*, *equally likely*, *trial* and more. Note that mathematical objects are defined as ordered pairs of mathematical signifiers and its realizations in commognitive theory. For emphasizing that a series of word (or symbol) is not a realization (or an object) but a signifier, I append '*S*[]' to the word such as *S*[probability], *S*[equally likely], and *S*[trial]. *Visual mediators* are visible realizations of objects of discourses. For example, in probabilistic discourses these are dies, tree diagrams, 1/2 as a signifier, and more. For emphasizing that a series of word or symbol is not a signifier (or an object) but a realization which is signified by the signifier, I append '*R*[]' to the word or symbol such as *R*[dies], *R*[tree diagrams], *R*[*S*[1/2]]. *Endorsed narratives* in a discourse are sequences of utterances that are regarded as true by participants of the discourse. For example, in probabilistic discourses these are *S*[1/2+1/2=1/4], *R*[definition of probability], and more. *Routines* are sets of meta-discursive rules that are not rules on behavior of objects of discourses but of participants of discourses such as *R*[rules of trails] and *R*[rules of computation] in case of probabilistic discourses.

METHODOLOGY

I analyse Japanese school mathematics textbooks for clarifying different natures of the concept of probability in this paper. The term of *concept* in the commognitive theory is defined as signifiers together with its discursive uses. From this definition, I define *natures of the concept of probability* as discursive uses of *S*[probability]. I think that identifying a textbook's discursive use of *S*[probability] is answering following questions. What kinds of objects around *S*[probability] can we find in the textbook? And what kinds of rules about *S*[probability] can we find in the textbook? The former question can be answered by describing vocabularies, visual mediators, and narratives because it is about kinds of signifiers and realizations used in probabilistic discourses. The later question can be answered by describing routines because it is about kinds of meta-discursive rules worked in probabilistic discourses. Thus, I define discursive uses of *S*[probability] as vocabularies, visual mediators, endorsed narratives, and routines in relation to it. Based on above-mentioned consideration, I can identify different natures of the concept of probability by describe discourses in which *S*[probability] is used.

In this paper, two Japanese mathematics textbooks are analysed. The one is *Mirai e hirogaru suugaku 2* (*MHS*; Okamoto et al., 2012), which is one of the most popular lower secondary textbooks. The other is *Koutougakkou suugaku A* (*KS*; Okabe et al., 2012), which is one of the most popular upper secondary textbooks. I analyse probabilistic contents, *Kakuritsu*, that is Japanese translation of *probability*, on these textbooks from their vocabularies, visual mediators, endorsed narratives, and routines in accordance with following methods:

- *Vocabularies* are described by gathering emphasized words in the textbooks. For emphasizing keywords, these textbooks change locally their fonts from Mintyou style to Gothic style. Thus, I pick up generally Gothic style words. However, I ignore these words in chapter titles and other headlines for the sake of brevity.
- *Visual mediators* are described by gathering figures and tables in the textbooks. I count them which emergent more than two times because I would like to exclude unimportant them from this analysis.
- *Endorsed narratives* are regarded as definitions of probability in the textbooks, because several communicational conflicts within discourses about probability stem from differences between definitions of probability (cf. Gillies, 2000).
- *Routines* focused in this analysis are *main* procedures on problem solving because Japanese school mathematics textbooks developers adopt problem solving learning as a fundamental textbooks design principle. The procedures appear on problems in textbooks only implicitly. So, We must prepare some method in order to find explicitly them. Here, I get help of a following characteristic of Japanese probabilistic curriculum: it deals with the frequentistic probability and the classical one. We can identify easily problems on the classical probability whether the problems assume that events are *equally likely*. Thus, in case of Japanese curriculum, we can identify problems on the frequentistic probability by remarking that problems are *not* related to 'equally likely.'

RESULTS

Results of analysis on *MHS* and *KS* are presented in table 1 and table 2. The elements of these tables are displayed generally in accordance with first emergence orders. Here, I must explain my identifications of endorsed narratives and routines in the tables, because they are not only picked up but also interpreted by analysts while vocabularies and visual mediators are only selected systematically. In *MHS*, probability is defined as following:

> Number of representing an expected degree of an event occurring is called *probability* of the event occurring. (*MHS*, p. 138, my translation)

In the textbook, this definition is related to a convergence of relative frequency on two coins throwing trials by means of some visual mediators which are *R*[figure of concrete objects] and *R*[table of frequency] in the table 1. In addition, it does not assume that the event is equally likely. Based on these facts, I identify an endorsed narrative in *MHS* as a *frequentistic definition of probability*. On the contrary, the *KS*'s probability is defined as a ratio about equally likely events. So, I identify an endorsed narrative in *KS* as a *classical definition of probability*.

MHS has two routines in problem solving procedures on probability. The one is a *many trials experiment*, which is related to the frequentistic probability. The other is a *combinatorial computation*, which is related to the classical probability. There are these two routines in *MHS*, but the many trials experiment routine is almost never used to solve problems. It is no exaggeration to say that this routine only help for defining probability as the frequentistic. Most problems are solved by combinatorial computation. Thus, I identify a routine of solving problems on probability in *MHS* as the *combinatorial computation on the classical* *probability*. The fact that the last two vocabularies in Table 1, that is *S*[equally likely] and *S*[tree diagram], are on the classical probability and combinatorial computations is may be related to this characteristics of the routine. In contrast, a *KS*'s main routine can be found easily because a following comment is described on an opening part of probabilistic contents in the textbook:

> As following, in all trials in this textbook, *all elementary events* in a whole event *U are equally likely*. (*KS*, p. 32, my translation)

Based on this statement and on all the vocabularies, visual mediators, and endorsed narrative in the Table 2, I also identify a routine of solving problems on probability in *KS* as a *combinatorial computation on the classical probability*.

Table 1. Results of discursive analysis on probabilistic contents in MHS

Discursive indicators	Results of analysis
Vocabulary	*S*[probability], *S*[equally likely], *S*[tree diagram] (my translations)
Visual mediator	*R*[figure of concrete objects], *R*[table of frequency], *R*[tree diagram], *R*[table of direct products]
Endorsed narrative	*R*[frequentistic definition of probability]
Routine	*R*[combinatorial computation on the classical probability]

Table 2. Results of discursive analysis on probabilistic contents in *KS*

Discursive indicators	Results of analysis
Vocabulary	*S*[trial], *S*[event], *S*[whole event], *S*[elementary event], *S*[equally likely], *S*[probability], *S*[sum event], *S*[product event], *S*[exclusive], *S*[exclusive events], *S*[empty event], *S*[addition law], *S*[complement event], *S*[independence], *S*[repeated trial], *S*[conditional probability], *S*[multiplication law], *S*[probability of cause] (my translations)
Visual mediator	*R*[figure of concrete objects], *R*[figure of sample space], *R*[table of direct products]
Endorsed narrative	*R*[classical definition of probability]
Routine	*R*[combinatorial calculation on the classical probability]

From these results of analysis, we can point out that a nature of the concept of probability in Japanese lower secondary school mathematics textbook *MHS* is a *hybridity* which includes not only components about the frequentistic probability but also about the classical probability, while a nature of the concept of probability in Japanese upper secondary school mathematics textbook *KS* is a *purity* which only includes components about the classical probability. However, a more interesting finding is that the lower secondary school's classical nature of the concept of probability is different from the upper secondary school's one. The former one does not have defining role but has a problem-solving one, while later one has both roles. Also the former one is realized by *R*[tree diagram] but not by *R*[figure of sample space], while the case of later one is opposite. These facts suggest that the words of *classical probability* play plural roles in teachers' or researchers' discourses in mathematics education. In other words, it seems that we have called different *didactical* natures of the concept of probability the same name.

CLOSING REMARKS

In this paper, I reported two differences between lower secondary school's natures of the concept of probability and upper ones in Japan at the present day. This work must be saw as a first step of studies about different natures of the concept of probability from perspectives of mathematics education. I can list two future tasks at least: *synchronically comparative studies in global contexts* and *diachronically comparative studies in local contexts*. The former example is an *inter*-nationally comparative study and the later example is an *intra*-nationally historical study. In addition, we may need to extent subjects of research from natures of the concept of probability *to be taught* like textbooks to *actually taught* them like classroom lessons and to *actually learned* them like students' performances.

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JOHN DEWEY IN MATHEMATICS TEXTBOOKS: END OF THE 19TH CENTURY AND THE EARLY 20TH CENTURY

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The following work is focused on a discussion about the contributions of John Dewey to mathematics education. It is based on the articulation of Dewey's ideas in different mathematics books/textbooks, between the end of the 19th century and the early 20th century. The purpose is to characterize the referred books and the differences among them, and to identify how Dewey's ideas – in special the notion of experience – are mobilized. More specifically, the present study is part of a doctoral research project that investigates the appropriations of John Dewey's and Edward Thorndike's ideas in the mathematics education field in Brazil between the 1930s and the 1960s. The discussion held in the present paper is based on the work of such authors as Roger Chartier and Michel de Certeau, regarding the social places of texts production, and concepts as appropriation and strategy. Some of the sources adopted were Dewey's books, such as The Psychology of Number (McLellan & Dewey, 1895); the series of arithmetic books written by James A. McLellan and Albert F. Ames, published between the 1890s and the 1900s; and the series of Georgia Alexander, published in the 1920s (both series had the involvement of Dewey). Based on this study it is possible to highlight the changes in mathematics teaching approaches in the analysed books as it relates to the changes in Dewey's own ideas. It is also possible to chart the evolution of Dewey's thinking from the general concepts conveyed in The Psychology of Number to a more focused attention on examples of daily life and references to cultural aspects regarding American context or other countries in Georgia Alexander's series.

Keywords: John Dewey, psychology of number, history of mathematics education, Alexander-Dewey arithmetic

INTRODUCTION

Considered one of the most important American philosophers, and remembered by works such as *How we think* (1910), *Democracy and Education* (1916) and *Experience and Education* (1938), the ideas and work of John Dewey (1859-1952) circulated in different countries, as it is possible to verify in works such as Schriewer and Bruno-Jofré (2012) and Popkewitz (2008). Widely studied, there are many researches and books published about Dewey. Nonetheless, there are approaches that have only been explored at a shallow level, like the contributions of Dewey in the field of mathematics education and how his ideas have circulated and been appropriated in the field, for example.

The following work focuses specifically on a book on mathematics teaching and two series of mathematics textbooks, published between the end of 19th century and the early 20th century. Such books had the (in)direct participation of Dewey. The discussion presented in this paper aimed to identify Dewey's participation in the mentioned books, characterizing and establishing a parallel among them.

The discussion is based in such authors as Roger Chartier and Michel de Certeau, concerning the social place of texts production, appropriation of ideas and strategy. The sources were the book *The Psychology of Number* (McLellan & Dewey, 1895); the series of arithmetic textbooks written by James A. McLellan and Albert F. Ames, published between the 1890s and the 1900s; and the series of Georgia Alexander, published in the 1920s. Due to the page limit of this paper, I will focus on a general characterization of the books and the notion of experience.

This work is part of the partial results of the doctoral research (FAPESP Process # 2012/11361-1), which investigates the appropriations of John Dewey's and Edward Lee Thorndike's ideas in mathematics education in Brazil between the 1930s and the 1960s.

VESTIGES OF DEWEY'S INVOLVEMENT WITH MATHEMATICS TEACHING

The book *The psychology of number and its applications to methods of teaching arithmetic* (referred as TPN in this paper) is perhaps the most emblematic example of Dewey's involvement with mathematics teaching. Published in 1895 in the USA by the D. Appleton and Company, the book was co-authored by Dewey in collaboration with James Alexander McLellan. What would have lead Dewey to get involved in such a project? Would it be an isolated project or did he have other works focused on mathematics teaching?

When the book TPN was first published, Dewey was Head Professor of Philosophy at the University of Chicago, and McLellan was Principal of the Ontario School of Pedagogy, Toronto. The relation between Dewey and McLellan dates back the writing of TPN, probably in the 1880s, when McLellan invited Dewey to write the introduction of one of his books. Based on correspondence of Dewey (Hickman, 1992) that mentions TPN, it is possible to conclude that McLellan gave the first step in order to write such a book. McLellan had already published mathematics textbooks in Canada. McLellan and Dewey each contributed to the book, though it is not clear how they divided the work. Dewey says in a letter sent to his wife Alice in 1894 he would be responsible for the psychology portion and McLellan for the methods.

Throughout his life, Dewey was involved in many different issues, and reflected this in his works, which cover a wide variety of subjects. If in his first works the interest in discussions about psychological aspects were visible, it spread to education, the arts, and social and political issues. Mathematics is one more of the aspects and can be explained perhaps by the logic branch in the philosophy field, which Dewey kept interest in since his first works. As idealism and Hegelian philosophy played an important part in Dewey's first phase, it is important to point out that logic figured as one of the topics discussed by Hegel. In addition, in the first school Dewey worked, he taught algebra for two years.

THE BOOK "THE PSYCHOLOGY OF NUMBER"

Published originally in the USA, TPN was volume 33 of *International Education Series*. It was translated in at least two countries: Japan in 1902, and Turkey in 1928. There are also vestiges of its circulation in other countries. Divided in 16 chapters and 2 prefaces – the first written by the editor William Torrey Harris and the second by the authors – the first chapters focus on the discussion of the importance of psychology in education, the psychological method and the concept of number. The other chapters focus on mathematical concepts – such as addition, subtraction, decimals – comparing what they call the psychological method with other methods.

Through the chapters, concepts such as value, ethics and experience are discussed. For example, in the first paragraph of chapter one, the authors discuss the value of facts and theories related to human activities, that the value is determined by practical applications in order to accomplish certain purposes, and such discussion about value is applied to the relation between psychology and education (McLellan & Dewey, 1895, p.02). They also criticize the study through facts, and methods that emphasize memorization, as the Grube's method.

The authors talk about the process of reflection that involves abstraction and generalization. Great focus is given to the importance of experience, not any kind of experience, but one made of a certain kind of practice based on rational principles. The role of psychology would be to turn into rational the experience.

Concerning the origin of number, they state that the number has a psychical nature; therefore, it is a rational process and is not just mere perception. The objects and their perceptions are not numbers, as the perception of multiplicity of things do not imply the perception of number. The consciousness of number consists in the capacity to quantify, count, measure. "Number is not a property of the objects which can be realized through the mere use of the senses, or impressed upon the mind by so-called external energies or attributes" (McLellan & Dewey, 1895, p.23-24).

Dewey explained to his wife Alice, in a letter written in 1894, the he was "trying to turn the Hegelian logic of quantity over into psychology & then turn that over into [a] method for teaching arithmetic" in TPN (fragment of letter reproduced by Martin, 2002, p.191).

THE SERIES OF MCLELLAN AND AMES

The series of McLellan and Ames consists of four books published by Macmillan Company: *The public school arithmetic* (1897); *The primary public school arithmetic* (1898); *The public school mental arithmetic* (1899); *The public school arithmetic for grammar grades* (1902). The inside cover announces the books are based on *The Psychology of Number*, which differentiates the series from other arithmetic textbooks, according to the authors. The following statement emphasizes the importance of TPN: "Teachers are recommended to study with care Dewey and McLellan's 'Psychology of Number', and the 'Public School Arithmetic', which illustrates so many points in the 'Psychology of Number'" (McLellan & Ames, 1898, p. viii).

It follows that, even not being a series written by Dewey, it is the result of the appropriation of one of his works. Besides, the prefaces of the books mention the name and insert pieces of Dewey's speech to reiterate the importance of the series, what works as a kind of strategy to promote the referred textbooks. It is important to note the first author of the series, McLellan, is the same of TPN.

In a general way, the four books have an approach that tends more to deduction than induction, although the prefaces and suggestions to teachers mention neither. For example, it is common for lessons to start with definitions followed by different problems. The prefaces highlight the importance of TPN in the building of the lessons, and the adoption of key notions as the idea of unity, group, count and measure, that lead to the development of the concept of number.

Concerning the suggestions to teachers, the only teachers' edition found is *The primary public school arithmetic*. There are three sections for the teacher at the beginning of the book: *Suggestions to Teachers*, which outlines suggestions regarding the first lesson (17 pages at total); *Suggestive Lessons*, with possible dialogues between teachers and students for all lessons (36 pages); *Suggestions*, with extra proposals (20 pages).

In the section *Suggestions to Teachers* the authors mention the notion of experience, emphasizing the importance of connecting "the practical work with the child's own experience as closely as possible."

THE SERIES OF GEORGIA ALEXANDER

The Alexander's series, entitled *The Alexander-Dewey arithmetic*, consists of three books (elementary, intermediate and advanced), published in 1921 by Longmans, Green and Company. Besides being editor, Dewey lends his name to the series title, in what can be considered a strategy to validate and promote the series. The author, Georgia Alexander, highlights in the preface – the same in the three books – the importance of Dewey in the elaboration of the material, stating that he reviewed and helped with suggestions.

Based on the data available on the inside cover of the books it is possible to say Georgia Alexander was District Superintendent of Indianapolis Schools. Concerning the relation with Dewey, the correspondence of Dewey (Hickman, 1992) points to a friendship between the families that dates back the publishing of the series.

A detail in the preface that stands out is the expression "schools of a democracy", in which mathematics teaching should focus on the preparation for business, science and industry; skill in mathematical computation; and civic responsibility promoting the welfare of the community. The author also highlights: "The subject matter is of contemporary interest which brings into the otherwise isolated school-room the great world where mathematics are found in every basic activity" (Alexander, 1921, p. iii).

The table of contents of the three books brings a piece of information in the head of the table stating the book is based on induction. The analysed copies conclude with suggestions to teachers. Through the lessons there are brief directions and hints, probably the copies are teachers' editions, although the inside cover does not inform. The books of the series have a similar structure, but the following comments are related to *The Alexander-Dewey arithmetic: elementary book.*

The lessons start with a brief story or dialogue introducing the concepts that will be studied. The situations presented are related to activities of daily life, probably common in students' family life or school contexts, for example: going to the grocery, farm activities, and national holidays. There are also stories that make reference to other countries, such as Japan.

The stories and problems in general place the child as the actor of the actions, and many illustrations in the book even picture children in the scenery. The active participation required of pupils in the activities of the book and the accessible language mirroring their reality are aspects that stand out.

The section *Suggestions to Teachers*, distributed in five pages, begins detaching the importance of the preface in order to elucidate "principles underlying the method." Besides, it suggests the use of dramatization, exploring different answers of students and elaborating problems based on students' experience. Another aspect is the interaction of pupils, advising the teacher to teach them "to help one another without telling the answer" (p. xi).

Regarding experience, the author does not refer to it explicitly, but it is visibly mobilized in the activities of the lessons, and in preface the following fragment points to elements common in the notion of experience: "The arithmetical ideas gained through this social introduction are made automatic through scientific practice which later culminates in their application to new concrete situations".

SOME REMARKS

It is possible to point to some differences between the two arithmetic textbook series. In the series of McLellan and Ames, some principles discussed in TPN, as the notions of unity and multiplicity, the whole and the part, counting and measuring, are present. Nevertheless, there are other elements to be analysed yet.

The language in the Alexander series is more accessible to students, making use of brief stories and examples that stimulate the participation. The series of McLellan and Ames has an approach more deductive, while Alexander's is assuredly inductive. The introduction of the content is also different: while in McLellan's series the lessons begin in general with definitions, Alexander's begin with situations related to daily life that imply the participation of students.

The notion of experience is present, in different degrees, in TPN and the two series of textbooks. That is a key concept in Dewey's work, and in the analysed books, there is a convergence to the idea that not every experience is educational, but the ones that make possible other experiences. These elements agree with another emblematic Dewey book, *Experience and Education*, published in 1938.

The use of Dewey's name in the two series is another aspect that stands out. Although Dewey contributed in the two series – as a reference in McLellan's series and with suggestions in Alexander's series - his name is also mentioned in the prefaces and even in the title as a strategy to promote the series.

There are still other elements to be analysed on the books explored in this paper, including notions discussed by Dewey such as value, ethic, democracy, the number concept, and how they are mobilized in the mathematics textbooks.

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A CROSS-CULTURAL ANALYSIS OF THE VOICE OF CURRICULUM MATERIALS[1]

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This paper presents a cross-cultural analysis of how authors of elementary mathematics curriculum materials communicate with teachers and what they communicate about, focusing on six teacher's guides from three distinct school systems, Flanders, U.S. and Sweden. Findings revealed distinct differences between approaches common to each cultural context that relate to different educational traditions. These findings point to differing assumptions about the knowledge needed by teachers to enact instruction. Further research is needed to explore these patterns qualitatively and consider teachers' use of these materials when planning and enacting instruction.

Keywords: textbook analysis, teacher guides, cross-cultural study, Flanders, USA, Sweden

INTRODUCTION

Mathematics curriculum materials and textbooks are used by elementary teachers around the world. They are commonly viewed as a primary tool for teachers' instructional design and as "the links between the ideas presented in the intended curriculum and the very different world of the classroom" (Valverde et al., 2002, p. 55). Designed for use by teachers, these materials represent assumptions about what mathematics instruction should look like and how teachers might be supported to enact instructional designs. As such, they stand as cultural artifacts (Pepin, Gueudet, Trouche, 2013). Cross-cultural analyses of curriculum materials can uncover cultural similarities and provide insight into differences.

This paper presents a cross-cultural analysis of curriculum materials, focusing on the teacher's guide, in three distinct school systems: the United States; Flanders, the Dutch speaking part of Belgium; and Sweden. The focus of our analysis was on the *voice* of the text, defined as the ways curriculum authors communicate with teachers and what they communicate about (Remillard, 2005). Our analysis focused on what different approaches to communicating with teachers reveals about: a) how curriculum materials support teachers; b) assumptions about what teachers need to know to enact instruction; and c) differences in cultural traditions and educational practices. We also wondered about patterns that cut across cultural boundaries and their implications for future research.

BACKGROUND AND FRAMEWORK

Our analysis rests an adaptive view of curriculum use, which holds that teachers actively interpret and construct curriculum in the classroom. (Remillard, 2005). This perspective raises questions about the type of guidance curriculum materials might provide. Ball and Cohen (1996) argue that, rather than simply scripting instruction, "curriculum materials could contribute to professional practices if they were created with closer attention to processes of *curriculum enactment*" (p. 7). Building on this idea, Davis and Kajcik (2005) propose that curriculum designed to be *educative* for teachers in this way might help teachers a) attend to student thinking, b) engage the content and make connections within the discipline, c) understand curriculum designers' rationale for pedagogical choices, and d) mobilize curricular materials within a specific classroom context.

METHODS

We analysed a sample of lessons from teacher's guides from 6 curriculum programs, 2 distinct programs selected from each cultural context. In order to examine how the authors communicate with the teacher, we coded each unit (sentence, phrase, figure) in a sample of 72 lessons, using a coding scheme designed to study how curriculum materials support teachers. The 72 lessons were evenly distributed among grades 3-5 and 2 programs for each country. Table 1 provides a brief overview of the coding scheme, adopted from the ICUBiT study in the U.S. and based on Davis and Krajcik's (2005) design principles. Codes 2-4 and D are viewed as educative. See online version of paper for details about the 6 programs.

Table 1: Overview of Coding Scheme

Code	Short Title	Abbreviated Description
0	Providing Referential Information	Information about the lesson without simultaneously accomplishing aims specified in other categories
1	Directing Actions	Indicates what teachers and students should do or say
2	Design Transparency	Communicates author's intent behind design decisions
3	Anticipating Student Thinking	Indicates intended student understanding or likely misconceptions and how to respond
4	Explaining Mathematical Ideas	Describes key mathematical concepts, relationships, definitions, or properties or their importance
D	Decision Making	Indicates that teacher should make a decision

Results of coding analysis were compiled and are presented in Table 2. Within and across country comparisons are discussed in the four sections that follow.

HOW THE U.S. CURRICULA COMMUNICATE WITH TEACHERS

Development and programme philosophy

The two U.S. curriculum programs, *Everyday Mathematics (EM)* and *Math in Focus (MiF),* represent two different instructional traditions. *EM* was developed with NSF-funding to reflect the vision presented in the NCTM *Standards*. The aim is to build conceptual understanding by gradually building on students' informal knowledge. *MiF* was adapted from one of the mathematics programs developed and used in Singapore. *MiF* also emphasizes conceptual understanding, but takes a more directive pedagogical approach.

Table 2: Variations in Types of Guidance for Teachers across Six Curriculum Programs

Percent of total number of units devoted to…

	Mean units per lesson	Referential Information	Directing Action only	Dir. action & Ed. Support[a]	Design Transparency[b]	Student Thinking[b]	Explaining Math Ideas[b]	Tot. Educative Support[c]	Decision Making
EM (US)	180	28	27	42	23	19	15	45	5
MiF(US)	109	22	27	55	21	28	21	52	5
NT (Fl)	97	26	35	52	19	18	14	39	4
KP (Fl)	121	16	43	66	18	19	18	41	17
MD (Sw)	21	10	36	51	38	11	11	54	15
ME (Sw)	32	15	18	42	29	32	23	68	20

Note. [a] Units coded as directing action and containing educative support; [b] Educative supports; [c] Total percent of units coded as educative supports, excluding multiply codes.

Differences in how the programmes guide the teacher

When it comes to communicating with and guiding the teacher, the two curricula are comparable in the proportion of units written to direct teacher actions (27%). A notable difference between the two curricula is evident when examining the use of two types of educative features, particularly features designed to communicate about mathematics concepts and student thinking. Proportionally, *MiF* devotes about 50% more attention to communicating with the teacher about student thinking and mathematics concepts. These differences can be accounted for when examining the proportion of directing-action units also coded as hybrids (*EM*=42%; *MiF*=55%). In addition to communicating to the teacher what to do or say, these units communicate details about the design of the curriculum, student thinking, or the mathematics.

HOW THE FLANDERS CURRICULA COMMUNICATE WITH TEACHERS

Development and programme philosophy

Both *Nieuwe Tal-rijk (NT)* and *Kompas (KP)* are frequently used in Flanders and are representative for the spectrum of curriculum programmes. *NT* and *KP* were developed in response of the launch of the Attainment targets in 1998. In line with the philosophy of the targets, both *NT* and *KP* stress the importance of conceptual understanding, realistic contexts, and communication. They differ in the specific approach toward communication. *KP* recurrently stresses the importance of correct use of mathematical language. *NT* promotes discussion of mathematical ideas and strategies through small group collaboration.

Structure

Both programs consist of student texts and a teacher's guide. The teacher's guide includes lesson plans and print material for assessment and differentiation. *KP* also includes electronic material for additional practice. Lessons in both guides are grouped in units that address several domains. Lessons in *NT* and *KP* contain considerable detail; they have a fairly high mean number of units per lesson (*NT*=97; *KP*=121). Besides the main body of the lesson, which contains detailed guidance for enacting the lesson (*NT*=71%; *KP*=93%), both programs also include an introductory section (*NT*=23%; *KP*=7%). Lessons in *NT* also contain a number of optional items (7%). Whereas the main body of the lesson in *KP* has a fixed structure for all lessons, the structure of the main body of lessons in *NT* varies.

Differences in how the programmes guide the teacher

Both *NT* and *KP* contain a high proportion of directive guidance (*NT*=52%; *KP*=66%). In contrast, the proportion of educative guidelines is among the lowest in our sample (*NT*=39%; *KP*=41%). Both programs contain among the highest proportion in our sample of guidelines that are merely directive, containing no additional supports (*NT*=35%; *KP*=43%). *KP* is more directive, both in number of units that are merely directive and in the units that intertwine directive and educative guidance. *KP* also includes a higher proportion of instances that indicate that the teacher should make a decision (*NT*=4%; *KP*=17%).

HOW THE SWEDISH CURRICULA COMMUNICATE WITH TEACHERS

Development and programme philosophy

Matte Direkt (MD) is a traditional curriculum programme (in the Swedish system), with a new issue adapted for the new national curriculum in 2011. In *MD*, students work alone or in pairs for most of the lesson. The role of the teacher is to introduce the lesson and get the students working. Goals for each lesson are listed in a box on the first page for each chapter in the teacher's guide and the student book. *Matte Eldorado (ME)* is a new curriculum programme that builds directly on the 2011 national curriculum. Each chapter begins with two pages that contain the unit goals, the authors' interpretation of the goals, the pre-knowledge the students should have, and a discussion of how each goal is met in the chapter.

Similarities in structure

The two Swedish curriculum programmes share several common traits even if they differ in some important aspects. Both programmes give information on each page in the student textbook, and both have a system for differentiating instruction using optional tracks. Both programmes also present goals in the beginning of each chapter, although in rather different ways. Compared to the programmes from the other countries in this study, the two Swedish teacher's guides are very short, on average 21 (*MD*) and 32 (*ME*) coded units per lesson.

Differences in how the programmes guide the teacher

MD has a higher proportion of units (sentences or images) that direct teachers' actions, whereas *ME* communicates more about the mathematical content and student thinking. Of all six programs in our sample, *ME* has the smallest percentage of units coded as merely *directing action*. *ME* also has the highest proportion of units coded as educative support and the highest proportion of units indicating the teacher should make a decision. Both programs contain a high proportion of units that communicate about design transparency (*MD* more than *ME*). A distinct feature of *ME* is that it asks the teacher questions concerning the students' work (e.g.; "Are the students' own expressions correct and on what level of difficulty are they?"). These rhetorical questions are used to raise the teachers' awareness of certain aspects of student learning.

DISCUSSION AND CONCLUSION

Our cross-cultural analysis has revealed a number of differences, both across and within the cultural contexts. They also point to fruitful areas for future research.

One difference is the amount of guidance offered in the lessons of the teacher's guide. In contrast to lessons in the U.S. or Flanders teacher's guides, lessons in the Swedish guides contain rather limited detail. For instance, *EM (US)* contains nearly 9 times as many units per lesson than *MD (Sw)*. Another difference relates to the balance between directive and educative guidance. The Flanders lessons are much more directive than educative. The US lessons and *MD (Sw)* are quite balanced, and lessons in *ME (Sw)* are more educative than directive. These differences may reflect and relate to differences in educational traditions. In Swedish elementary math education, student texts have a central position. The teacher's role is to facilitate the student-text interaction. The teacher's guide indicates what the teacher might look for and expect. In Flanders and the U.S., teachers play a directive role, leading instruction. The teacher's guides, in turn, offer directive guidance for this role. In the U.S., there is also a strong commitment to student-student collaboration and some of the educative features in U.S. guides may be aimed supporting this less directive role. It is worth noting that the dominant instructional mode in each culture is reflected in the mode by which text authors communicate with teachers.

These observations raise several questions about assumptions about the knowledge teachers need to enact instruction. Might it be that the Swedish curriculum authors assume teachers know how to engage students with the content, whereas the U.S. and Flanders curriculum authors assume teachers need more prescription? It is interesting to note that, although the proportion of indications to make a decision differs across the curriculum programs, except for *KP (Fl)*, the number of decisions to be made per lesson is similar (an average between 3 and 8 decisions per lessons). Another interesting variation to analyse more deeply is the difference in guidance that is merely directive and guidance that combines directive guidance with educative guidance. This hybrid approach appears to assume that teachers benefit from directive guidance when it is accompanied by educative explanations.

To summarize, this cross-cultural analysis has pointed to within and cross-cultural differences that appear to reflect educational contexts and values in each culture. Further research might explore these differences qualitatively and in greater depth and consider differences in how teachers use the guidance their curriculum guides provide.

Note

[1] A full version of this paper can be downloaded at http://www.gse.upenn.edu/icubit/

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TEACHER GUIDES AS INSTRUMENTS FOR TEACHING MATHEMATICS: A CASE STUDY

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In Germany, teacher guides to secondary mathematics textbooks were merely solution manuals for the past 20 years. A new textbooks series for secondary school now offers a teacher guide, with a comprehensive didactical and methodological commentary to each page of the mathematics textbook. The teacher guide is structured in terms of blocks with different functions. In order to understand how teachers use these teacher guides to prepare mathematics lessons a case study with two teachers (cases) was conducted. The instrumental genesis (Rabardel) of these two cases was analysed. The study shows that among different kinds of resources the teacher guides are the most important and are heavily used for preparing mathematics lessons. However, the analysis of utilization schemes shows that both teachers show different instrumentations of the teacher guides. Whereas one teacher is clearly instrumented by the block structure of the teacher guide the other is not. Furthermore, the results indicate possible factors that are likely to be responsible for teachers' following or deviating from the textbook and the intended implementation of it.

Keywords: teacher guides, instrumental genesis, user study, Germany

INTRODUCTION

Studies on secondary mathematics teachers' preparatory work draw a homogeneous picture concerning the role textbooks: 'The textbook has a central role in teacher's planning activities, in fact not only the currently introduced one, but also several others' (Bromme & Hömberg, 1981, translated from German by SR). According to Hopf (1980) about 50% of German secondary school teachers use the mathematics textbook as a guide for teaching strategies. Tietze (1986) shows that more than 75% of the German secondary school teachers follow the structure of the mathematics textbook closely. These studies proof the importance of mathematics textbooks for lesson preparation activities of secondary school teachers in Germany. However, meanwhile, a lot of these studies are over 30 years old and therefore derive from a time, where computers and the Internet were not common resources. Concerning the omnipresence of digital resources, the question of the role of textbooks and the corresponding teachers' guides for teacher's preparatory work has to be answered in a completely new context. Furthermore, compared to mathematics textbooks, the relevance of teachers' guides for lesson preparation has not been studied to the same extent.

As digital resources are omnipresent in our daily life, a new development is rising from the German textbook market. After a period of time, where teachers' guides for secondary level textbooks were basically solution manuals, a detailed teachers' guide is offered for the new mathematics textbook series 'mathewerkstatt' (Barzel et al., 2012). This teachers' guide is structured in a novel way for secondary school mathematics textbooks. Every page of the students' textbook is guided by elaborate didactical hints and recommendations for implementation.

Given that teachers in the digital era actually use textbooks and corresponding teachers' guides for lesson planning the question is, how teachers actually use textbooks and teachers' guides for lesson planning. This paper presents an explorative case study with two cases' use of the innovative teachers' guide for lesson preparation.

SHORT DESCRIPTION OF THE TEACHERS' GUIDE

The teachers' guide of the textbook series 'mathewerkstatt' is characterized by a consistent modular structure. Information about every class unit is divided into blocks in the sense of Valverde et al. (2002): a *tabular overview* about the structure of the unit, a *general access* and a *page access*. The *general access* summarizes central questions and didactical approaches of the unit. The *page access* is made of a double page. The right page shows the actual textbook page and adds some selective comments; the left page refers only to the image of the textbook page on the right and is divided into a *quick access*, which informs about goals, references to other parts of the textbook, preparative material and recommendations for implementation, and an *intensive access*. The *intensive access* offers further advice for implementation and alternatives, information about model answers, expected outcomes, diagnosis and differentiation possibilities.

THEORETICAL FRAMEWORK

In order to understand how teachers use the teachers' guide the instrumental approach (Rabardel, 2002) is used as a theoretical lens. According to Rabardel (2002, p. 86), an instrument is 'a composite entity made up of an artefact component (an artefact, a fraction of an artefact or a set of artefacts) and a scheme component (one or more utilization schemes, often linked to more general action schemes)'. The user develops the instrument in use. While the user uses the artefact (here: the teachers' guide) for certain purposes (*instrumentalization*), the artefact's features influence the development of utilization schemes by the subject (*instrumentation*). In order to characterize utilization schemes Rabardel refers to Vergnaud's (1998) notion of schemes. According to Vergnaud (1998, p. 168) "A scheme is the invariant organization of behavior for a certain class of situations". Vergnaud (1998) identifies "four different kinds of ingredients in a scheme: 1. goals and anticipations; 2. rules of action, information seeking, and control; 3. operational invariants; 4. possibilities of inference" (p. 173). Out of these four ingredients, operational invariants are the most characteristic, because they refer to the knowledge, which is incorporated in schemes. This knowledge can be described in terms of concepts and theorems. Since these concepts and theorems are usually not made explicit, but expressed through actions, Vergnaud calls them *concepts-in-action* and *theorems-in-action* and defines: "A theorem-in-action is a proposition which is held to be true; […] A concept in action is an object, a predicate, or a category which is held to be relevant" (Vergnaud 1998, p. 168). This means that *concepts-in-action* can be identified by their relevance in a situation whereas *theorems-in-action* are either true or false in a situation. As opposed to Vergnaud, concepts- and theorems-in-action do not refer to mathematical concepts and theorems, but to propositions and relevant concepts that guide teacher's lesson planning activities.

This study analyses two teachers' instrumental geneses (*instrumentalization/ instrumentation*) of the teachers' guide. On the one hand, the modular structure of the artefact (the teachers' guide) offers many possibilities for *instrumentalization*, as teachers can choose blocks especially for their own purpose in lesson planning activities. On the other hand, the use of the blocks shape teacher's planning activities, which is mirrored in their utilization schemes of the teachers' guide.

DESIGN OF THE STUDY

The two teachers – called Mr. B und Mrs. B. – that participated in this study are teachers in a comprehensive school in Germany in the state of North Rhine-Westphalia. Mrs. B is between 40 and 50 years old and her professional experience as a teacher is between 5 and 10 years. Mr. B. is aged under 30 years and his professional experience as a teacher is between 0 and 5 years.

The method of data collection draws on the method developed and applied in a study on student's use of their mathematics textbooks (Rezat 2009): Both teachers were asked to highlight passages they used in class preparation of the unit "Brüche verstehen" (understanding fractions) in 5th grade and to answer an online-questionnaire after each preparation. The questionnaire included questions concerning topic, goals and central didactical principles of the planned lesson. In order to make sure that the prerequisite for the study is satisfied, namely that the teachers' guide is actually a relevant resource for lesson planning, questions were asked about used resources (cf. Adler 2000) (textbook, teachers' guide, internet, other books, journals, any additional material, experience, conversations with colleagues) while preparing the class, explaining why or why not specific passages were used and about how useful and how subjectively relevant these passages were.

The time frame of the study lasted as long as it took the teachers to teach the unit, which was 6 weeks in the end. Afterwards both teachers were interviewed, partially based on the stimulated recall of highlighted passages or statements in the survey, to gain a better understanding of their way of proceeding. During the whole time of the study, the use and mediation of the textbook during class was observed and documented in field notes.

RESULTS

In both cases, the data show (Fig. 1) that both teachers judge both textbook and corresponding teachers' guide as very relevant or relevant resources for most of the prepared lessons. The Internet, other books and journals are by trend the less relevant or are *de facto* not used resources. A mixed result can be seen in the other resources, as the relevance seems to be dependent on the particular class.

Even though both teachers use the highlighting differently (Mr. B. highlights everything he has read, while Mrs. B. highlighted what seemed important to her), the marks show that both teachers read the teachers' guide nearly completely during class preparation. The interview data shows, that Mr. B has developed the following *instrumentalization* of the *page access* of the teachers' guide for lesson preparation.

- He always uses the commented textbook pages with advice on the right side of the *page access* (comment: "that even works in class"[1]).
- He usually reads the *quick access*; the goals he reads to assure himself of the mathematical matter. He adopts the suggestions, if he agrees with them. If he sees problems with the suggested structure of the lesson, he reads the *intensive access*, especially the further advices for implementation, the alternatives and proposals for differentiation.
- During class, he always keeps the relevant double page open.

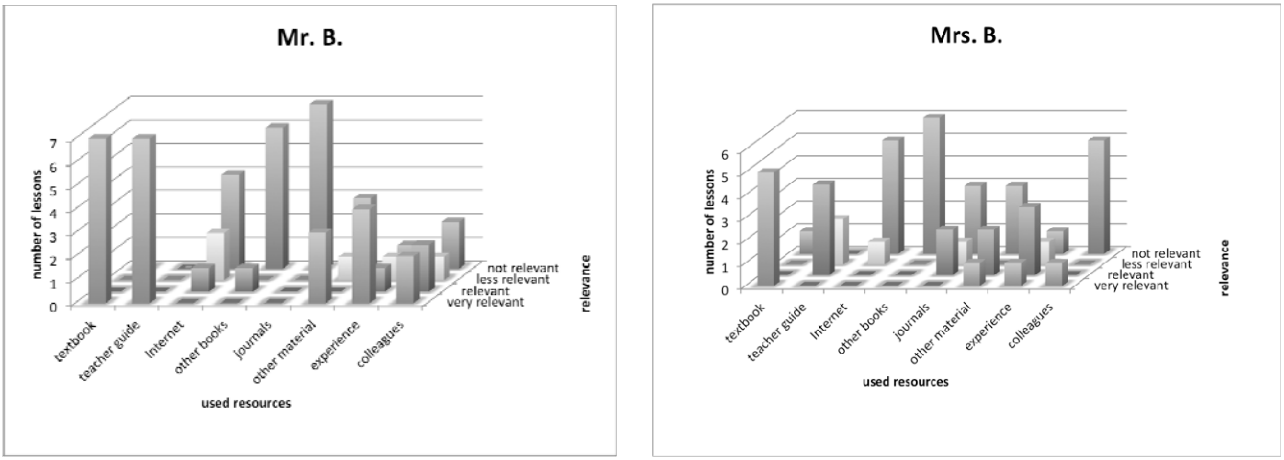

Figure 1: Mr. and Mrs. B.s relevance of resources in the number of prepared lessons

This *instrumentalization* of the particular blocks of the *page access* indicates that Mr. B.s utilization scheme of the teachers' guide seems to be based on the *theorem-in-action* that the different blocks of the *page access* are of different scope. His idea of the scope of each block guides his use of them in lesson preparation.

As opposed to Mr. B., Mrs. B. does not seem to have a distinct use of individual blocks. That can be seen by the way she is highlighting, but also by the way she answers the questionnaire, about what is especially relevant. Here, she does not refer to special blocks, but to content-related aspects like e.g. 'impulse questions' and 'visualizations', which are no distinct blocks, but appear within different blocks. It seems as if she is choosing parts from the guide, which she recognizes as essential. In the interview it also becomes clear that she uses the teachers' guide as a source of inspiration: "Sometimes as a teacher, when you are planning your lessons, then you don't get these ideas immediately and then I found it helpful, because you can use this as a starting point to think about further questions. It is simply facilitating the planning. You might get the same ideas, but it would take you more time".

It is apparent in the interview that 'available time' and her 'beliefs about her group of learners' seem to be the two interrelated prominent *concepts-in-action*, which guide her utilization scheme. Mrs. B. refers to "available time" her "group of learners" in several

[1] All quotes from teachers are originally in German and were translated by the author.

episodes in the interview, e.g. “it is not possible time wise, to strictly walk this given path”, “you haven't always got the time to let them work through the topic themselves and then let them neatly copy what they have worked out”, “The teachers’ guide suggests lesson units, which are full of conversation, and our pupils are not able to do it”. These *concepts-in-action* are complemented by *theorems-in-action* about the textbook, which Mrs. B. even makes explicit in the interview: “the exploration phases in the book are lengthy”, “the proposed lessons in the teachers’ guide focus on conversation”, “there are also suggested time ranges in the book, which are not long enough for our pupils”. Consequently, deviations from the proposals in the teachers’ guide she mostly explains by taking up too much time or not matching the group of learners, e.g. “Therefore, I can’t always use the principle of ‘Exploring’, ‘Organizing’, ‘Deepening’ in this case, as the ‘Exploring’ part in this book is structured very lengthy and the pupils would have lost their concentration” It also appears in the interview that the question ‘Where can I cut it short?’ is very relevant during Mrs. B.s planning activities.

CONCLUSIONS

Both analysed cases show an intensive use of the teachers’ guide. However, the instrumental geneses of both teachers are different. Mr. B. seems to follow the book closely. In order to do so, he reads the teachers’ guide intensively. In that, he shows a clear *instrumentalization* of the blocks of the teachers’ guide: Particular blocks are used for specific purposes. That he only reads particular blocks of the teachers’ guide if he sees problems with the suggested implementation of the textbook supports his general adherence to the book.

Mrs. B.s utilization scheme does not seem to be as clearly instrumented by the features of the teachers’ guide, but is guided by her individual system of beliefs about time constraints and her group of learners. Although she is reading the teachers’ guide intensively she does not seems to aim at following the textbook closely, but to take from the textbook what is conform with her ideas, her group of learners and the time-constraints.

These two cases show that two teachers use one artefact very differently. This could be an indication that it might be possible to distinguish different types of users in terms of a user-typology. The two cases in this study might be representatives of two different user-types of teachers’ guides: a ‘textbook conform type’, who uses the teachers’ guide to understand the aims and intentions of the textbook in order to follow the book as close as possible, and a ‘teachers’ guide inspired type’, who uses the teachers’ guide to get some additional inspiration and helpful ideas. In the latter case selection is guided by the individual beliefs of the user. In order to verify these hypothetical user-types and to further develop the typology further cases would need to be studied.

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DIFFERENTIAL AND INTEGRAL CALCULUS IN TEXTBOOKS: AN ANALYSIS FROM THE POINT OF VIEW OF DIGITAL TECHNOLOGIES

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Worldwide, courses of Differential and Integral Calculus have as a basic subsidy the use of textbooks. These constitute an important support for the course, either for the first reading of the students, to supplement the lessons taught, motor of research conductions by students or even for solving exercises. Considering the way the concepts of Calculus are presented in textbooks, and the concern of mathematics education community, which began with the movement of Calculus Reformation, which proposes the integration of digital technologies as a way to make concepts meaningful for a larger number of students, we conducted an exercise to understand how the technological component has been privileged in Calculation books, enabling approaches that go beyond the concepts of algebraic approach. Thus, we took textbooks adopted in mathematics courses (in the form degree and bachelor's degree) in six state university units from São Paulo doing a comparative analysis of the work, looking for evidence of how the visual approach is privileged. After we developed the analysis, based on the methodology of content analysis of Bardin (2008), we observed fragmentations in the presentation of some topics in some of the work and a sudden movement of change in more current works, to bring some approaches considering the use of technology.

Key-words: calculus, technology, Brazil

INTRODUCTION

Worldwide, courses of Differential and Integral Calculus (DIC) have as a basic subsidy the use of textbooks. These constitute an important support for the course, either for the first reading of students, either to supplement the lessons taught, either as a driver of conduct of research by students or even for solving exercises (Weinberg, 2010).

The approach of the concepts in a book of Calculus reveals evidence of how the concepts can be built. Considering the way the concepts of Calculus are presented in textbooks, and the concern of mathematics education community that began with the Calculus Reform movement from the twentieth century emphasizing the quality of the teaching and learning process and by proposing the integration of technology as a way to make the concepts more significant to a larger number of students (Tall, Smith and Piez, 2008), we dedicated to examining how the visual approach, based on the use of technology, has been privileged in differential and Integral Calculus textbooks.

As Richit (2010), the changes related to the teaching of DIC anchored in the Calculus Reform movement suggest: a) change in the focus of teaching of Calculus, focusing on fundamental ideas rather than emphasizing rules, techniques and procedures; b) show the importance and application of the Course of Calculus in various fields of knowledge as well as in the field of mathematics education and c) introduction of information and communication technologies in the curriculum of Calculus (Frid, 1994).

However, as we suggest that the process of teaching and learning of Calculus needs to be modified according to which proposes the reform of calculus, we are not suggesting that such courses should simply be modernized in order to use graphing calculators, computers, software or any technologies resources of graphic computing. We believe rather that in a calculus course, the CAS (Computer Algebra System) or other technological resources will not transform students with major difficulties in Mathematics in great mathematicians, but they can provide better understandings of the studied concepts (Richit, 2010).

From this perspective, Miskulin, Escher and Silva (2007) emphasize that the implementation of activities that take into account the use of technological resources, rescues exploration of mathematical concepts through a differentiated methodological approach that assists in the process of exploration, visualization and representation of the mathematical concept.

Considering the dynamic nature of the Calculus, we believe that this feature is hardly worked into a traditional learning environment, in which priority is given to studies of algebraic nature, where the focus of activities concentrates on finding solutions to the problems presented, expressed by closed formulas and specific techniques for solving certain problems. Regarding the use of technological resources in the context of teaching and learning mathematics, Villarreal (1999, p.362) says that:

> [...] o computador pode ser tanto um reorganizador quanto um suplemento nas atividades dos estudantes para aprender Matemática, dependendo da abordagem que eles desenvolvam nesse ambiente computacional. Do tipo de atividades propostas, das relações que for estabelecida com o computador, da frequência no uso e da familiaridade que se tenha com ele.

It is observed from this perspective that literature in general and researchers in the field, whose interest focuses on the coordination of information and communication technology (ICT) to the processes of teaching and learning technologies DIC, have argued that ICTs are extremely important in the discussion of activities that can be worked in algebraic mode as well as those that cannot, bringing the possibility of removing some of the algebraic “burden” inherent this discipline.

Moreover, ICT are characterized as resources which favor the establishment of investigative learning environments the extent that aim at strengthening relations between the subjects of the process, as well as providing questions, reflections, analyses and make that the classroom becomes a space for dialogue, allowing the student to propose and verify conjectures, thereby building knowledge of CDI.

Although we visualize in the academic scenario, a pivotal movement of ICT to the processes of teaching and learning IDUs, this trend is due to individual initiatives. Thus, some textbooks have been permeated, incipiently, for pedagogical approaches that allude to using some mathematical software in the study of concepts of CDI. It is this point that we explore in this paper.

Considering the whole movement of joining ICT to the processes of learning and teaching CDI, we conducted an exercise to understand how the technological component has been privileged in Calculus books, enabling approaches that go beyond the algebraic approach of concepts. Thus, this paper is structured as follows: initially presents a brief review of the literature discussing the potential of ICT in teaching and learning processes of CDI; following, it provides a description of the methodological processuality on the use of ICT in the study of Calculus; therefore presents an analysis of textbooks of Calculus, focusing on the presence of ICT in addressing the concepts; finally exposes some final considerations concerning the analysis undertaken.

Differential and integral calculus and information and communication technology: Leveraging the approach of concepts

Miskulin, Escher and Silva (2007) emphasize that the use of the activities that make use of technological resources in addition to the concepts of CDI approach provides a differentiated focus, which enables the exploration and visualization, as well as coordination of different representations of mathematical concepts. For example, coordination and mobility of representations in a semiotic perspective, the focus of the dissertation Farias (2007), were found in an activity involving the concept of continuity that had the Winplot. This activity was to assess the continuity of the function $f(x) = \begin{cases} kx^2 + 1 \; se \; x \leq 1 \\ 2x - 3 \; se \; x > 1 \end{cases}$. At the dawn of the discussions on this activity students said intuitively that this was a continuous function. Following, they made use of the resource "animation" of winplot software to verify, from the perspective of the function representation, what they had initially conjectured. However, only when they animate function, by varying the parameter k, they were able to visualize that for k = - 2 the function became continuous. Also, they were based on the algebraic approach to prove the initial conjecture.

We observe, therefore, that in the face of mathematical research and exploration processes carried out by students with Winplot software, doubts were clearing up. Moreover, they could confront the findings produced from investigation conducted with the software with that obtained by algebraic manipulation of the function; i.e., through the coordination of mathematical representations they might rethink about the “mistake” made about the statement originally made.

Menk (2005) develops a research on Maximum and Minimum of functions, especially those that are related to concepts and geometric properties, supported by the Géomètre Cabri-II software. Among the proposed activities, one of which was to find the most economical way to install a power cable, connecting a hydroelectric plant situated on a river 900 meters wide to a factory located on the other side of the river, 3000 yards downstream of the plant. In this problem, it was considered that the cost of installation of the submerged cable was R$ 25.00 per meter, while land was R$ 20.00 per meter.

After the construction of this situation in Géomètre Cabri-II, after an extensive process of mathematics experimentation and intense discussion, the students present a proposal on how the cable should be connected to minimize the cost of installation. We note that these conjectures were only possible when students compared the two situations representing the minimum and maximum cost for installation of the cable, linking the graphical representations produced in Cabri software and also the algebraic representation sketched on paper.

At the end of the investigation Menk (2005) points out that these procedures can create conditions that enable ease of interpretation, observation, analysis and resolution of problems considered. The way the activities were developed, focusing on the simulation and visualization, have created situations in which they could “see” the process of how it developed students’ thinking in various situations.

In his doctoral Javaroni (2007) examined possibilities for learning and teaching Introduction to Ordinary Differential Equations (ODE) with students of mathematics, working on a qualitative approach with prospect of some mathematical models (models falling object, growth population Malthus, population growth and Verhulst law of cooling), aided Excel spreadsheets, and Maple Winplot of software and some applets. From the study, the author emphasizes that the work following this approach becomes evident that the interaction of students with ICT has provided new possibilities for qualitative approach of the studied models. Moreover, it suggests the need to rethink the teaching of EDO, emphasizing the geometric aspect of mathematical models, and the algebraic aspect.

In summary, the studies mentioned in this section, although supported by different theoretical biases, highlight the important role of the visual aspect of the approach of mathematical concepts, whose graphical representation provides a broader perspective of these concepts and the notion of totality of mathematical knowledge. Continuing our discussion we bring in the next section, the theoretical perspective underlying to analysis used in our study.

THEORETICAL FRAMEWORK

In this study took by theoretical presuppose the understanding that the visual aspect that is privileged in the approach of DIC’s concepts by graphical representation of functions, which allow to analyse important characteristics of situation under study , possibility an comprehensive analyse of a mathematical concept. About this Dugdale (1993) and Schwartz et al. (1993) recognized the necessity of a better understanding of the student about graphical representations of functions, which enable comprehensions that derivate of a visual aspect.

About the question of visual aspect, embodied by graphical representations, this oftentimes follow an approach only of observation, in other words, the students make the construction and just the observed, unable to think or glimpse other possibilities from this graphic. This is also a procedure quite often in this text books of DIC, where the students don’t know why they do what they do and the activity become mechanical. Moreover in the DIC courses it approaches functions’ family, whose graphical treatment it show in a flowed way, between many other concepts that have dynamic nature, but the representation follows a static nature.

METHODOLOGY

This study was guided by the following question: What the textbooks used in mathematics courses suggest regarding the construction of concepts and strategies for teaching and learning of the Differential and Integral Calculus from ICT? In other words, we are concerned about how textbooks DIC qualitative to approaches concepts of the aforementioned discipline considering the technological component, analyzing the type of pedagogical approach proposed, among other things.

For this, we took the summaries of the discipline of Differential and Integral Calculus of six state university of Sao Paulo (UNESP), namely: Bauru, Guaratinguetá, Ilha Solteira, Presidente Prudente, Rio Claro and São José do Rio Preto. These summaries, found a total of 25 books listed as reference textbooks for the course of DIC.

Of these books, we took for analysis those that propose some approach of the mathematical concept considering the use of graphics software, graphing calculators or spreadsheets. The analysis was performed according to the methodology of Content Analysis from Bardin (2008), because this theory allows analyze any document’s types.

Table 1 shows the distribution of the textbooks in accordance with the six academic units and was organized with a view to ordering the textbooks cited in all summaries until those referenced in only one.

Textbook/Number of units that adopting	Textbook/Number of units that adopting	Textbook/Number of units that adopting
Guidorizzi / 6	Ávila / 1	Courant and John / 1
Swokowski / 5	Boulos / 1	Aguiar, Xavier, and Rodrigues / 1
Leithold / 5	Al Shenk / 1	Anton and Bivens and Davis / 1
Flemming and Gonçalves / 5	Táboas / 1	Larson, Hostetler and Edwards / 1
Stewart / 4	Braun / 1	Edwards and Penney and / 1
Thomas / 3	Medeiros / 1	Finney, Weir and Giordano / 1
Munen-Foulis / 2	Boulos / 1	Hostetler / 1
Anton / 2	Safier / 1	
Hughes-Hallet et al/ 1	Courant / 1	

Table 1: distribution of the textbooks in accordance with the six academic units

The analysis undertaken on the 25 textbooks listed in the summaries of courses DIC in six state university of Sao Paulo, Brazil, guiding us in the methodology of Content Analysis from Bardin (2008), reveals that only one of them mentions the use of some technology in approach to concepts: James Stewart. In this textbook we identified some propositions of mathematical activities that take the ICT in the approaches of concepts' DIC. The author emphasizes that the availability of technologies such as graphing calculators and computers are favorable tools to the discovery and understanding of concepts. Under this understanding, in some exercises there is some reference to the use of this resource is not being specified, and at other times, it is suggested to use CAS such as Derive and Mathematica software or TI-82 calculator. However, the author makes clear that the use of these resources does not exclude the use of pencil and paper and algebraic techniques commonly employed in DIC classes.

Analyzing the activities that suggest the use of CAS in the study of concepts of DIC, we verify that these activities, as proposed, do not satisfactorily exploit the potential of ICT, these resources that enable conjecture and test ideas, besides providing the mathematical experience (Villarreal, 1999). In this perspective we consider that the role of ICT in the proposed activities in this textbook serves only as a setting for "observation" to the student, without that he experiment and observe what happens with the situation in study. In other words, the proposition of ICT use in these activities constitutes in the illustration of any idea or mathematical concept.

The analysis shows also that ICT are not yet taken the approach of concepts in most textbooks. Furthermore, we observed that older editions of textbooks do not mention ICT, aspect which may be linked to the fact that this movement started around 80's, with the Reform movement's Calculus, since the Calculus was the area from Mathematics which received its first investments in this direction. With respect to textbooks that bring some investigation to take the use of technology, especially more current books, are those who follow this methodological approach. However, textbooks generally have not brought the investigation of concepts such as Limits, Derivatives and Integrals that goes beyond just observation.

Finally, believe in the possibilities of an approach of "construction" of concepts, in which the student may have a comprehensive understanding of the content, and not just observational, how come only if the ICT to improve the construction of graphs that can be made in pencil and paper, we argue in favour of the incorporation of these resources in teaching practices in DIC and the need to revise the textbooks adopted in these courses, as a way to foster new ways of teaching and learning Calculus.

FINAL REMARKS

This article points to a methodological approach in which the algebrism is not the only way of representing problems of DIC and this proposal assumes that the concepts of DIC's approach fosters a more holistic understanding considering the visual aspect as well as the coordination of multiple representations, in that where possible the problems must be approached in algebraic, graphical and numerical contexts. Strategies that prioritize the mathematical representation multiples enables a more complete understanding of the problems, which in turn can be favourable by insertion of software like mathematical investigation environments.

As already pointed out, we tried to show among textbooks used in courses DIC in universities of state of Sao Paulo, those who did mention the use of ICT in their concepts approach. Of the

25 books, only 3 of them mention the use of ICT, and only 1 proposes activities based on the use of these resources. Given this scenario and considering that ICT enable qualitative changes in terms of mathematical approaches, we argue in favour of the inclusion of these resources in teaching practices in DIC and the need to revise the textbooks adopted in these courses, as a way to foster new ways of teaching and learn Calculus.

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AN INTERNATIONAL COMPARISON OF MATHEMATICAL TEXTBOOKS

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In this paper we report partial results of a study that compares Brazilian mathematics textbooks with textbooks from seven other countries (France, Germany, Italy, Japan, Portugal, Singapore and Spain) trying to assess whether they are suitably designed to support students on attaining a full range of mathematical abilities and to construct a view of mathematics as a science based on deduction. The analysis is based on the attributes of textbooks ranked by Howson (2013, p.653-654), including: mathematical coherence; clarity and accuracy of explanations; clarity on the presentation of kernels. Additionally, we consider one particularly important aspect: Do the textbooks allow the student the opportunity of experiencing abstract mathematical thinking? The partial results are presented through excerpts from textbooks. Those excerpts are examples of what may lead to misconceptions, misinterpretations, contradictions, or give the student an unclear idea of mathematical deduction.

Keywords: textbook comparison, mathematical accuracy, mathematical coherence, mathematical deduction

INTRODUCTION

As Howson (2013, p. 657) comments, mathematical textbooks “have played and will continue to play a vital role in mathematics education objectives, and not merely examination success.” In Brazil it is not different. As Borba (2013) points out, mentioning a government document, “textbooks are the main resources used by Brazilian teachers for planning classroom activities, so they have a very strong influence on what happens at school, and may affect students’ mathematical performance and understanding.”

Therefore, attention should be paid, for instance, to the accuracy and coherence of textbooks, as a criterion to assess whether they are suitably designed to support students on attaining a full range of mathematical abilities and constructing a view of mathematics as a science. And even more attention should be paid to the textbooks that are distributed by the government to the majority of the students in a country.

Since 1997 the Brazilian Ministry of Education has been providing textbook analysis and approval by specialists in different subjects (mathematics is one of them[1]) which are published in guides (Guia Nacional do Livro Didático). Each state-run school must choose

[1] From now on in this paper, for brevity, we just use ‘textbook’ to refer to ‘mathematics textbook’

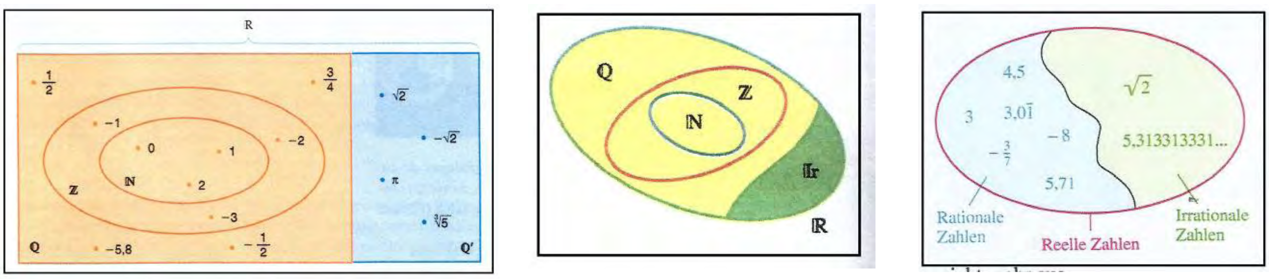
three titles among the approved ones. Then, for the next year, it receives from the government one of those three titles and each student receives a copy.[2]

Despite the analysis provided by the Brazilian government, one can still find aspects that can be criticized in the distributed textbooks mainly with respect to the attributes mentioned in the abstract. The question “Does any of those aspects to be criticized also appear in foreign textbooks?” was considered by the author and partial results comparing textbooks from Brazil with other countries are reported in this paper. All the textbooks under analysis are for 11 to 14-year-old-students, except the one from Portugal and Italy, which are for 4th grade students and for 15-year-old students, respectively.

SOME EXCERPTS CONFIRMING SIMILARITIES

The diagrams illustrating the numerical sets

The apparently naive diagrams illustrating the numerical sets that were found in Brazilian textbooks (Figure 1, (*a*) and (*b*)) as well as in German ones (Figure 1(*c*)) may contribute to the misconception that the irrational numbers are exceptions in the set of real numbers by suggesting that the sets of rational numbers and of the irrational numbers are of the same “size” and/or that there are less irrational numbers than rational ones.[3] Additionally, the author could find no discussion about the irrationality of $2\sqrt{2}$ or $\sqrt{2}+\sqrt{3}$ in the section usually called *computing with radicals* and which can be found in many textbooks.



(*a*) (*b*) (*c*)

Figure 1: Diagrams illustrating numerical sets in (a) Paiva (2009), p.45; (b) Dante (2009, Vol. 8), p.30; (c) Griesel et all (2007), p.96

The definition of an irrational number

A conflict in thinking[4] may occur with some definitions of irrational numbers found in the textbooks. The assertion “an irrational number is a number the decimal expansion of which is

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[2] A more detailed description of this Program can be found in Borba (2013)

[3] By the author´s experience working with students in their first semester at the University, it is not rare that Brazilian students by the end of High School have difficulties in giving six examples of irrational numbers besides $\sqrt{2}, \sqrt{3}, \pi$ and sometimes e or ϕ.

[4] “A conflict (or inconsistency) in thinking occurs when there are two (or more) distinct ways of interpreting data that are not coherent. (…) It occurs in particular when experience in one context leads to incidental properties that do not carry over to other concepts. For instance, in counting whole numbers, after each number there is a 'next' number and there are no numbers between one number and the next. In fractions,

neither finite nor periodic" (Dante (2013, Volume 8), p.26) suggests that all the numbers have a decimal expansion, which is not true: this property does not carry over to the complex numbers. An analogous remark fits to the following excerpt from a Singaporean textbook (Hong et all (2012), p.44): "The decimal 0.5 can be expressed as ½ and thus is also rational. However, there are numbers which cannot be written as exact fractions. Such numbers are called irrational numbers."

Such a remark also emphasizes one of the principles of the mathematics as a science, namely, being based on accuracy (of definitions, in this case).

One can compare both definitions with the one found in a German textbook (Schmid (1996), p.14) in which the verb "can" prevents the imaginary numbers from being irrational: "Numbers that can be represented by non-terminating non-periodic decimal fractions are called irrational numbers."

The definition of π

One can notice in some Brazilian textbooks a lack of presentation of kernels while dealing with the number π. They inform that π is an irrational number approximately equal to 3.14, but then treat approximations with equalities, what gives rise to an incoherence: π is an irrational number and π=3.14. This fact can be found in Dante (2013, Vol.8, p. 28), but also in a Japanese textbook (Isoda (2012), p.42)[5] where the formula "Circumference = Diameter x 3.14" appears. We also point out that this substitution constitutes an obstacle in the learning of irrational numbers, if we intend that the student realizes that an irrational number can be arbitrarily approximated by rational numbers.

The definition of angle

The textbooks under analysis deal with the concept and the definition of angle in three different ways: *i)* using as definition "angle is the union of two rays with the same origin" as a Brazilian textbook (Figure 2(*a*); *ii)* using as definition "angle is a portion of the plane which is limited by two rays with the same origin" as a Brazilian (Ferrari (2000), p.47), an Italian (Cassina & Bondonno (2012), p.670) a Portuguese (Rodrigues et all (2009), p.85) and a Singaporean (Figure 2(*b*)) textbooks; *iii)* presenting only the idea of angle with illustrations that suggest the second definition (Brazilian and French textbook – Figures 3(a) and (b) resp.).

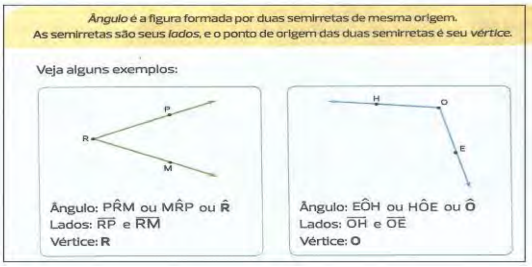
Two rays with a common origin divide the plane into two sectors. Hence the assertion in *(i)* does not serve as definition of angle if one intends to *measure* this set in the way usually done in school, namely, measuring in fact the portion of the plane limited by those two rays. No student would be sure which portion of the plane in Figure 2(*a*) should he consider in measuring. One can notice in Figure 2(*b*) an attempt to indicate it in the picture. Nevertheless, in the next page of this Singaporean book, one can also find the phrase "An angle can be

there is no 'next' number and between any two fractional numbers there is an infinite number of fractions" Tall (2014)

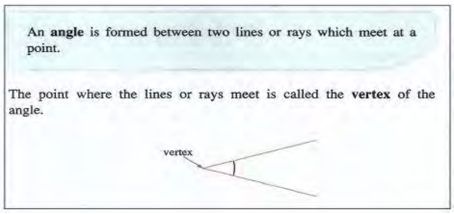
[5] This is a Japanese collection translated into Spanish.

named using the letters of the points on the rays, using the letter of the vertex in the middle", a notation that can be ambiguous if nothing more is said about the region.

In the Portuguese textbook (Rodrigues et al, 2009, p.85) one can find the definition indicated in *(ii)*, but also the following phrase: "Two rays with the same origin form an/one angle" which also makes this text ambiguous / incoherent[6].

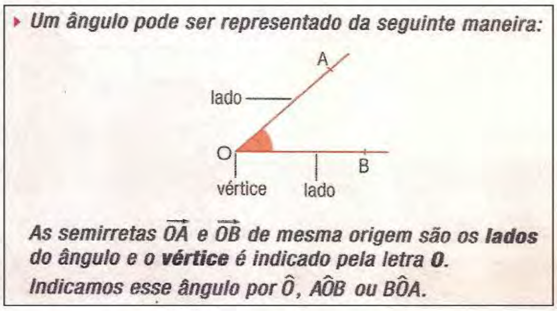


(*a*)

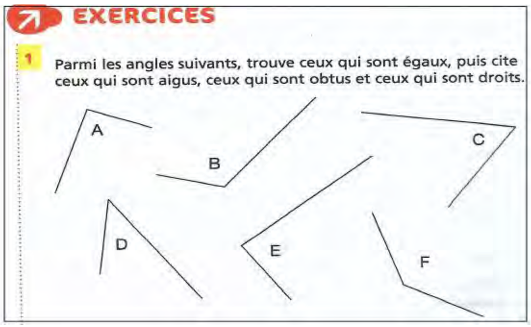


(*b*)

Figure 2: The definition of angle in (*a*) Dante (2013, Volume 7), p.168 and Figure 2(*b*) Hong et all (2012), p.189



(*a*)



(*b*)

Figure 3: The idea of angle in (*a*) Souza & Pataro (2012), p.206 and in (*b*) Peltier et all. (2009), p.58

Despite only presenting the idea of angle to the students, we remark that, in the teachers' guide of the French textbook (Peltier et al, 2009; Livre du Professeur, p.104) of the French textbook, the authors are very clear about the definition of angle: "The notion of angle is complex: it´s common that one identifies angle with its measure, forgetting that *angle is a geometric object.* It is a portion of the plane limited by two rays with the same origin, also called sometimes *angle sector*."

The opportunity of experiencing abstract mathematical thinking and arguing

Many properties of the numbers or of the operations with numbers are sometimes inadequately established, being based only on its validity in two or three instances. This strategy (inductive reasoning) does not give the student a clear idea of a fundamental principle of the mathematics as a science, namely, the deductive reasoning. This could be found in Brazilian (e.g. Souza, J. R. & Pataro, P.M. (2012), p.121) and Spanish textbooks

[6] "Duas semi-rectas com a mesma origem formam um ângulo". In Portuguese, the indefinite article "a" and the cardinal "one" are translated into the same word: *um*. Hence, depending on the interpretation of the "um" mentioned in the phrase, the text becomes either ambiguous or incoherent.

(Colera & Gaztelu (2010), p.68): “Compare the two following expressions and remark that one obtains the same result.

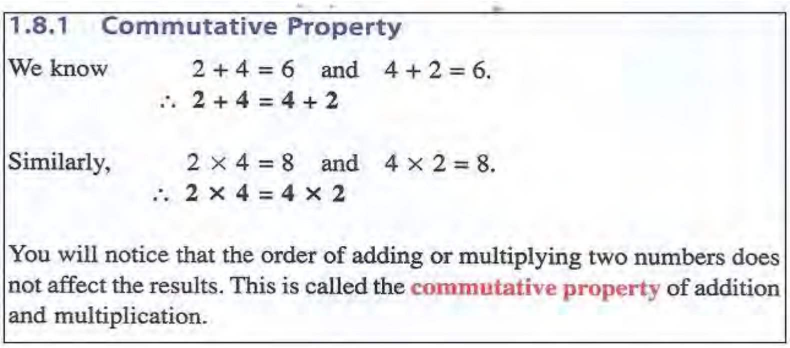
Example:

$$(2\cdot 3)^3 = 6^3 = 6\cdot 6\cdot 6 = 216$$

$$2^3\cdot 3^3 = (2\cdot 2\cdot 2)(3\cdot 3\cdot 3) = 8\cdot 27 = 216$$

The power of a product is equal to the product of the power of the factors.”

Tall (2014) refers to *generic thinking* as “thinking of specific instances of a concept as representing the general idea itself.” The excerpt from the Singaporean textbook (Figure 4) seems to be an attempt to make use of the generic thinking.



1.8.1 Commutative Property

We know $2 + 4 = 6$ and $4 + 2 = 6$.

$\therefore$ $\mathbf{2 + 4 = 4 + 2}$

Similarly, $2 \times 4 = 8$ and $4 \times 2 = 8$.

$\therefore$ $\mathbf{2 \times 4 = 4 \times 2}$

You will notice that the order of adding or multiplying two numbers does not affect the results. This is called the **commutative property** of addition and multiplication.

Figure 4: a slight allusion to generic thinking in Hong et all (2012), p.19

We do not disagree with that strategy, but remark that perhaps the students do not become aware of the necessity of the generality aspect of the given example without being previously exposed to assertions that are in some instances true and in some instances false. Such assertions were not found in the analysed textbooks.

Undoubtedly in a German textbook (Griesel et al (2007), p.66) the students are invited to the abstract mathematical thinking and it is offered to them an opportunity to be engaged in reasoning-and-proving. After one example, one can find the following reasoning:

We assert:	*We justify:*
$a^m . a^n = a^{m+n}$	$a^m\cdot a^n = \underbrace{a\cdot a\cdot \ldots\cdot a}_{m\ factors}\cdot\underbrace{a\cdot a\cdot\ldots\cdot a}_{n\ factors} = \underbrace{a\cdot a\cdot\ldots\cdot a}_{m+n\ factors}$

We also remark that the above reasoning could be translated into colloquial language without loss of abstract mathematical thinking, i.e., it is not the symbolic language that we are emphasizing in the German textbook. There are many other instances in which we can exercise abstract mathematical thinking without making use of the symbolic language or of what Tall calls generic thinking.

CONCLUSIONS

All the topics mentioned in the previous section emerged from the analysis of Brazilian textbooks. Altogether, including others not mentioned here, they reveal, in the opinion of the author, a necessity of paying more attention to mathematics as a science at least in the textbooks distributed by the government in Brazil.

The fact that some of them (angle and irrational numbers, for instance) are also not adequately treated in other countries could indicate that this is effectively a difficult topic to be taught in school. In the opinion of the author, this is indeed the case for irrational numbers and angles, but for different reasons.

Finally, with respect to the foreign textbooks, it is obvious that the present analysis should not be taken as an overall evaluation of the textbooks here mentioned.

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RULES OF INDICES IN UNITED KINGDOM TEXTBOOKS 1800-2000

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This paper reports a textbook analysis which examined how the shift from integer to rational exponents in the rules of indices are discussed in mathematics textbooks. Rules of indices are centrally important to algebra, both theoretically and as a computational tool. The topic occurs at a number of levels in school and university curricula. The data set is a selection of historic textbooks published in the United Kingdom during the period 1800-2000. Historic textbooks potentially avoid the problem of high-stakes examinations driving textbook design, and they deal with algebra as a separate and self-contained subject. These books were in print for many years, were popular in terms of numbers of books sold and used, and were influential. The majority of books in the corpus were in print for more than 25 years, with some continuously in print for more than 80. The analysis seeks to understand the justification given by textbook authors for the shift from integer to rational exponents in the rules of indices. A coding scheme is developed to group justifications into categories. It is intriguing that this topic is justified in so many different ways. Some of the epistemological and didactic implications of this analysis are discussed.

Keywords: rules of indices, history of mathematics education, historical analysis, UK

INTRODUCTION

In this paper I report the results of a content analysis I undertook to examine how authors explain the shift from integer to rational exponents in rules of indices. The notation a^n is usually defined as repeated multiplication of a with itself n times. In older books this process is called *involution*. The processes of ‘undoing’ squaring is the search for square roots. In older books this process, and in general nth roots, is called *evolution*. This leads to an extension of the definition of the notation a^n where n is now a rational number. In defining $a^{1/2}$ a meaning is given which seeks to preserve “rules of indices” such as

$$a^n a^m = a^{n+m}, \quad (a^n)^m = a^{nm}, \quad a^n b^n = (ab)^n. \qquad (1)$$

We currently also have parallel, but much older, notation $\sqrt{a}$ which may, or may not, be defined to be exactly the same as $a^{1/2}$. I examine, and compare, the meaning given to rational powers and examine the justifications provided for this extension of meaning.

Rules of indices are centrally important to algebra, both theoretically and as a computational tool. The use of exponential notation enables multiplication to be rephrased in terms of addition of indices. I am interested in how textbook authors have chosen to sequence the introduction of these mathematical ideas, and address the consequences. These choices are clearly intended, and very likely, to have an effect on how students view algebra.

THE CORPUS OF TEXTS

I have taken a historical selection of texts from the period 1800–2000. Contemporary textbooks are tied to particular curricula or targeted at a specific examination in one jurisdiction. By looking at historic textbooks I seek to avoid the problem of high-stakes examinations driving textbook design, see (Howson 2013, p. 652). Each textbook is written in its own historic and sociocultural context, and therefore a historic approach also provides an opportunity for a wider variety of conceptions to be evident than might be the case with a selection of contemporary books. Another significant reason for looking at historical texts is that they deal with algebra as a separate and self-contained subject. I have chosen not to look at algebra texts prior to 1800 because what is now elementary algebra was then the subject of mathematical research. Another reason I decided not to consider earlier works on algebra is that mathematical notation differed significantly.

I have chosen to look at textbooks which either seek to give a comprehensive and complete treatment of “elementary algebra” or those intended for use by “advanced school students”. Criteria for selecting texts included popularity (i.e. sales), longevity (duration in print & number of editions) and influence (e.g. acknowledgements from subsequent authors). Experts in both history of algebra and mathematics education were consulted for suggestions of texts but I only accepted the recommendations of experts when other corroborating evidence warranted the inclusion of a text. The complete text of the majority of books published prior to 1900 is now readily available online which facilitates cross-referencing of editions.

In many cases the work is a single book but in other cases it is not so clear what constitutes a single “work”. Many textbooks are published as a series, in multiple volumes. In these cases it is appropriate to treat more than one book as a single work for the purposes of this study. Hence I have used some discretion in deciding what constitutes a single work. In some cases authorship is not entirely clear. For example, Wood, the original author of (Lund 1852), died in 1839 and Todhunter says *“the tenth edition [1835] was the last issued in Dr Wood’s life-time”* (Todhunter 1897, p. vi). Lund took over editing this work and between the 13th and 14th editions there are significant changes to the treatment of the rules of indices. I treat Wood (1st) and Lund (14th) as separate publications. (Hall & Knight 1962) and (Bonnycastle 1836) also have others contributing after the death of one author.

METHODOLOGY

The purpose of my analysis is to compare and classify the stated justifications for the rules of indices for rational exponents. My goal was not to read the whole book in detail, but rather to work as a reader trying to resolve a question satisfactory. To record my results I needed to classify the form of the justification, if any, provided by the book authors. Many previous writers have considered forms of argument, including for example (Harel & Sowder 2007) and (Stacey & Vincent 2009). I compiled excerpts from 12 textbooks, together with brief descriptions of the codes and invited 6 colleagues to code the data. Ultimately the following coding scheme was developed and used. No argument or external authority (NL), use of real-world physical model (MO), empirical, or use of specific cases (EX), general deductive argument (DI), formal Proof (PF), analogy (AG).

RESULTS

The texts selected had numerous editions over more than 25 years, some over 80 years. It is remarkable how consistent algebraic notation is throughout the period 1800–2000. The choice of letters *a*, *n* and *m* in $(a^n)^m$ is ubiquitous. It is remarkable that parallel surd notation and fractional powers has persisted. For many authors these are the same, but for others they are different, e.g. see (DeMorgan 1837, p.122). Two notations is potentially confusing, and results in many exercises simply converting from one form to the other without adding any new concepts. (Euler 1822, §200) makes interesting comments on this issue.

Table 1: Textbooks and their modes reasoning

1st pub	Last ed.			Code
1797	1840	(Euler 1822)		EX+AG
1782	1836	(Bonnycastle 1836)		NL
1795	⋯	(Wood 1801)	(1–13)	DI
⋯	1876	(Lund 1852)	(14–)	DI+PF
1798	1841	(Hutton 1836)		NL
1828	1837	(De Morgan 1837)		AG
1858	1870	(Todhunter 1897)		DI+AG
1885	1962	(Hall & Knight 1962)	Elementary	NL+AG
1887	1940	(Hall & Knight 1896)	Higher	AG
1886	1964	(Chrystal 1893)		NL+PF
1930	1963	(Durell 1930*b*)		AG
1957	1985	(Backhouse & Houldsworth 1976)		AG
1964	⋯		SMP [11-16]	MO
⋯	2008	(Thwaites 2012)	SMP [16-18]	EX

Euler: This text is unique (in the corpus selected) in introducing complex numbers before algebra. See (Euler 1822, §145) Euler extends the notation to algebra later with a discussion of "doing" and "undoing". (Euler 1822, §196) is typical using repetitive examples and working inductively, which I coded as empirical. In §200 he uses an analogy in justifying exponential notion *"because it manifestly corresponds with the nature of the thing"*. Since complex numbers have already been introduced, there is no reason to specially highlight the case $a<0$, so Euler escapes a case by case analysis of the following form. *"§146. If the root to be extracted be expressed by an odd number, the sign of the root will be the same with the sign of the proposed quantity, as appears by Art. 137.[...]"* (Lund 1852, p. 70).

Wood, Lund and Todhunter: These texts show remarkable similarities in presenting the rules of indices. Their argument is coded as a general deduction, e.g. see (Wood 1801, §119). In 1852 between 13th & 14th edition a section on *Theory of indices*, is added preceding work on surds. There then follow symbol pushing proofs of the laws of indices, an example is (Lund 1852, p. 83). Hence I code (Lund 1852) as deduction from a general case and proof. (Todhunter 1897 p. 149) retains the chapter on the *Theory of Indices*, but works by analogy.

De Morgan contains a conscious and explicit extension of meaning which is coded as a full analogy. His is the first in my corpus to discuss a progression of extensions in the meaning of number from negative, through rational and surds to real numbers and finally the meaning of $\sqrt{-1}$; see (De Morgan 1837, p. 110–111).

Hall and Knight return to introducing surds (of numbers) before discussing fractional powers. They are therefore able to define fractional powers in terms of roots, e.g. (Hall & Knight 1962, p. 260). They justify these rules in a later book, (Hall & Knight 1896, p. 431), which I interpreted as defining the meaning by analogy.

Chrystal: Rather than extending by analogy, (Chrystal 1893) takes the laws as *axiomatic*, see (Chrystal 1893, p. 29). Students are expected to undertake mechanical manipulation without interpretation so initially this is an appeal to authority, (NL). When Chrystal returns to this subject he provides a proof, working from the definition, that the rational exponent is related to the idea of roots. See (Chrystal 1893, p. 181).

Durell: Durell provides an extension of meaning by analogy; see (Durell 1930*b*, v. 3, p. 2). His definition of complex numbers is not included in the algebra texts, but rather as part of *trigonometry*, (Durell 1930*a*). Durell defines a complex number as an ordered pair $[a,b]$ together with formal rules, from which he derives various algebraic and geometric properties.

Backhouse and Houldsworth (Backhouse & Houldsworth 1976) take the laws of indices as axiomatic for the rational numbers, (Backhouse & Houldsworth 1976, Vol. 1, p. 169).

SMP New Books 1–5 The SMP New Books represent a significant departure. They are the only books to work from a continuous model. In this sense they avoid the algebraic extensions of their predecessors.

> If the Megalogean tree grows steadily and doubles its height every year, by what factor is its height multiplied in half a year? In n years its height is multiplied by 2^n, so in half a year its height ought to be multiplied by $2^{\frac{1}{2}}$, but what does this mean? (SMP 1983, p. 65–66)

In appealing to a physical system, i.e. the size of a tree, the justification makes use of a model.

SMP Advanced Mathematics Books 1–4 also represent a significant departure. These books ask the student to undertake sequences of exercises *before* sumarising the rules of indices as a synopsis of the previous work. The justification, provided by the students in this case, is experimental. See (Howson & Dodd 1967, p. 167–170).

DISCUSSION

The majority of books articulated the need to define rational powers. The rules of indices (1) are a theorem on the natural numbers, but then become defining axioms on the rationals. In this sense the definitions are not arbitrary conventions but are chosen to preserve a structural property. What the textbooks actually do is give rules for computation with fractional exponents. That is, they take a representation of a rational number as a fraction and explain how to compute with formal rules. The separation of interpretation or meaning from the computational rules is clear and conscious in some books, but not so in many. In others, a mixture of formal computational rules, their meaning and any justification is confused. This contributed to the difficulty in assigning a single classification code to the justifications from a fine-grained scheme.

It is common to find statements which are simply too general, and hence not true. There is a paradoxical inconsistency on the focus of increasing one domain, but ignoring the other. I find this particularly surprising. We are changing the domain of the exponent from the natural numbers to (at least) the rational numbers. To preserve previous computational rules of indices, the extension of the domain to rational coincides with a (potential) contraction of the domain of the base to positive numbers.

Some of the books did not consider the case $a<0$, or alert the reader that this may be problematic. None of the books examined contained a discussion of the problematic case with fractions not in lowest terms when $a<0$. For example, $-2=(-8)^{\frac{1}{3}}=(-8)^{\frac{2}{6}}=\left[(-8)^2\right]^{\frac{1}{6}}=2.$

This illustrates why the extension of meaning of a^n to rational powers is so difficult: the result of computation can depend on the fractional representation which is highly unsatisfactory.

CONCLUSION

There is a significant variety of approaches to extending the meaning of exponential notation to include rational powers, including appeal to authority, deduction from specific cases, deduction from general cases and formal proof. Many books provide more than one justification, and many books blur the difference between a definition and a theorem. The most common approach was to accept rules of indices, as given, and to derive meaning by an analogy. In doing this most texts quietly ignored the possibility of a negative base.

There is a remarkable longevity in many of the texts considered, however there is gradual change over the period 1800–2000 with complex numbers increasingly accepted, and integrated. The tables of contents will show very similar overall structural approaches to algebra as a broader subject. The algebraic notation is also remarkably stable.

The texts increasingly recognize the interconnected roles of *trigonometry*, *algebra*, and *analysis*. A modern treatment of complex numbers requires substantial trigonometry. The modern exponential function forms part of analysis and usually comes relatively late in a university mathematics course. It also seems remarkable that only Euler introduces complex numbers first, and really tries to use them effectively before algebra is introduced at all. Only Durell, with his ordered pairs [0,1] for i, and SMP with their continuous model, are significantly different from the main corpus on the issue of rational powers.

It seems strange that surds continue to be introduced and manipulated after rational powers. Some books contain extensive practice of converting from one notation to the other. Why do these authors retain two notations for the same thing? Sometimes the surd notation and exponential notation are not defined to be exactly the same, which seems to be an especially subtle distinction and one which is certainly not universally accepted in mathematical practice.

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MODES OF REASONING IN ISRAELI SEVENTH GRADE MATHEMATICS TEXTBOOK EXPLANATIONS

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The goal of this study was to characterize the justifications and explanations offered in 7th grade Israeli textbooks for mathematical statements. Eight mathematics textbooks were analysed: six intended for the general student population, and two for students with low achievements. The justifications and explanations offered in the textbooks for eight selected mathematical statements were classified using the modes of reasoning framework (Stacey & Vincent, 2009). Our findings suggest that 7th grade Israeli textbooks provide justifications for all of the analysed statements (but one statement in one textbook), commonly using several modes of reasoning in explanations for each statement. Most of the explanations are included in the explanatory texts; few in tasks intended for student individual or small-group work. Almost every justification is deductive or empirical, yet different modes of reasoning are used for geometric and algebraic statements. Also, empirical justifications are more prevalent in textbooks intended for students with low achievements, whereas deductive justifications are typically offered in textbooks intended for the general student population.

Keywords: mathematical reasoning, mathematical justifications, mathematical explanations, Israel

INTRODUCTION

Proving, justifying, and explaining are important components of doing and learning mathematics. Yet, the extensive research on students’ conceptions of proof and ways of justifying mathematical claims reveals students' difficulties in understanding the need for justification and in distinguishing between deductive and other types of justifications (e.g., Harel & Sowder, 1998). One factor that considerably influences classroom instruction and students’ opportunities to learn mathematics is the textbook used in class (Eisenmann & Even, 2011; Haggarty & Pepin, 2002). To better understand students’ opportunities to develop the habit of justifying, and to learn how to justify in mathematics, this study examines the justifications and explanations to key mathematical statements offered in mathematics textbooks, centering on 7th grade Israeli textbooks.

BACKGROUND

Justifications of mathematical statements vary in nature, from intuitive, informal explanations to rigorous deductive proofs (e.g., Blum & Kirsch, 1991; Harel & Sowder, 2007; Sierpinska, 1994). Research on the issue of justifications in school mathematics attends to various aspects. Some researchers focus on the formality of justifications (Blum & Kirsch, 1991), others consider the community addressed (Sierpinska, 1994), and yet others focus on the proof scheme of justifications (Harel & Sowder, 2007). For example, Harel and Sowder's (2007) classification system distinguishes among three types of justifications: justifications based on deductive processes; on empirical processes; and on external sources such as a textbook, a teacher, or a famous mathematician.

Studies of the opportunities for students to read justifications and explanations in student textbooks show that textbooks justify mathematical statements in various ways and that valid proofs are rare. For example, building on Harel and Sowder's (2007) framework, Stacey and Vincent (2009) developed the *modes of reasoning* framework for analysing textbook explanations and used it to analyse Australian textbooks. Their framework includes seven modes of reasoning in textbook explanations:

- *Appeal to authority*: reliance on an external source of authority (e.g., a calculator).
- *Qualitative analogy*: reliance on a surface similarity with non-mathematical situations.
- *Experimental demonstration*: identifying a pattern after checking selected examples.
- *Concordance of a rule with a model*: comparing specific results of a rule and a model.
- *Deduction using a model*: a model that serves to illustrate a mathematical structure.
- *Deduction using a specific case*: an inference process conducted using a special case.
- *Deduction using a general case*: an inference process conducted using a general case.

Stacey and Vincent found that justifications offered in the analysed textbooks used several modes of reasoning, yet the students were given no indication regarding which can be classified as deductive proofs and which can only serve as supportive empirical evidence at best. Drawing on Stacey and Vincent's (2009) conceptual framework, Dolev (2011) analysed the modes of reasoning in justifications offered to three mathematical claims in six 7th grade Israeli textbooks (experimental version). She found that all the textbooks offered justifications for the sampled claims, at times using several modes of reasoning. Additionally, Dolev found a difference between the modes of reasoning used in algebra and in geometry – geometry claims were often justified by *deduction using a general case*, whereas algebra claims were generally justified in other ways. Similarly, Hanna and de Bruyn (1999) found differences between justifications in algebra and in geometry.

Our study builds on these studies, and expands their scope. Our research objective is to examine the justifications and explanations to key mathematical statements offered in mathematics textbooks, centering on 7th grade Israeli textbooks (approved version). We attend to three issues: (1) the modes of reasoning offered, (2) the nature of justifications of algebra and geometry statements, and (3) the nature of justifications in textbooks intended for the general student population and in textbooks intended for students with low achievements.

METHODOLOGY

Eight key mathematical statements were selected for analysis from the Israeli 7th grade mathematics national curriculum (Israeli Ministry of Education, 2009), across several curricular topics. Each selected statement was chosen because it contains a mathematical idea or concept that requires justification in the national mathematics curriculum, and is considered to be an important result in the mathematics education literature. The statements are listed in the following:

- The product of two negative numbers is a positive number.
- Performing a basic operation on both sides of an equation maintains their balance.
- Division by zero is undefined.
- The distributive property – for every three numbers a,b,c: a(b+c)=ab+ac.
- Angle sum of a triangle is 180^o
- The area formula for a trapezium with bases a,b and altitude h is $(a+b)h/2$.
- The area formula for a circle with radius r is πr^2.
- The corresponding angles between parallel lines are equal.

Analysis included all eight approved 7^{th} grade textbooks for Hebrew speakers, along with their accompanying teacher guides and supplementary materials. Six textbooks are intended for the general student population and two for students with low achievements. Data analysis included the textbook chapters introducing the statements – a total of 549 textbook pages; 48-94 pages (7.5-13%) from each textbook. We analysed the explanations and justifications in the explanatory texts and in tasks and problems in the related exercise pools. First, we identified distinct justifications of the statements in each section of the textbooks (i.e., explanatory texts and exercise pools). Then, we classified each justification for its mode of reasoning (following Stacey & Vincent, 2009) and compared frequencies of various classifications.

FINDINGS

A total of 156 justifications of statements were found in the textbooks. Comparison of the distinct justifications by textbook section reveals that, for each textbook, most of the explanations were included in the explanatory texts, and few in tasks intended for student individual or small-group work, as shown in Table 1. Moreover, no differences were found in this regard between textbooks intended for the general student population (A-F) and those intended for students with low achievements (G-H).

Table 1: Number of distinct justifications by textbook section for each textbook

	Textbook								
Section	A	B	C	D	E	F	G	H	Total
Explanatory texts	12	16	20	14	18	24	14	17	135
Exercise pools	12	3	0	5	0	0	1	0	21
Total	24	19	20	19	18	24	15	17	156

The textbooks provided justifications for all[1] analysed statements, commonly using several modes of reasoning in explanations for each statement. Figure 1 illustrates three distinct justifications in textbook A for the statement "The product of two negative numbers is a positive number" (translated from Hebrew): two of the mode *deduction using a special case* (Figures 1 (a) and (b)), and one of the mode *deduction using a model* (Figure 1(c)).

[1] Exception: one textbook provides a reminder for the area formula for a circle, without explanation.

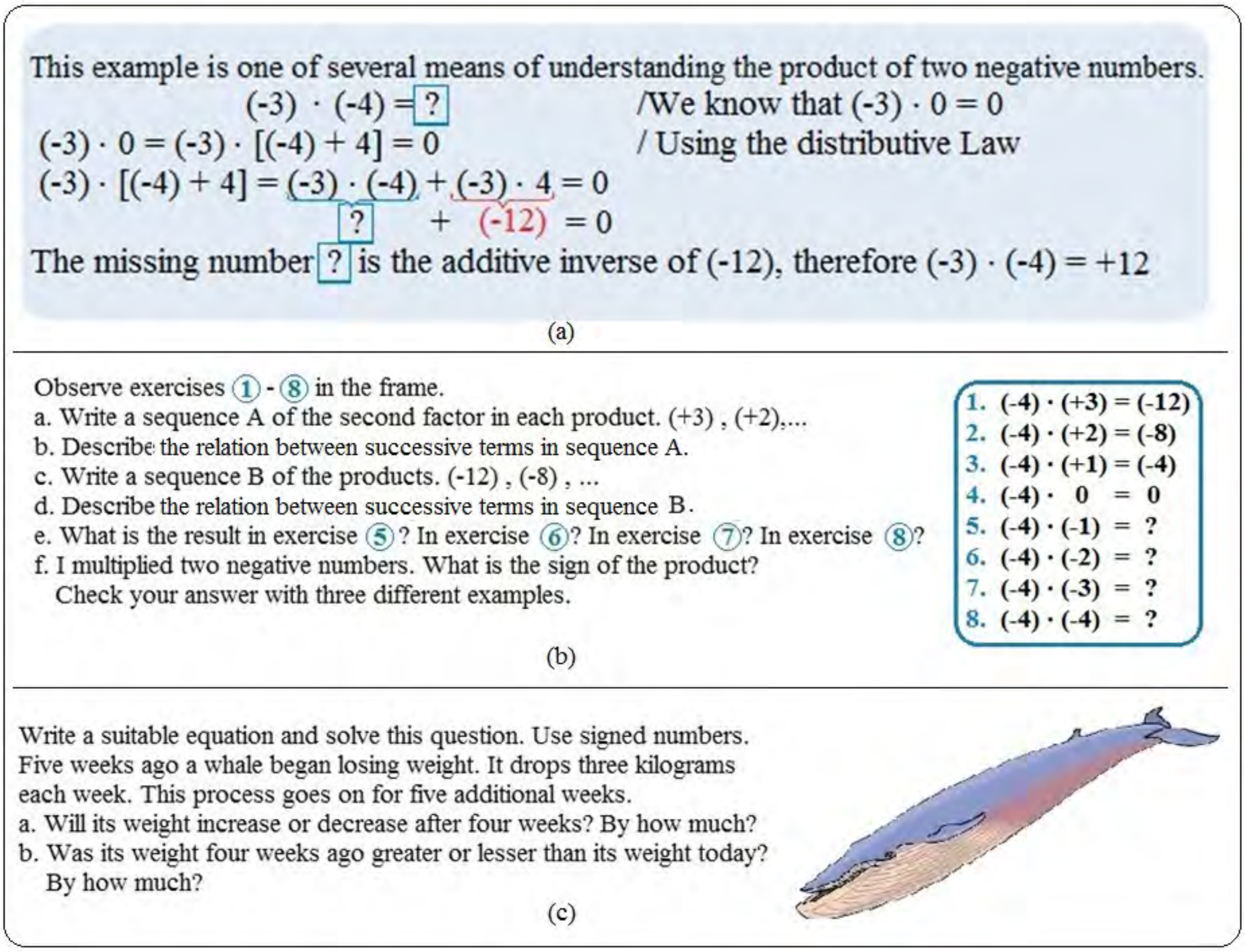

Figure 1: Different justifications and modes of reasoning of a single statement

Analysis of the modes of reasoning in the explanations offered in the textbooks revealed that algebra statements were typically justified by deductive modes of reasoning, whereas geometry statements were usually justified both by deductive modes of reasoning and by an empirical mode of reasoning (i.e., *experimental demonstration*). Still, *deduction using a general case* appears more in justifications of geometry statements than in those of algebra statements. This is illustrated in Figure 2, which presents the frequency of modes of reasoning in textbook explanations by in each textbook by content topic – algebra (Figure 2(a)) and geometry (Figure 2(b)).

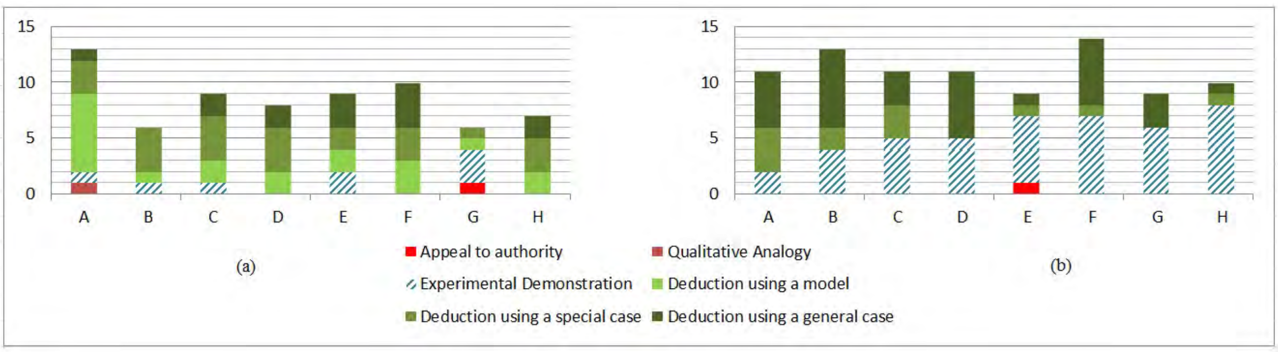

Figure 2. Frequency of modes of reasoning by textbook and topic: (a) algebra, (b) geometry

Comparison of the explanations offered in textbooks intended for the general student population with the explanations offered in textbooks intended for students with low achievements was conducted on six of the eight statements due to the textbooks' structure. Figure 3 illustrates the distribution of the modes of reasoning in textbook explanations for these six statements. We found a greater ratio of empirical justifications (i.e., *experimental demonstration*) in textbooks intended for students with low achievements (G-H) compared with textbooks intended for the general student population (A-F), and a greater ratio of deductive justifications in textbooks intended for the general student population compared with textbooks intended for students with low achievements.

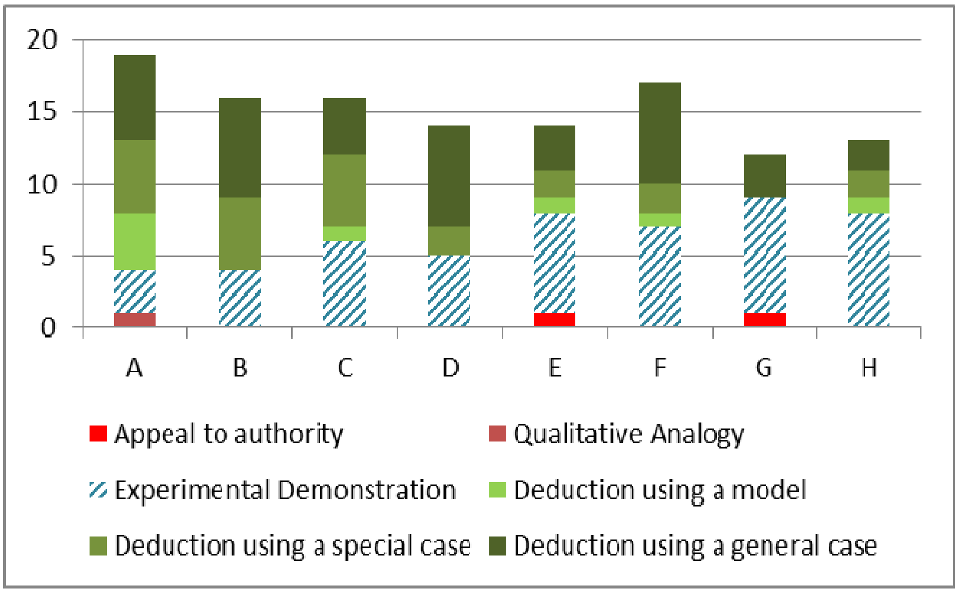

Figure 3. Frequency of modes of reasoning by textbook.

DISCUSSION

Our findings reveal that Israeli 7th grade mathematics textbooks provide justifications for all the analysed key statements (but one statement in one textbook), commonly using several modes of reasoning in explanations for each statement. These results are in line with Dolev's (2011) results, and correspond to Stacey and Vincent's (2009) results on similar topics in Australian textbooks. However, one of the seven modes of reasoning in Stacey and Vincent's framework – *Concordance of a Rule with a Model* – did not appear in any of the 156 justifications we identified. The inclusion of multiple modes of reasoning in textbooks appears to indicate attentiveness of the textbook authors to the complexity of developing mathematical understanding.

Additionally, we found that almost all explanations in the Israeli textbooks are deductive or empirical. This finding does not comply with the findings reported in Stacey and Vincent's (2009) study, where 17% of the explanations were neither deductive nor empirical.

Moreover, our study shows that justifications for geometry statements and for algebra statements typically used different modes of reasoning: deductive for algebra statements and both deductive and empirical for geometry statements. This finding is surprising at first glance considering the historic bias towards geometry as a subject suitable for teaching proof.

Yet, careful analysis shows that the mode closest to formal proof – *deduction using a general case* – appears more in justifications of geometry statements than in justifications of algebra statements. Similar results were reported in Dolev (2011).

Our study also shows that empirical explanations were more prevalent in textbooks intended for students with low achievements, whereas deductive explanations were more prevalent in textbooks intended for the general student population. These differences have the potential to considerably limit the opportunities of students with low achievements to learn how to justify in mathematics, because teachers often follow teaching sequences suggested by textbooks (Eisenmann & Even, 2011; Haggarty & Pepin, 2002). Additional research is needed in order to find how textbooks intended for higher grades address this issue.

Finally, a note about methodology: Our study shows that most of the justifications and explanations to the analysed mathematical statements in 7th grade Israeli textbooks were included in the explanatory texts. Yet, some explanations were embedded in the exercise pools intended for student individual or small-group work. Previous studies of the justifications and explanations offered in textbooks analysed only explanatory texts (e.g., Dolev, 2011; Stacey & Vincent, 2009). Our study suggests that including exercise pools in the analysis provides a more complete picture of the justifications offered in textbooks.

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THE EIFFEL TOWER AS A CONTEXT FOR WORD PROBLEMS IN TEXTBOOKS FOR SCHOOL MATHEMATICS AND PHYSICS: WHY AUTHORS HAVE A *LICENTIA POETICA* AND WHAT ARE POSSIBLE CONSEQUENCES FOR STUDENTS' LEARNING AND BELIEFS?

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The Eiffel Tower is a popular context of word problems in school mathematics and physics textbooks. A detailed analysis of a few formulations of those problems, carried out within the framework of Palm's theory of authentic tasks (2009), shows that they are examples of invented events which can rarely or never happen in real world. In addition, in some cases the mathematical model of the Eiffel Tower used for calculations implies it only has one dimension (the height). Being so, one is obliged to ask two important questions: (1) why do textbook authors have such a licentia poetica *for inventing artificial contexts as supposed examples of mathematics and physics applications in real world? (2) what are possible consequences of artificial problems contextualization for students' learning and beliefs? The answer to the first question is related to different aspects of the "teaching culture", going from historic examples of contextualized problems formulated by Fibonacci to the absence of rigorous quality control mechanisms for textbook production, similar to those applied in research journals. The base for answering the second question is the idea that the students behave as sense-seekers in their authentic real-world activities. In school setting, when they are exposed systematically to non-authentic learning practices, the students are prone to conclude that school mathematics and physics deal with problems that are senseless from the point of view of normal human beings. In this way, known "suspension of sense-making" syndrome might be an understandable consequence of artificial problems contextualization.*

Keywords: word problems, problem posing, physics, student beliefs

INTRODUCTION

A recent survey (Fan, Zhu & Miao, 2013) revealed that important progress has been made over the last few decades in mathematics textbook research. The major achievement has been concentrated in the areas of textbook analysis (including textbook comparison), and the use of textbooks in teaching and learning. Being so, now would be incorrect to claim that the research related to mathematics textbook is 'scattered, inconclusive, and often trivial' as it seemed to some researchers six decades ago. Nevertheless, the development of research on mathematics textbooks has not been uniform, leaving some areas with insufficient research attention and results.

In addition, mathematics textbook research as a field of research is still at an early stage of development, and its philosophical foundations, theoretical frameworks and research methods for disciplined inquiry are still lacking or fundamentally underdeveloped (Fan, 2013). To make a further progress, Fan (2013) suggests that it would be necessary to put forward a conceptual framework which conceptualizes textbooks as an intermediate variable in the context of education and hence defines mathematics textbook research as disciplined inquiry into issues about mathematics textbooks and the relationships between mathematics textbooks and other factors in mathematics education.

According to Fan (2013), to advance the field of mathematics textbook investigations, researchers need to expand research issues from descriptive issues such as how a special topic is treated in textbooks to correlational issues and, especially, causal issues including how they are affected by other factors and how they affect other factors concerning education.

In this article, I try to call attention, by citing a few examples (some of them are incredible perverse), to disturbing issue of artificial contextualization of textbook problems in school physics and mathematics. Although the issue of inadequate image of ‘mathematics applications’ in education was raised many times before (Pollak, 1968; Pollak, 1969; Pollak, 1978; Korsunsky, 2002), my hope is that these examples, all related to the same real-world context (Eiffel Tower), taken together might cause some ‘waking up’ effect in the mathematics textbook research community.

THE EIFFEL TOWER AS A CONTEXT FOR WORD PROBLEMS IN SCHOOL PHYSICS AND MATHEMATICS

Before presenting and analyzing the selection of examples related to it, it is in places to say a few words about the famous tower. The Eiffel Tower (Figure 1) is an iron lattice structure located on the Champ de Mars in Paris. It was named after the engineer Gustave Eiffel, whose company designed and built it in 1889.

Figure 1. The Eiffel Tower

Tourists can visit the tower’s three levels. There are restaurants on the first and second levels. The highest, third observatory platform is 276 m (906 ft) above street level. Visitors can ascend by stairs or by elevator to the first and second levels. About 300 steps are needed to climb from street level to the first platform and from there to the second level. To get to the third platform it almost always necessary to use the lift, a fact that implies a strict security control, which makes impossible that someone with a bow and arrow, or with a cannon ball, to get to the top of the tower (see examples cited below).

As with Pollak (1978) and Korsunsky (2002), I do not reveal the identity of the authors who created and published the cited examples. This is in resonance with my view that artificial problem contextualization, found in mathematics and physics textbooks, is not just a personal extravaganza of a few eccentric individuals but a serious collective phenomenon which should be properly researched.

Examples from Physics textbooks

Launching an arrow upwards: The Eiffel Tower has a height of 335 meters. How many seconds will pass until an arrow, launched vertically upwards from the tower with a speed of 4,000 cm per second, reach the ground?

Superman saving Lois Lane: Superman is flying at treetop level near Paris when he sees the Eiffel Tower elevator start to fall (the cable snapped). His x-ray vision tells him Lois Lane is inside. If Superman is 1.00 km away from the tower and the elevator falls from a height of 240 m, how long does have to save Lois, and what must his average speed be?

Deadly free fall of a boy: The Eiffel Tower is 300 m high. A boy at the top slips on a banana skin and falls over the side. How long has he got to live?

Launching an arrow to hit a falling melon: You decide to drop a melon from rest from the first observation platform of the Eiffel Tower. The initial height h from which the melon is released is 58.3 m above the head of your French friend Pierre, who is standing on the ground right below you. At the same instant you release the melon, Pierre shoots an arrow straight up with an initial velocity of 251.m/s. (a) At what height above Pierre's head do the melon and the arrow collide? (b) What are the velocities of the melon and the arrow at the moment of collision?

Free-falling scientists making measurements: A beam of X-rays having energies of 5.9 keV is fired from the top of the Eiffel tower, vertically downwards. The height of the Eiffel tower is 324 m. A scientist makes a measurement of the wavelength of the X-rays as he jumps off the tower in the direction of the beam. He falls freely to the base of the tower (assume that there is no air resistance), a just before he hits the ground measures the wavelengths of the X-rays again. He discovers whilst still falling that the wave length of the X-rays in his reference frame is exactly the same as it was at the top of the tower. What is the wavelength of the X-rays at the top of the tower measured by the scientist before he jumps?

Examples from Mathematics textbooks

Cannon ball falling down: How many seconds would it take for a cannon ball to reach the ground if it were dropped from the top of the Eiffel Tower, which is 984 feet tall? How many seconds would it take for the cannonball to reach the ground if it were dropped from a point that is halfway to the top?

The Statute of Liberty on the top of the tower: The Eiffel Tower is about 1063 feet high. The Statue of Liberty along with its foundation and pedestal is about 305 feet. If you could put the Statue of Liberty on top of the Eiffel Tower, how high up in heaven will two monuments reach?

Superman saves Lois Lane (again): Superman is flying at treetop level near Paris when he sees the Eiffel Tower elevator start to fall (the cable snapped). His x-ray vision tells him Lois Lane is inside. If Superman is 2 km away from the tower, and the elevator falls from a height of 350 m, how long does Superman have to save Lois, and what must be his average velocity? Solve this problem with and without assuming air resistance.

Spitting from the top of the tower: While up on the Eiffel Tower in Paris, France, you spit straight down (even though you will be arrested by Paris Police). Nonetheless, the velocity (speed) of your spit is 0.75 meters per second (m/s). How long will it take your spit to reach the pavement 327 meters (m) below? Set up the Quadratic Equation. Solve the Quadratic Equation.

A detailed analysis of the formulations of these problems, carried out within the framework of Palm's theory of authentic tasks (Palm, 2009), shows that they are examples of invented events which can rarely or never happen in real world. What is worst is a dangerous possibility that the types of the events and questions suggested in these 'problems' have very little or nothing to do with what normal persons would think as something worth doing or knowing.

In addition, in some cases the mathematical model of the Eiffel Tower used for calculations implies it only has one dimension (the height). It is impossible to launch an arrow vertically upwards and to think that its motion upwards and downwards motion would be the same as if the tower does not exist.

Being so, one is obliged to ask two important questions:

1) Why do textbook authors have such a *licentia poetica* for inventing artificial contexts as supposed examples of mathematics and physics applications in real world?
2) What are possible consequences of artificial problems contextualization for students' learning and beliefs?

WHY ARE ARTIFICIAL CONTEXTULIZATIONS TOLERATED?

The answer to the first question above is related to different aspects of the 'teaching culture', going from historic examples of contextualized problems formulated by Fibonacci (lion climbing and sliding 1,575 days in a well or lion, tiger and wolf eating a sheep) to the absence of rigorous quality control mechanisms for textbook production, similar to those applied in research journals.

Some authors very likely think that humorous formulations (for example, skipping from the top of the tower) might be more interesting to the students and could eventually increase learning. In fact, some research results could lead to such conclusion.

> When properly used, humor can be an effective tool to make a class more enjoyable, reduce anxiety, and improve the learning setting. The 'ha-ha' of humor in the classroom may indeed contribute to the 'aha!' of learning from the student. (Garner, 2006)

> Humor appropriately used has the potential to humanize, illustrate, defuse, encourage, reduce anxiety, and keep people thinking (Torok, McMorris & Lin, 2004).

An additional example of supposedly funny problems might be the following one:

> A student at a window on the second floor of a dorm sees his math professor walking on the sidewalk beside the building. He drops a water balloon from 18.0 m above the ground when the professor is 1.00 m from the point directly beneath the window. If the professor is 170 cm tall and walks at a rate of 0.450 m/s, does the balloon hit her? If not, how close does it come? (Wilson, Buffa & Lou, 2007, p. 64, Problem 104)

To add fine details to a potential definition of what is funny in the case of physics and mathematics learning, it would be interesting to get some research-based answers on the questions:

- How many students find this and similar problems funny and why?
- How many students do not find those problems funny and why?
- To what extent do these problems lead students to change their negative image of physics and mathematics?
- To what extent do these problems result in an improvement in physics and mathematics learning by the students?

THE POSSIBLE CONSEQUENCES OF ARTIFICIAL CONTEXTUALIZATION

The base for answering the second question is the idea that the students behave as sense-seekers in their authentic real-world activities. In school setting, when they are exposed systematically to non-authentic learning practices, the students are prone to conclude that school mathematics and physics deal with problems that are senseless from the point of view of normal human beings.

In this way, known ‘suspension of sense-making’ syndrome might be an understandable consequence of artificial problems contextualization or, in other words, some kind of auto-defence to keep undamaged mind for out-of-school uses.

An informal research study (Slisko & Dykstra, 2011) was carried out by analyzing the comments, provoked by a YouTube video in which an allegedly humorous formulation of a textbook optics problem has been staged. The ‘funny’ problem was the following:

> You lie on your back in a bathtub, without any water in it, and look at a cup resting on a soap dish. Now somebody fills up the tub with water so the water level is over your head, exactly at the top of the soap dish and the bottom of the cup. Sketch what your view of the cup and soap dish is after the water is added and before you drown. To help you do this, you should draw a side view, showing how the rays from the cup and soap dish reach your eye (Falk, Brill & Stork, 1986, p. 71)

Many comments showed that expectations ‘funny problems makes learning enjoyable’ might be quite wrong, revealing that the above ‘humorous formulation’, in fact, caused profoundly negative attitudes towards physics, physics learning and physics textbook authors.

It seems that a research agenda about learning and affective effects caused by artificial contextualization of mathematics problems in textbooks and internet should be formulated very soon.

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THE BROKEN-TREE PROBLEM: FORMULATIONS IN MEXICAN MIDDLE-SCHOOL TEXTBOOKS AND STUDENTS' CONSTRUCTIONS OF THE CORRESPONDING SITUATION MODEL

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The broken-tree problem is one of the most popular tasks in mathematics textbooks, both on large temporal and geographic scales. This article has two parts. First, we have analyzed how this problem was formulated in seven Mexican middle-school textbooks. The formulations differ in a few features: (a) the type of the broken object; (b) mention of its historic origin; (c) presence or absence of a drawing with the problem wording; (d) explicit drawing task as a part of solution path. Second, we used one formulation without explicit drawing task to explore middle school students' performances when they were asked to draw how they imagined the situation described in the problem. The results show that the construction of the situation model was not an easy task for many students. In fact, only one out of 30 involved students was able to correctly construct the situation model. The implication of these results is: construction of a situation model and its simplification and idealization, leading to the related mathematical model, should be reserved as an explicit task for students.

Keywords: broken-tree problem, mathematical drawings, situation model, mathematical modelling, Mexico

INTRODUCTION

Many mathematical problems have quite a long and rich poly-cultural history (Smith, 1917; Singmaster, 2004; Swetz, 2012). A good example is the "broken-tree problem", which is still one of the most popular tasks in mathematics textbooks on large temporal and geographic scales. Although the bibliography of the manuscripts and books related to this problem is really amazing (Singmaster, 2004), Smith, in his famous "History of Mathematics" (Smith, 1958), mentioned explicitly only two versions of the problem in their English translations. The first version is attributed to Chinese mathematician Ch'ang Ts'ang who lived in 2nd century BCE (Smith, 1958, p. 139):

> Among his problems is that of finding the height of the trunk of a tree, the upper part of which was 10 feet high but has fallen over and reaches the ground 3 feet from the base.

The second version is related to the seventh-century Indian mathematician Brahmagupta (Smith, 1958, p. 159):

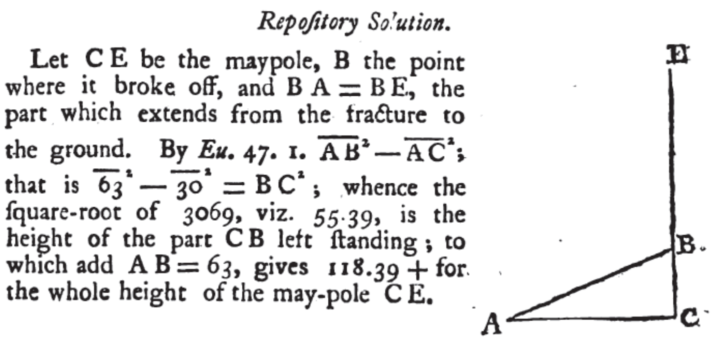
> A bamboo 18 cubits high was broken by the wind. Its tip touched the ground 6 cubits from the root. Tell the lengths of the segments of the bamboo.

The third version, one which appeared in the arithmetic book of Calandri in 1491, was presented by Smith in Italian, without bothering himself with English translation. It is might be understandable because it served as an example of illustrated mathematical problems (Smith, 1958, p. 255). Here comes an English translation of the problem from a later edition of Calandri's (1543, p. 15) book:

> There is a tree on the bank of a river, that is 50 fathoms tall and the river is 30 fathoms wide. By chance, it broke down at such a place that the top of the tree touched the bank of the river. I want to know how many fathoms of it were broken and how much it stands it straight.

The drawing of the problem situation in the Calandri's book is given in the Figure 1.

Figure 1. The drawing corresponding to the broken-tree problem in Calandri´s book



Repository Solution.

Let CE be the maypole, B the point where it broke off, and BA = BE, the part which extends from the fracture to the ground. By *Eu.* 47. 1. $\overline{AB}^2 - \overline{AC}^2$; that is $\overline{63}^2 - \overline{30}^2 = BC^2$; whence the square-root of 3069, viz. 55.39, is the height of the part CB left standing; to which add AB = 63, gives 118.39 + for the whole height of the may-pole CE.

Figure 2. The repository solution of the problem

It is important to note that the drawing is surprisingly imprecise because the distance between the top of three touching the ground and the vertical part of the tree is much bigger than the wide of the river. If Calandri himself approved the artist's work, it might mean that he didn't care about this discrepancy because he saw the drawing only as a decorative appendix. From the mathematical modelling framework, whose first steps will be briefly described later, there is a complete disconnection between an imprecise visual representation of the problem situation and an implicit mathematical model on which calculations were based on.

Two and half century after Calandri, the problem formulation and solution changed a lot, as can be seen in the "Diarian Repository" (A Society of Mathematicians, 1774). The Question LXII, posed by Mr. William Taylor for the year 1715, is a rhymed formulation of a variant of the broken tree problem (p. 65):

"As I was walking out one day, / Which happen'd on the first of May,

As luck would have it, I did spy, / A maypole raised up on high,

The which at first me much surpriz'd, / Not being before-hand advertiz'd,

Of such a strange uncommon sight; / I swore I would not stir that night,

Nor rest content, until I'd found / Its height exact off the ground:

But when these words I just had spoke, / A blast of wind the maypole broke, Whose broken piece I found to be / Exact in length yards sixty-tree;

Which, by its fall, broke up a hole, / Twice fifteen yards from off the pole;

But this being all that I can do, / The maypole now being broke in two

Unequal parts; to aid a friend, / Ye ladies, pray an answer send."

The repository solution of the problem is given in the Figure 2.

As it can be noted from the Figure 2, corresponding drawing contains no real-world details, being it a totally opposite approach when compared with the drawing in the Calandri's book. Here a pure mathematical model of the situation was drawn and used to make clearer the solving strategy used and calculations carried out.

THE BROKEN-TREE PROBLEM IN MEXICAN MIDDLE-SCHOOL MATHEMATICS TEXTBOOKS

For this article, we have briefly analyzed how the broken-tree problem or some of its variants were formulated in seven Mexican middle-school mathematics textbooks (correspond the grade IX). The formulations differ in various features:

a) Type of the broken object;
b) Mention of its historical origin;
c) Presence or absence of a drawing with the problem wording;
d) Explicit drawing task for students as a suggested part of solution path.

Regarding the type of the broken object used in problem formulations, the results are the following:

In three textbooks "a tree" is used (Mancera Martínez, 2008; Waldegg et al., 2008; Briseño et al., 2007), while in three textbooks, the problem is related to "a bambú" (Pérez Rivas y Pérez Ruiz, 2008; Arteaga García y Sánchez Marmolejo, 2008; Sánchez Sandoval, 2008). Only in one textbook, the problem was formulated for "a light pole" (Farfán Márquez et al., 2008).

Authors of three textbooks do mention that the problem has a long history (Pérez Rivas y Pérez Ruiz, 2008, pp. 227-228; Arteaga García y Sánchez Marmolejo, 2008, p. 75; Sánchez Sandoval, 2008, p. 216). All of them used "a bamboo" in the problem formulations.

In three textbooks, the problem formulations are complemented with a drawing that refers to the problem situation (Briseño et al., 2007; Sánchez Sandoval, 2008; Waldegg et al., 2008).

The authors of four textbooks did not consider necessary to provide students with a visual representation of the problem situations (Pérez Rivas y Pérez Ruiz, 2008; Arteaga García y Sánchez Marmolejo, 2008; Mancera Martínez, 2008; Farfán Márquez et al., 2008). Nevertheless, only in one of those textbooks, the students were asked explicitly to generate a visual representation of the situation as a step in problem solving (Farfán Márquez et el., 2008).

STUDENTS' DRAWING TASKS RELATED TO THE BROKEN-TREE PROBLEM

The mentioned difference in the treatments of the drawing task in the solving procedures of the broken-tree problem was our main inspiration for this small-scale pencil-and-paper initial research. We wanted to explore experimentally whether the drawing the problem situation is an easy or difficult task for middle-school students.

Between problem formulations with "a tree", "a bamboo" and "a light pole", we have chosen the first one. So, the students' drawing tasks were related to the following formulation (Mancera Martínez, 2008, p. 333):

> A tree was broken by the wind in the way that its two parts form with the Earth a rectangle triangle. The upper part forms with the ground an angle of 350, and the distance, measured over the ground, from the trunk to the fallen peak is 5 m. Find the height that the tree had.

Accordingly to the rapidly-growing modelling approach to mathematics education, the first steps in mathematical modelling cycle are constructions of a "situation model" and "a mathematical model" (Borromeo-Ferri, 2006; Blum & Borromeo-Ferri, 2009). The first should be constructed out from the real situation, while the second comes out from the "situation model" (via a in intermediary "real model"), by performing adequately demanding mathematical-thinking processes of "simplifying", "structuring" and "mathematising".

In this research, the students had two drawing tasks. In the first one, they were asked to draw, "with all (important) details", the situation described in the problem formulation. The expected results were imagined "situation models". The second task was designed to explore students' ideas about the corresponding "mathematical models":

> Eliminating all what is unnecessary, draw only the triangle that would help you solve the problem. Add also the numbers and data that appear in the problem formulation.

STUDENT SAMPLE AND RESULTS

Our sample was made of 30 middle-school IX-grade students (mean age 15.5 years), who should have had curriculum-required trigonometry knowledge, although they were not supposed to find the numerical answer.

The main result is rather alarming: Only 10 of 30 students were able to draw the situation described in the problem text. From those 10 students, only one could relate in a correct way the numerical data with the drawing.

The nine students, although they were able to draw correctly two parts of the tree and the ground, introduced the numerical data (350 and 5 m) wrongly.

Twenty students were not able to draw to situation correctly. Some of them did not draw the broken tree (Figure 3) or, if the broken tree was drawn, its top did not touch the ground (Figure 4).

Modest good news is that even the worst performing students apparently understand that the mathematical model, necessary for further problem solving steps, should be got in a simplification processes in which irrelevant details of the situation model should be left out (Figure 5 and Figure 6).

CONCLUSIONS AND IMPLICATIONS

This initial research shows that only very few students are able correctly to imagine and draw the situation stipulated in the broken-tree problem. Too many of them have alternative interpretations of the problem formulation, leading them to incorrect drawings and numerical information attributions.

Figure 3. Unbroken tree and an unrelated triangle

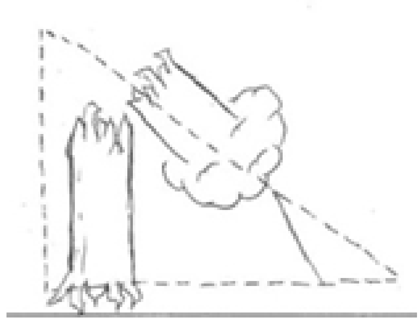


Figure 4. Broken tree and a loosely related triangle

Figure 5. Situation and mathematical models, drawn by a student (note that the angle of 350 is formed by the broken tree parts and that the “5 m” is added ad hoc in the mathematical model)

Figure 6. Situation and mathematical models, drawn by a student (note that the triangular shape is connected with the tree’s body, the top of the tree is cut vertically, and numerical data in the mathematical model are added ad hoc)

The implication of these results is that the construction of situation model and its simplification and idealization, leading to the related mathematical model, should be reserved as explicit, systematic and carefully designed tasks for students. Although in some simple arithmetic problems students’ dealing with the situation model might seem unrelated to their performance (Voyer, 2010), in trigonometric problem solving, students would be unable to get an adequate mathematical model without construction of the corresponding situation model.

To help students grasp the fundamental relationship between a situation and mathematical models, common textbook practice of giving to students their uncommented mixture should be avoided.

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MODERN DESCRIPTIVE GEOMETRY SUPPORTED BY 3D COMPUTER MODELLING

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In my research, I investigate innovative methods of explaining complex concepts in descriptive geometry. These novel methods are employed in my upcoming textbook on descriptive geometry for undergraduate students. The innovation in explanation and didactic methods includes 3D computer modelling and interactive software visualization. In this paper, I present my recent advances in teaching aspects of several descriptive geometry topics – parallel and central projections, especially linear perspective or the geometry of curves and the geometry of surfaces in technical practice. For selected topics, I provide examples of usage of 3D computer modelling. My aim is to stimulate students' interest in the study of geometry, motivate them, improve their understanding of geometry, innovate the methods of teaching geometry, achieve better results in examinations and put emphasis on practical use of geometry. I would like to attract students to the traditional topics of descriptive geometry by using modern methods. At the same time, since I would like to motivate students to think about concepts in descriptive geometry, the suggested teaching aids should not suppress independent thinking. In other words, these aids should not be designed so as to solve all questions for my students – some should be left for the students to address. Surfaces used in technical practice are very suitable for presentation by means of 3D printing. Therefore, I plan to model these surfaces in the 3D modelling computer software in cooperation with my students so that these computer models can subsequently be printed on a 3D printer to foster spatial imagination.

Keywords: geometry, curves and surfaces, 3D computer modelling, 3D printing

INTRODUCTION AND MOTIVATION

Descriptive geometry is an area of classical geometry dealing with the representation of three-dimensional objects in two dimensions where 3D computer modelling and interactive software visualization can be applied with potentially significant impacts. Hence, the typical task in descriptive geometry is to represent three-dimensional objects on a two-dimensional display planar surface and to reconstruct 3D objects from the two-dimensional result of the projection. Descriptive geometry deals with those representations which are one-to-one correspondent. In order to gain deep understanding of descriptive geometry it is necessary to have knowledge of the fundamentals of geometry, the properties of geometrical objects in the plane and in the space, and their relations. This means that, in addition to geometrical projection, descriptive geometry should focus on special types of technically important curves and surfaces in engineering practice.

In general, geometry can be conceived as an independent discipline comprising various branches (descriptive, Euclidean, differential, algebraic, no-Euclidian, computational, applied geometry and so on) and it also forms the basis for many modern applications. The motivation for studying geometry can be found in building practice, engineering and construction practice, architectural and industrial design, production industries, export of real interiors and exteriors into the virtual worlds of computer games, digitization of real objects by 3D scanning, digital surface reconstruction from point clouds, replication of the shapes of real objects using 3D printing, computer graphics and many more, (Pottmann et al., 2007). The common basis of all these modern applications is the combination of geometric principles and knowledge. Applied methods are often based on elementary geometry.

From a historical point of view, the development of descriptive geometry reached its greatest height in the last century. Nevertheless, even despite today’s innovative approaches and continuous development of modern computer technology and equipment, descriptive geometry has not lost its importance. The role of descriptive geometry in practice is irreplaceable in such branches in which correct visualization is crucial. Owing to the fact that all of the mentioned application fields are dependent on clear illustrations and visualization, descriptive geometry has become the language of designers, engineers and architects. Overall, geometry in the plane and in the space, i.e. the properties of geometrical objects and their relations, form a part of many modern and contemporary scientific fields.

Geometry represents one of the highly demanding fields of mathematical science which require logical thinking and which also strongly stimulates spatial imagination. The study of geometry, and especially descriptive geometry, represents an ongoing challenge in terms of research and practice.

On the top level, the paper is organized into two parts. The first part explains possible novel methods of teaching descriptive geometry which include 3D computer modelling and interactive software visualization and presents new study materials and web support for descriptive geometry. This part is largely a summary of the existing concepts used in my lessons. The second part contains the main contribution of my work: the description of the upcoming textbook on descriptive geometry for undergraduate students. The conclusion is devoted to my future work and research in geometric fields.

3D COMPUTER MODELLING IN TEACHING AND STUDYING GEOMETRY

Descriptive geometry, and geometry in general, is a rather unpopular subject in secondary and undergraduate education due to its level of difficulty. It has been an unfortunate tendency in the recent years in the Czech Republic that the interest in studying this engaging mathematical discipline in on the decrease. In some cases, teachers of mathematics also show a lack of interest in teaching geometry. But primarily, it is necessary to offer education of geometry of a high quality and sufficient degree at elementary schools because the development of spatial imagination in early childhood is crucial.

The study of descriptive geometry includes, both at secondary schools and colleges, sketching and drawing activities. It may seem to be useless due to widespread availability of computers and modern software. Of course, we have to follow general trends and adapt teaching methods to the real practice, which, however, does not imply that classical drawing is outdated. We rely on these tools when developing of our initial ideas and finding solutions to geometrical problems. Drawing and sketching helps us develop our precision skills and patience.

Through modernization of descriptive geometry to better results

Currently, computer aided design (CAD) is a common tool used in the process of designing, design documentation and construction for modelling and drawing, and generally throughout the entire design process, (Farin et al., 2002). There exist professional graphics software applications and environments which provide the required user input tools, and speed up production. Similar software can be used in teaching traditional geometric subjects, including descriptive geometry. As there is a wide range of inexpensive or free software applications for geometry and mathematics, it is not necessary to work with expensive CAD applications.

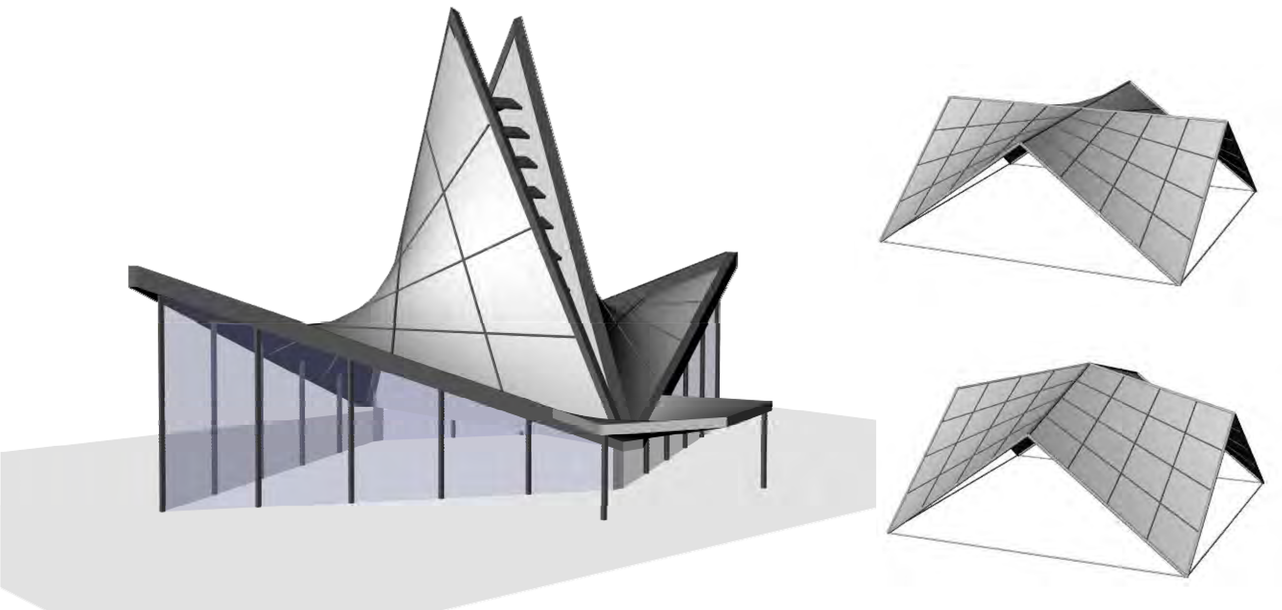


Figure 1. Example of 3D models created in Rhinoceros

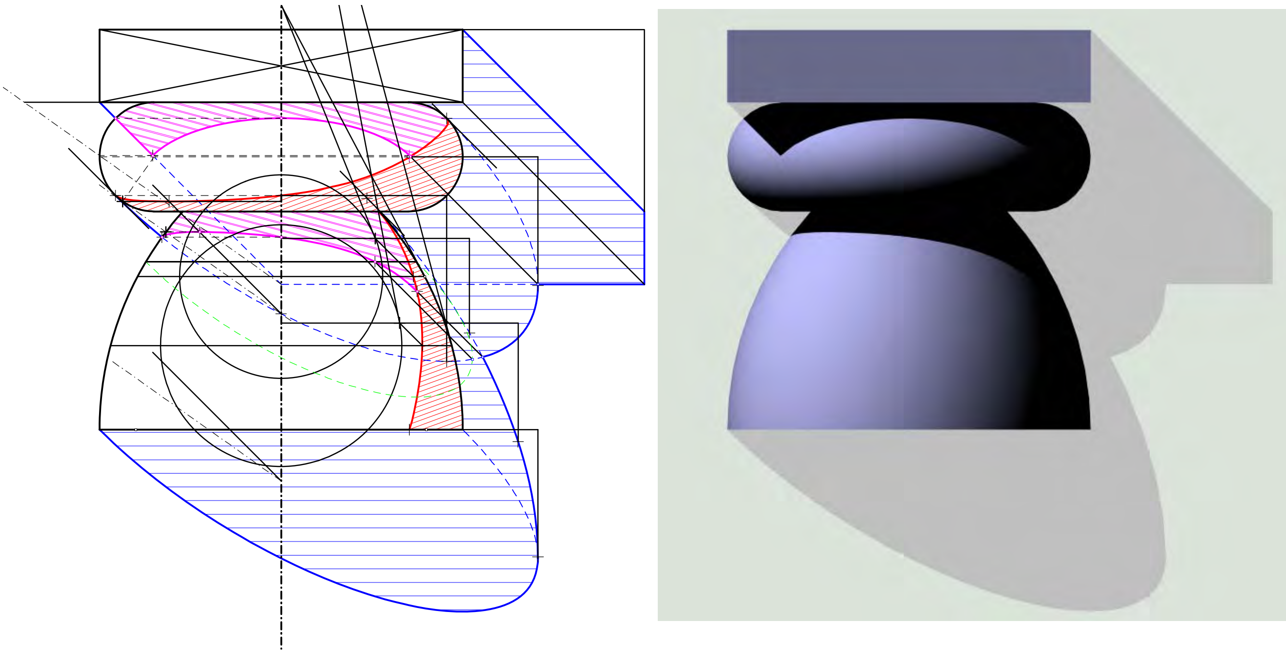


Figure 2. Examples of computer drawings created in Rhinoceros

In order to provide insight into more complex geometric problems and to increase the interest in geometry, I have integrated 3D computer modelling in my descriptive geometry lessons at the Faculty of Mathematics and Physics at Charles University in Prague. I work with the *Rhinoceros* (*NURBS Modelling for Windows*) software which is a commercial NURBS-based 3D modelling tool commonly used in the process of designing, design documentation and construction. I use Rhinoceros to create 3D models of geometric objects and situations in the space. It should also be noted that if we work with 3D modelling software, we can change the view of a designed object and see spatial geometric objects from another perspective which provides a clearer idea of the object. Examples of some 3D models are provided in Figure 1. We also use Rhinoceros to draw up constructions in the plane. As has already been pointed out, we do not intend to abandon traditional hand drawing methods because computer drafting is not efficient in developing our skill and thoroughness. Computer drafting is a modern auxiliary method which is also capable of yielding more precise results. Examples of computer drawings are shown in Figure 2.

The use of modelling and graphics software in teaching geometry increases students' interest in the subject and ensures their active involvement in the lessons, which is evident from the reactions of my students and also from their interest in these issues when dealing with their seminar projects or bachelor and master theses. 3D computer modelling is also an efficient aid in innovating the teaching of geometry and achieving better results.

I have been seeking to establish a stronger connection between descriptive geometry and its practical application and the extension of descriptive geometry with knowledge of computer graphics and computer geometry. The integration of descriptive geometry with 3D computer modeling appears to follow as a logical step.

Electronic study materials and web support for descriptive geometry

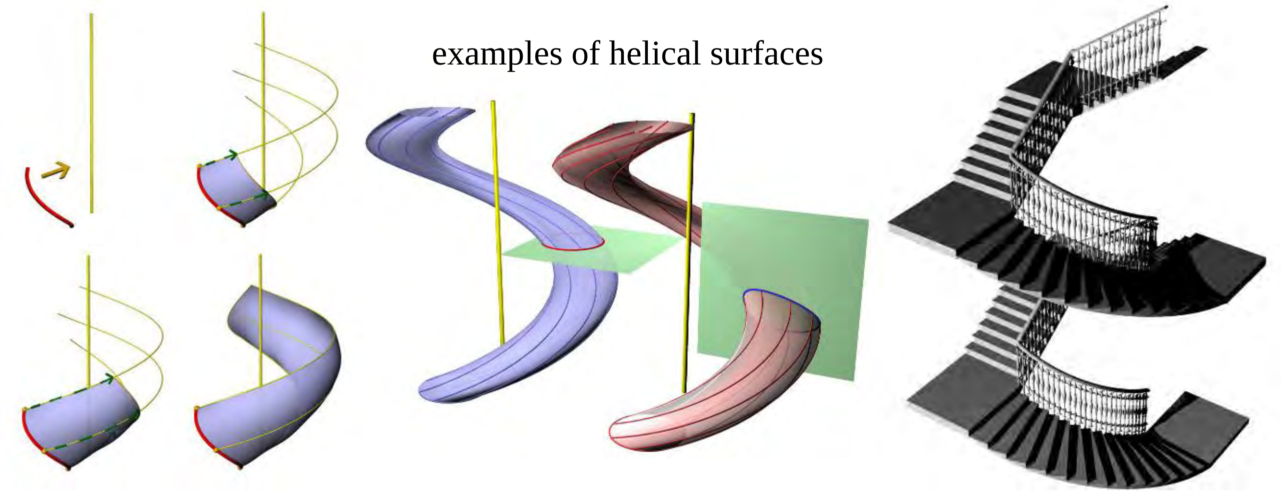
I have been gathering all of the aforementioned outputs obtained during the preparation of descriptive geometry lessons to create electronic collections of examples as well as for the purposes of new electronic methods of study of materials that relate to various geometric topics. All of these outputs are published on the website http://www.surynkova.info/, (Surynková, 2014). The site is continuously updated and it is intended not only for my students but also for everybody who is interested in geometry (some of the links are in English). New study materials and examples are dedicated to geometric constructions; there are also 3D computer models, examples of students' works and many more.

NEW TEXTBOOK ON DESCRIPTIVE GEOMETRY DESIGNED WITH 3D COMPUTER MODELING

This paper explicitly addresses the content and the design of a new printed textbook on descriptive geometry which I have been working on. This textbook is primarily dedicated to geometric topics such as curves and surfaces, solids, their definitions and properties, their parallel and central projections and the geometry of shadows. The textbook is intended mainly for students of the Faculty of Mathematics and Physics of Charles University in Prague and the first edition is planned to be published in Czech. The textbook will be illustrated using 3D computer modelling and modern software visualizations; I plan to use exclusively the already finished outputs. The important part of the publication is the collection of examples with solutions and examples for testing purposes.

Case study for textbook chapters

Let us now focus on some parts of the planned chapters in the upcoming book, and describe its expected design. I am currently working on the theoretical aspects of special groups of surfaces used in engineering practice. The book will define each regular geometric surface and introduce its properties. Let us, for instance, look at an example of a part dedicated to helical surfaces; the concept of the chapter is as follows.



the determination of helical surfaces defined by an axis and screwing spatial curves

practical application of helical surfaces

Figure 3. The illustration of helical surfaces

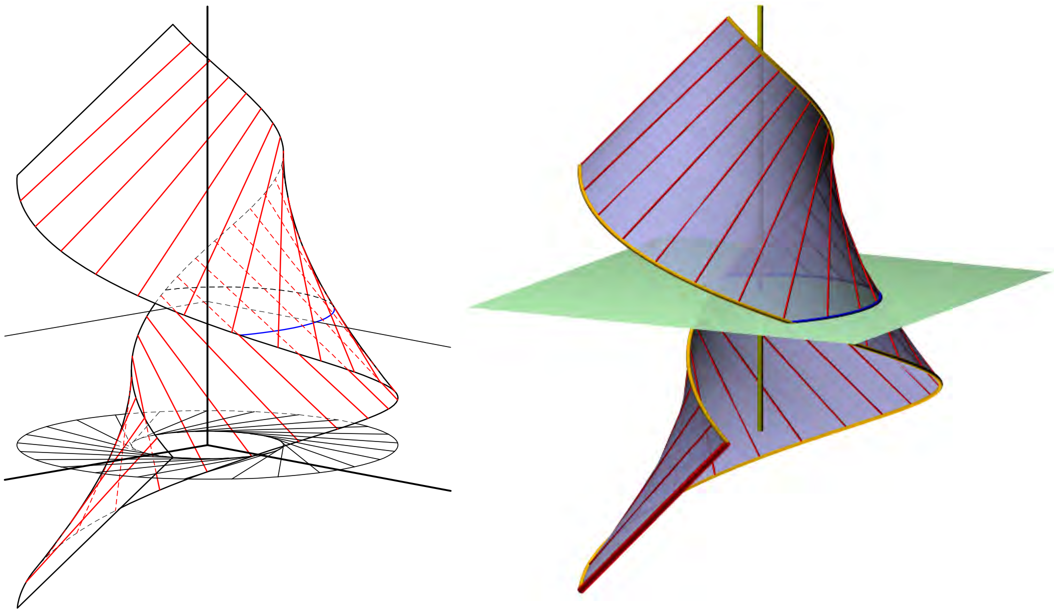


Figure 4. The result of the projection of helical surface and the situation in the space

First, a theoretical explication regarding the determination of helical surfaces is provided, accompanied with illustrations from the 3D computer modelling software. It is assumed that the source files of most pictures from the textbook are available on the attached removable media to allow practical exercises regarding the properties of the discussed surface or spatial situations directly in the software. The illustration of helical surfaces with a brief description is shown in Figure 3.

The second part of each chapter is devoted to parallel and central projections of the studied surfaces, accompanied with a typical example including a detailed step-by-step solution and illustration. The typical task is to construct a parallel or perspective view (a two-dimensional image) of a particular surface. Figure 4 shows an orthogonal axonometric projection of a helical surface, defined by an axis and screwing segment line. The result of the projection and also the situation in the space are visible. Every illustration is made using 3D computer modelling.

The last part of every chapter comprises a collection of examples for exercising the properties of surfaces in various projections. Students can solve the tasks using 3D modelling or graphics software or they can draw the solutions by hand. When using software, it is necessary to construct the silhouette of the surface; if drawn by hand, the aim is to depict some of the important curves on the surface. In both cases, the result is a planar image.

An interesting additional feature of these examples is the possibility to model the surfaces in 3D modelling computer software in space. The spatial situation and principles of projection can also be demonstrated. The virtual model of the spatial situation and 3D virtual models of surfaces make a significant contribution to the development of spatial imagination. Some examples in the book are added in the form of 3D models on attached removable media and additional 3D computer models can be created in cooperation with my students, for example, as part of their theses.

CONCLUSION AND FUTURE WORK

Two areas are addressed in this paper – the possible *methods of innovation in teaching descriptive geometry* (including 3D computer modelling and the creation of new study materials and web support for descriptive geometry) and *the description of the upcoming textbook on descriptive geometry* for undergraduate students.

A survey of contemporary mainstream fields of application of geometry is provided. The main aim is to improve and innovate the methods of teaching descriptive geometry by using 3D computer modelling and enabling connection with practice. It is planned to integrate the suggested outputs from 3D computer modelling software into my new textbook on descriptive geometry. In the future, it is envisaged to publish the textbook in English translation. Some parts of the textbook are also planned to be published on the Internet.

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29-31 July 2014, University of Southampton, UK

PROVIDING TEXTBOOK SUPPORTS FOR TEACHING MATHEMATICS THROUGH PROBLEM SOLVING: AN ANALYSIS OF RECENT JAPANESE MATHEMATICS TEXTBOOKS FOR ELEMENTARY GRADES

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Problem solving has been a major theme in Japanese mathematics curricula for nearly 50 years. Numerous teacher reference books and lesson plans using problem solving have been published since the 1960s. Government-authorized mathematics textbooks for elementary grades, published by six private companies, have had more and more problem solving over the years. As a result, almost every chapter in Japanese mathematics textbooks for elementary grades begins with problem solving as a way to introduce students to new concepts and to introduce new procedures. There has been a long tradition of teaching mathematics through problem solving in Japan, however; a large wave of teacher retirement in recent years has left newly hired teachers without the collegial support they need to develop the expertise to teach through problem solving. In order to overcome this challenge, the latest edition of a major mathematics textbook series in Japan includes more resources to help teachers teach through problem solving and to help students learn through problem solving. The book contains more alternative approaches to a problem, provides diagrams meant to help students solve problems independently, and includes pages that teach students how to take notes effectively. By comparing the latest two editions of this textbook series, the author will highlight how the text has increased its support of problem solving, and will relate the changes to recent trends of Japanese mathematics textbook designs.

Keywords: mathematical problem solving, alternative approaches, Japan

CURRENT ISSUES IN TEACHING MATHEMATICS IN JAPAN

Problem solving has been a major focus in Japanese mathematics curricula for nearly half a century. Numerous teacher reference books and lesson plans using problem solving have been published since the 1960s. Government-authorized mathematics textbooks for elementary grades, published by six private companies, have had more and more problem solving over the years. As a result, almost every chapter in recent Japanese mathematics textbooks for elementary grades begins with problem solving as a way to introduce students to new concepts, and even to procedures (Stigler & Hiebert, 1999; Takahashi, 2000, 2011a). In spite of the long tradition of teaching mathematics in Japan, novice teachers have two major challenges to use the textbook effectively.

The Ministry of Education, Culture, Sports, and Technology released the 2008 Course of Study (COS), the national curriculum in response to concerns about declining mathematics achievement due to a severe reduction in the content and number of class periods in the 1998 revision (see Table 1) As a result, the 2008 Course of Study re-aligned itself to the content and the number of class periods of the 1989 COS. Table 1 shows how the standard numbers of class periods required for mathematics by the law has changed. This change requires an additional 20% of mathematics content in grades 1-6. This content increase mostly impacts the teachers with less than 10 years of teaching experience. For those teachers who previously taught according to the 1989 COS, this change presented little challenge because the new contents were mostly taught under the 1989 COS. On the other hand, younger teachers see this revision as an overwhelming increase to their workload and it includes some mathematics they may never have taught before.

Table 1: Required number of mathematics class periods

Grade	Age (years old)	1951	1958	1977	1989	1998	2011	
Elementary school								
1	6	77	102	136	136	114	136	(4 periods per week)
2	7	123	140	175	175	155	175	(5 periods per week)
3	8	138	175	175	175	150	175	(5 periods per week)
4	9	160	210	175	175	150	175	(5 periods per week)
5	10	160	210	175	175	150	175	(5 periods per week)
6	11	160	210	175	175	150	175	(5 periods per week)
Lower secondary								
1	12	140	140	105	105	105	140	(4 periods per week)
2	13	140	140	140	140	105	105	(3 periods per week)
3	14	140	105-175	140	140	105	140	(4 periods per week)

Note: Total school days may be different among schools. About 845 Japanese elementary schools have 196-205 school days in each year

Another challenge is due to a large wave of teacher retirement in recent years. Based on the Ministry's report (2013), Japanese elementary schools have been experiencing a radical democratic transition. Moreover, it is expected that about 37% of elementary school teachers will be at the retirement age of 60 years in the next ten years.

This recent change in the schools has left newly hired teachers without the collegial support of experienced teachers necessary to develop skills needed to teach through problem solving.

In addition to the above major challenges, the 2008 COS increased the emphasis on mathematical processes such as thinking mathematically and expressing thoughts and ideas using mathematical representations such as diagrams and equations. In order to address these new concepts, all classroom teachers are expected to regularly provide each student with opportunities to think mathematically and to express their own thoughts.

In order to implement the new COS, Japanese teachers, especially those who have had less than 10 years of experience, need to consider not only how the amount of content has changed but also how to design lessons that will push all students to think mathematically and to communicate their ideas in such a way as to learn multiple ways of thinking mathematically.

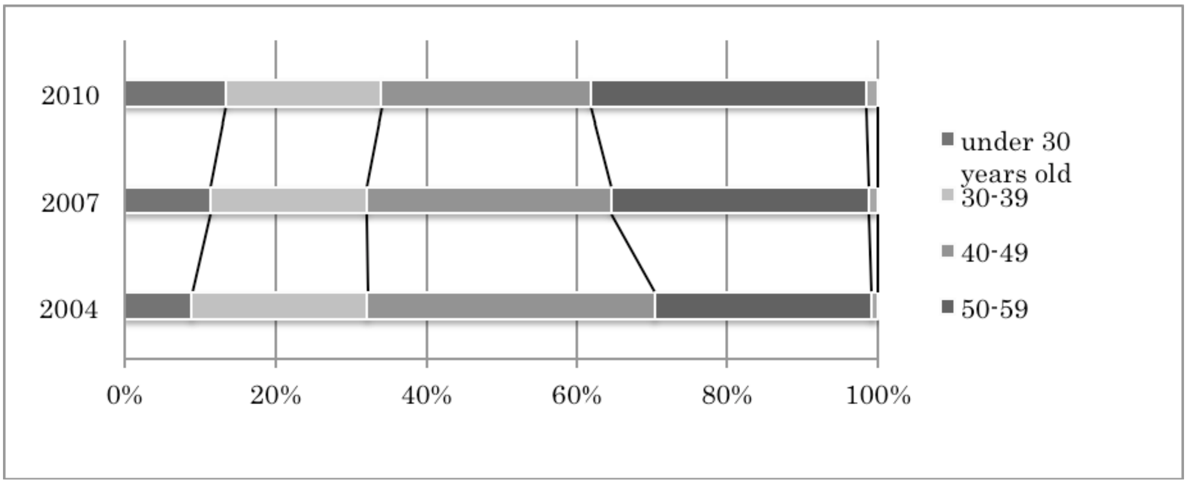

Figure 1. Percentage of Full-Time Teachers by Age in Elementary Schools
(Source: Ministry of Education, Culture, Science & Technology in Japan, 2013)

THE TEXTBOOK UPDATES

In order to overcome the above challenges, the latest edition of a major mathematics textbook series in Japan has been designed to include more resources to assist teachers in their instruction via problem solving and also helps students to learn through problem solving. The book-series contains more alternative approaches to solving a problem, it provides diagrams meant to help students solve problems independently, and it includes separate pages that teach students how to take notes effectively.

The following section will highlight how the text has increased its support of problem solving, and relate the changes to recent trends of Japanese mathematics textbook design by comparing the latest two editions of the best-selling elementary mathematics textbook series—the 2006 edition and the 2011 edition.

In order to highlight such changes, pages using similar problems have been compared. The problems are typical of those found in Japanese mathematics textbooks for 4th grade; they are presented to students who have just learned the formulas for finding the area of rectangles and squares (See Appendix 1 and 2). The objective is for students to understand how they might use formulas, which they have learned previously, to find the area of unfamiliar shapes. In order to use their prior knowledge to find the area of these new shapes, the students should use strategies like area-preserving transformations (cutting and re-arranging) or area-doubling transformations (copying and re-arranging). Thus, the teachers should be able to use this problem to help students learn general strategies for using previously-learned area formulas to find the area of unfamiliar shapes (Watanabe, Takahashi, & Yoshida, 2008).

Although both pages are designed for teachers to introduce the strategies to use the area formulas though problem solving and use very similar problems, the contents of the pages are very different. For example, the total number of pages that are assigned for a particular concept varies. The 2006 edition uses only one page including two exercise problems. On the other hand, the 2011 edition uses three pages, page B25, B26, and B27 with one exercise problem. Although both editions expect teachers to use the pages for one 45-minute-lesson, the contents in the 2011 edition includes more directions and diagrams. For example, the 2006 edition include the only two major directions:

1) Write the ways to find the area using the following diagrams.
2) Calculate the area in several ways.

It is clear that these directions are asking students to find the area of the shape in several different ways. These key directions may be enough for experienced teachers to facilitate a fruitful discussion in order to support students in accomplishing the goals of the lesson. However, novice teachers may have difficulty in teaching through problem solving and could end up using more teacher-centred approaches such as teaching-by-telling (Takahashi, 2011a).

On the other hand, the 2011 edition includes following five directions:

1) Write down the way you thought about doing it using pictures and math sentences.
2) Look at the math sentence Takumi wrote and explain how he thought about the problem.
3) Look at the mathematics sentence that Yumi wrote on the next page and explain how she thought about the problem.
4) What idea is common among the three students?

Different from the directions in the 2006 edition, these include what students can do by looking at other approaches for finding the area of the same shape. In order to do so, the textbook pages include more diagrams and/or equations of alternative approaches.

Another major change made for the 2011 edition was to include special pages to support students in developing note-taking skills throughout the grades. Starting during grade 2, each textbook includes some examples of how to take notes to foster students' mathematical thinking and problem solving skills. Appendix 3 shows the pages entitled "My Math Notes" which incorporates the contents of the area lesson to demonstrate how to organize notes when focusing on problem solving.

SUMMARY

Japanese educators have been using textbooks as a resource for teaching mathematics. Thus, they often emphasize that they can distinguish between "teaching the textbook" and "using the textbook to teach mathematics." In order to provide a better learning experience for their students, all the teachers should be able to use the textbook to teach mathematics effectively using their knowledge and expertise for teaching mathematics. This statement is still widely accepted in Japan, however, it can be seen that recent changes in elementary mathematics textbooks encourages novice teachers to "teach the textbook" rather than be more effective teachers and simply use it as a resource.

Further study on examining how Japanese textbook editors collect classroom data, and the mechanism used to address the needs of the students and the teachers in textbook revision would be useful to consider for improving textbooks as a major resource for implementing the curriculum.

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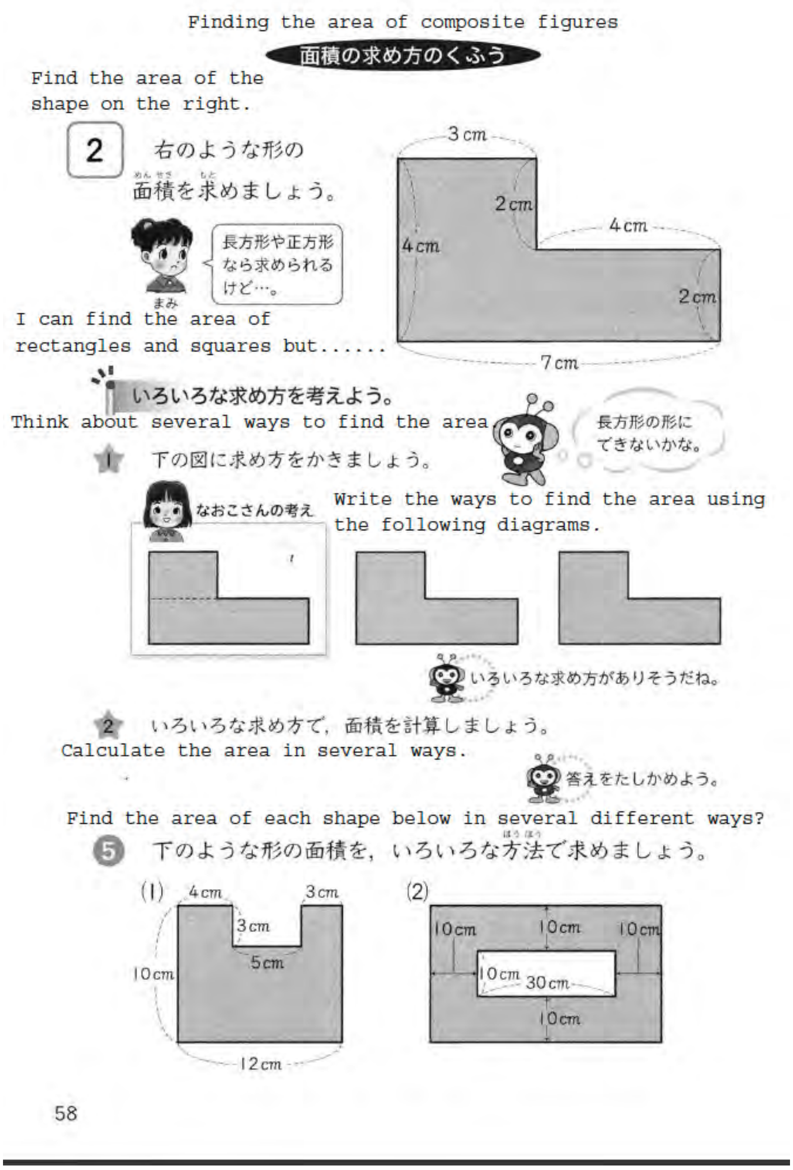
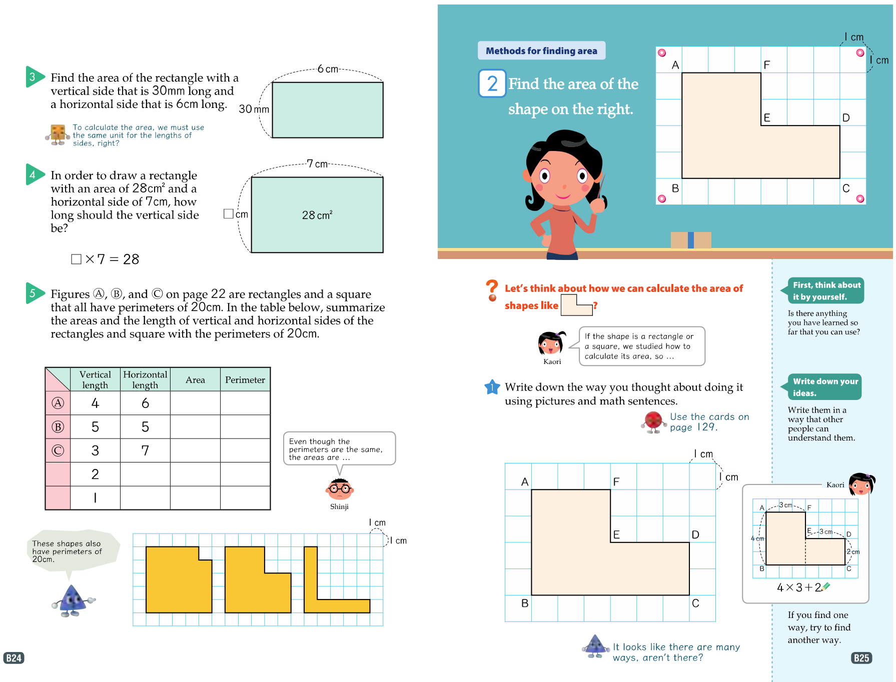
Appendix 1

Reprinted from Sugiyama et al. (2006) *Mathematics Textbooks for Elementary Grade 4th grade*, p.58. Reprinted with permission from Tokyo Shoseki Publishing Co. The English translations are added by the author.



Appendix 2

Reprint from Fujii, T. & Iitaka, S. (2012). *Mathematics International Grade 6*. Tokyo, Japan: Tokyo Shoseki, Grade 4th grade, pp.85- 87. www.globaledresources.com



3 Find the area of the rectangle with a vertical side that is 30mm long and a horizontal side that is 6cm long.

To calculate the area, we must use the same unit for the lengths of sides, right?

4 In order to draw a rectangle with an area of 28cm² and a horizontal side of 7cm, how long should the vertical side be?

□×7 = 28

5 Figures Ⓐ, Ⓑ, and Ⓒ on page 22 are rectangles and a square that all have perimeters of 20cm. In the table below, summarize the areas and the length of vertical and horizontal sides of the rectangles and square with the perimeters of 20cm.

	Vertical length	Horizontal length	Area	Perimeter
Ⓐ	4	6		
Ⓑ	5	5		
Ⓒ	3	7		
	2			
	1			

? Let's think about how we can calculate the area of shapes like ⌐?

If the shape is a rectangle or a square, we studied how to calculate its area, so ...

Kaori

1 Write down the way you thought about doing it using pictures and math sentences.

Use the cards on page 129.

$4 \times 3 + 2$

It looks like there are many ways, aren't there?

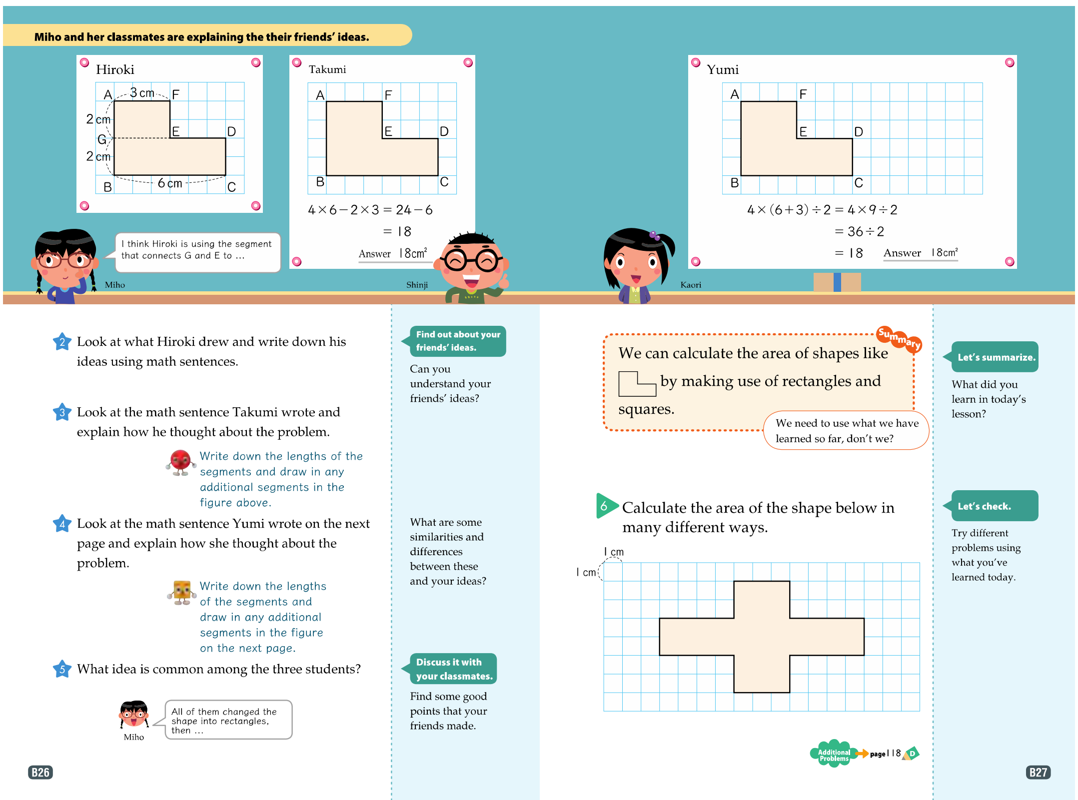
First, think about it by yourself.

Is there anything you have learned so far that you can use?

Write down your ideas.

Write them in a way that other people can understand them.

If you find one way, try to find another way.



Miho and her classmates are explaining the their friends' ideas.

2 Look at what Hiroki drew and write down his ideas using math sentences.

3 Look at the math sentence Takumi wrote and explain how he thought about the problem.

Write down the lengths of the segments and draw in any additional segments in the figure above.

4 Look at the math sentence Yumi wrote on the next page and explain how she thought about the problem.

Write down the lengths of the segments and draw in any additional segments in the figure on the next page.

5 What idea is common among the three students?

All of them changed the shape into rectangles, then ...

Miho

Find out about your friends' ideas.

Can you understand your friends' ideas?

What are some similarities and differences between these and your ideas?

Discuss it with your classmates.

Find some good points that your friends made.

Summary

We can calculate the area of shapes like ⌐ by making use of rectangles and squares.

We need to use what we have learned so far, don't we?

6 Calculate the area of the shape below in many different ways.

1 cm
1 cm

Additional Problems page 118

Let's summarize.

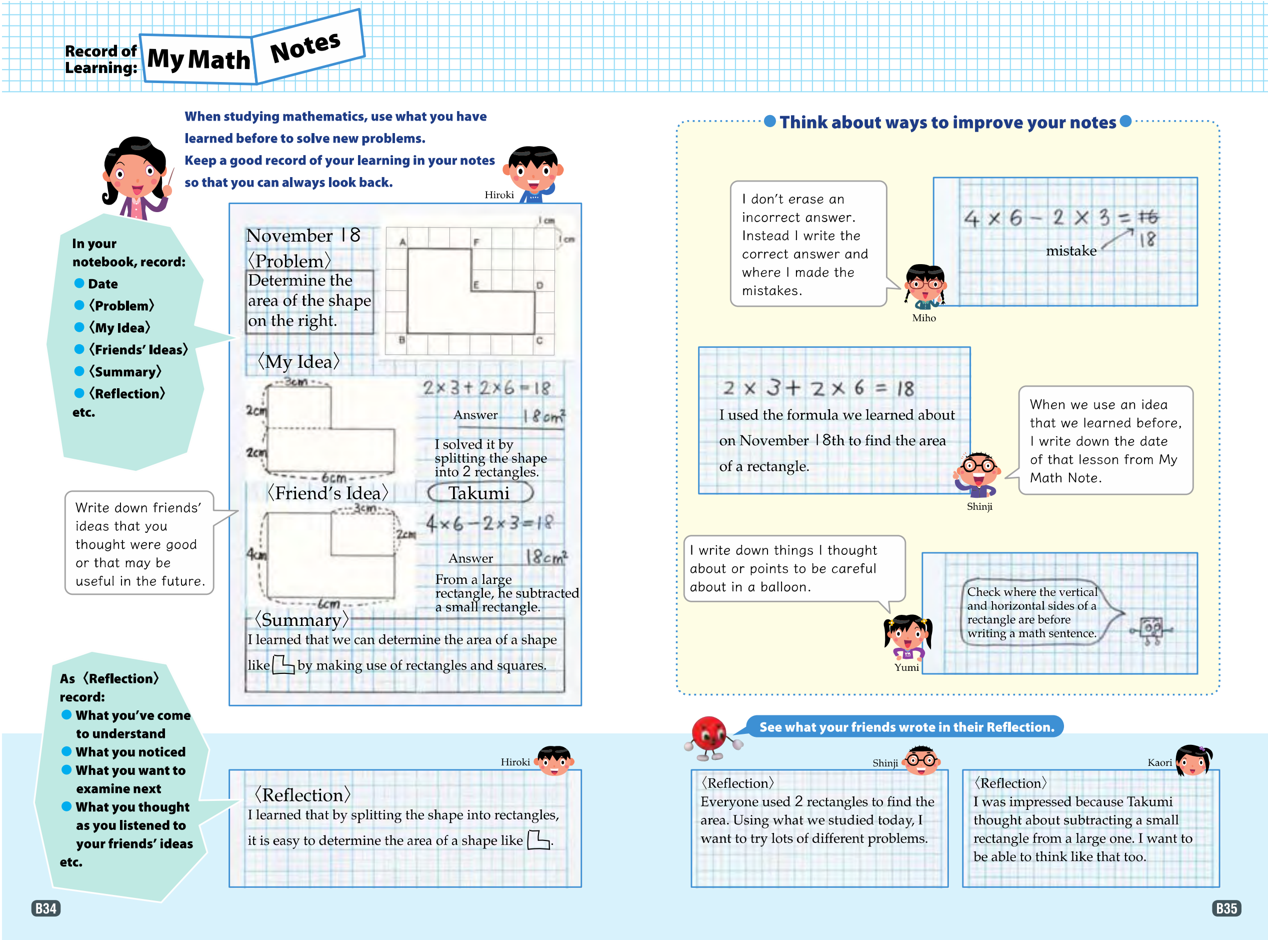
What did you learn in today's lesson?

Let's check.

Try different problems using what you've learned today.

Appendix 3

Reprinted from Fujii, T. & Iitaka, S. (2012). *Mathematics International Grade 6*. Tokyo, Japan: Tokyo Shoseki, Grade 4th grade, pp.90-91. www.globaledresources.com



BUILDING NEW TEACHING TOOLS IN MATHEMATICS: TEACHER AND TECHNOLOGY RESOURCES

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Using the paradigm of activity theory, the central problem is the characterization of the processes through which teachers replicate, adapt, and improvise tasks of textbooks with use of technological resources (CD- ROMs and web portals), in other words, we seek to identify teachers' use of schemes in actions mediated by these technological elements. Two of us accompanied Portuguese secondary mathematics teachers in the assessment of learning tasks involving the use of new technological resources and the analysis of feedback of teaching performance after implementation in the classroom. This feedback was obtained from their peers, trainers and teachers' reflection on the actions in classes and occurred during the sessions of the training activities. The study[1] *shows that teachers plan coordinated tasks that integrate technology resources and apply them in classes adjusting them to the technological environment of their schools. However, some difficulties in interpreting the returns are revealed.*

Keywords: technology, assessment of learning, activity theory, secondary school, Portugal

INTRODUCTION

Current textbooks include technological resources (CD- ROMs and web portals) and what teachers do with these new curriculum resources matters to the understanding of their professional development. In three in-service training workshops during the years 2009 and

[1] This work is supported by national funds through FCT - Foundation for Science and Technology in the context of the project *Promoting Success in Mathematics* -PTDC/CPE-CED/121774/2010

2011 (totalling 95 sessions) led by two of us, teachers analysed the technological materials that come with Portuguese mathematics textbooks from six different publishers. Voluntarily, 63 teachers from 24 basic and secondary schools participated in the sessions.

Oral and written productions by teachers allowed us to identify the *utilization schemes* that were developed during the construction process of *didactical exploitation scenarios* and their reflection on teaching performance.

FRAMEWORK

We distinguish between an *instrumental dimension* related to the electronic form of resources and a *documentational dimension* associated with the work of teachers with the technological materials and their use in class.

The instrumental dimension relates to the *instrumental genesis* (Rabardel, 1995) and was adapted to the study of teaching and learning mathematics by Artigue (2002), Ruthven (2002), Guin and Trouche (2002), particularly in technology-mediated learning. In this dimension we discuss *the instrumentation* and *instrumentalisation processes*, associated with the construction of *schemes of utilisation* — schemes for an organizing activity with an artifact associated with the performance of a particular task (Rabardel, 1995). Drijvers and Trouche (2008) go further and distinguish two types of schemes of utilisation: *usage schemes* oriented management of artefact and *instrumented action schemes*, entities oriented to perform specific tasks.

The documentational dimension is defined by the theoretical framework developed by Gueudet and Trouche (2012) (*documentational genesis*) specifically directed to study the activity of teachers and their professional development. In this dimension we discuss the interactions between mathematics teachers and curriculum resources, and the consequences for their professional development (Drijvers and others, 2010; Drijvers and Trouche, 2008).

DATA COLLECTION

Data included oral and written productions of teachers required during the workshops together with written accounts of the sessions produced by the researchers. Special attention was paid to *feedback*, which is defined by Hattie and Timperley (2007) as information provided to a student by an agent (e.g., a teacher, a colleague, a book, curriculum materials, by itself, an experience) about aspects of their learning, their performance or their understanding. They argue that feedbacks are one of the most powerful influences on learning and achievement. A teacher or trainer can provide corrective information, a couple can provide an alternative strategy, and a book can provide information to clarify ideas. Feedback is a result of performance. For these authors, the effective feedback should reduce the gap between current performance and performance towards a goal or objective.

Data were collected in three different occasions: 1) during the analysis of the proposals contained in the technological materials, 2) during teachers activity preparing tasks from textbooks or the materials, and 3) after its actual application in classroom. Particular attention was paid to the activities developed by students, and the reflections on the teachers' own performance.

INSTRUMENTAL GENESIS

Instrumental genesis is associated with the mediation actions of teachers with the resources as technological artefacts. Two processes are distinguished: *instrumentation* which is the action of the artefact over the subject (the teacher) — associated *usage schemes*—, and *instrumentalisation*, the action of the subject (the teacher) over the artefact — associated with *instrumented action schemes*.

Usage schemes arise from teachers' analysis of the strengths and limitations of the resources *per se*, before any conjecture about their relevance to the program or their usage in the classroom. The notion of usage here does not imply a teaching practice, but just an observation resulting from the first contact with the materials.

For example, when asked to describe the technological resources, one group of teachers simply listed the contents of the CD-ROM from a textbook of the 9th grade:

> "Programmatic contents of the 9th grade, suitable to the current program;
>
> Didactic approaches: introductory videos for all content; Interactive explanation; Interactive exercises; Tests at the end of each chapter; Global test."

Instrumented action schemes go further and result from the analysis of the strengths and limitations of the technological resources and are imbued with actions centred on their use in the classroom. Here, teachers show how they can build with it didactical exploitation scenarios, adapted to the prescribed curriculum, the means available in their schools or the characteristics of students in their classes.

The following is an example of instrumented action schemes centred in technology. The teacher summarizes the actions develop with the technological resources and explains how to overcome school limitations.

> In addition to the manual, I usually use the web site [of the publisher] to view animations related to different content and some interactive applications. While existing animations are sort of a show-off of the site, I usually use them to introduce the topics and then discuss in large group.
>
> I also have access to the eBook, where I can view the manual in electronic format, and flip through the book, looking for resources in the different pages. The projection of the manual is useful because I can use graphics or pictures to explain something related to what the students should notice and explain to students how to solve the exercises.
>
> To work, for example, with dynamic geometry software, at school there are some laptops that can be requested, but not always work correctly. Alternatively, I ask students to bring their laptops.

DOCUMENTATIONAL GENESIS

Also in the documentational geneses, associated with the development of actual didactical interventions, the utilisation schemes were separated on usage schemes and instrumented action schemes. Usage schemes by professors take the form of didactic scenarios whose components are: available technological artefacts, the mathematical situation with the produced artefact (a task) and instrumental orchestration. Instrumented action schemes result of teachers' reflection on their teaching performance, including technological artefacts and the artefacts produced.

To study the artefacts produced by teachers, was used the characterization proposed by Brown (2009), which defines three forms of interaction between teachers and curriculum materials: offloading, adapting, and improvising. In the first case the teacher just copies the proposed curriculum materials. In the second case the teacher follows the suggestions in the course material, but adapts them to his or her context and preferences. In the third case the teacher does not observe the suggestions made by the curriculum materials and follow their own ideas. In table 1 we present a summary of what happened with groups of teachers in the construction of didactical exploitations scenarios.

Table 1: Frequency of the forms of interaction between teachers and curriculum materials by workshop.

In-service workshop	Forms of interaction		
	offloading	adapting	improvising
A	6	5	
B	11		
C	7	2	5

In general teachers chose a task from the textbook or from the technological resource and only in a few cases adapted or improvised a task.

Instrumented action schemes obtained from written productions of teachers about their teaching performance were organized into six categories: 1) the teacher reflects on the technological artefact, 2) the teacher reflects on the artefact produced, 3) the teacher reflects on his role, 4) the teacher reflects on the students' productions obtained in class, 5) the teacher reflects on the prescribed curriculum, and 6) teacher presents students' productions. Sometimes written reflections involve more than one category.

In the following example the teacher reflects on the technological artefact. She points out the benefits of using the CD-ROM, but wonders if it was useful for students.

> The use of CD-ROM benefits student learning, when the presentation of concepts requires a visual support. However, the contents of the CD have proved insufficient whenever it was necessary to apply these concepts, since students were unable to solve the exercises, either on the CD or on the textbook without my support. Only displaying the CD has been clearly insufficient in obtaining any sort of learning.
>
> The fact that they are not used to work with this type of tool may have influenced the results of their work, namely the lack of autonomy of students. Note that, during these and other lessons, whenever students perform autonomous work constantly they resort to teachers for questions or to confirm whether their learning was the most correct, regardless of the task that is applied to them.

In the following example, the teacher reflects on the artefact produced and submits his decisions for future use of the worksheet built on the results obtained in implementation with the students.

> Seeing the results, I think the worksheet is well done because students easily run it and reach the desired results. Still, if re-use this worksheet, I intend to improve the following aspects: [lists several changes in the wording of the sheet].

CONCLUSION

The production of the documentational geneses associated documents as a product of the in-service sessions was achieved, namely: the technological resources studied were made flexible by the teachers and thus allowed them different educational choices; teachers developed didactical exploitation scenarios to support their practice; teachers considered the instrumental orchestrations conditions of their schools and the characteristics of students in their classes; teachers shared with peers accounts of this didactical experimentation. Therefore, the training workshops were facilitating the creation of conditions for teachers to transform the artefacts into documents.

The contents of technological resources, their format, the characteristics of students, the prescribed curriculum and school technological facilities, in particular, the number of computers per student and the distribution of the resources in the class, are considered in the schemes utilization of teachers.

The two processes of instrumental geneses (instrumentation and instrumentalization) occur simultaneously; teachers appropriate the resources and evaluate their constraints and potentials, and at the same time integrate the materials in specific didactic settings within the context of their school. The transformations of the action of the artefact towards the teacher and towards the action of the teacher in relation to the artefact are complete; they place the two processes of instrumental geneses.

This process is not identical for all teachers. The productions involving both usage schemes and instrumented action schemes show that teachers have developed particular schemes that allowed them to use the transformation of a technological artefact into an instrument. Some teachers developed limited instrumented action schemes. Some reflections about the didactical exploitation scenarios in practice were essentially descriptive of the actions developed in the classroom rather than reflective teaching experiences.

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POSSIBLE MISCONCEPTIONS FROM JAPANESE MATHEMATICS TEXTBOOKS WITH PARTICULAR REFERENCE TO THE FUNCTION CONCEPT

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The purpose of this paper is to partially answer the following general research questions: (a) Do mathematics textbooks have an influence on students' misconceptions? (b) What should the textbook writers pay attention to? In this paper, we particularly focus on the treatment of the function concept in Japanese mathematics textbooks, and point out that misconceptions are inherent in the way of presenting the examples of functions or the questions about functions in the textbooks. We took both of the constructivist and sociocultural perspectives, and carried out the conceptual analysis, which is one of the constructivist ways of discussing what conceptions can arise. As a result of the analysis, we found two possible misconceptions in Japanese representative textbooks. One conception seemed to come from the encapsulation of the process of formularizing a certain single algebraic rule for the given relationship between the variables x and y. Another conception seemed to come from the encapsulation of the process of fixing x and calculating y in the given formula. In addition, there was no chance around the presentation of the definition of a function in the textbooks to unify and generalize these two conceptions. As an explanation why the students may have difficulties to unify and generalize them, the idea of subjective randomness was introduced. As a possible direction of the future textbook development, to improve the insufficiency of subjective randomness, an example based on geometric construction was proposed.

Keywords: misconceptions, functions, role of examples, Japan

INTRODUCTION

The word *misconception* has been used to refer to the existence of an erroneous guiding rule (Nesher, 1987). The information provided by textbooks may be one of the important factors of misconceptions because it determines what should be taught. Thus, it is worth examining whether the textbook writings are naturally interpretable for students as the writers have intended. To do so, in this paper, we tackle to partially answer the following general research questions: (a) Do mathematics textbooks have an influence on students' misconceptions? (b) What should the textbook writers pay attention to? Since the answers of the research questions depend on which textbook and on what mathematical concept, we tentatively focus on the writings about the *function* concept in Japanese textbooks.

BACKGROUND

In this section, we review the literature about a misconception of function. Especially, we focus on recent Japanese situations.

As Vinner and Dreyfus (1989) reported, at least 22% of the participants tried to propose a single algebraic rule when they construct one example of functions satisfying the given condition. They tend to refer not to the concept definition but to their own concept images to construct a new function or to judge whether something is a function or not. Their behaviours will be observed as misconceptions from the observer point of view. In this paper, we will refer to this type of misconception as a *single algebraic rule conception*, in short, a *SARC*.

Although Japanese textbooks mention the functions which cannot be represented by a single algebraic rule, a SARC is also observed in Japan. Although all Japanese students learn the general definition of function at Grade 8, it is only 13.8% of Grade 9 students who can choose the correct answer (e) of the following question:

> Which of the following items define y as a function of x? Choose a correct one. (a) x is the number of students in a school and y m^2 is the area of its schoolyard; (b) x cm^2 is the area of the base of a rectangular parallelepiped and y cm^3 is its volume; (c) x cm is the height of a person and y kg is his/her weight; (d) a natural number x and its multiple y; and (e) an integer x and its absolute value y. (MEXT & NIER, 2013, p. 64, translated by the author)

34.1% wrongly chose (b), and 35.3% chose (d). MEXT & NIER (2013) suggest that the reason that they chose (b) or (d) may be that they associate the formula for the area of a rectangular parallelepiped or the word *multiple* with proportional functions (p. 65). This seems to be influenced by one of SARCs, and the students may not consider the notation of the absolute value $|x|$ as a single algebraic rule. Nunokawa (2014) pointed out that there are two inconsistent narratives in the textbooks: a function as a relationship and as an object. However, there are not enough discussions about the misconceptions.

THEORETICAL STANCE & RESEARCH QUESTION

Coordinating the constructivist and the sociocultural perspectives (Cobb, 1994) can be one of the promised approach to the present issue. Following Gray & Tall (1994), the constructivist perspective focuses on whether the students encapsulate an *appropriate* process into a new conception, but it does not tell us enough information on why some of them fail to encapsulate. On the other hand, following Lave and Wenger (1991), the sociocultural perspective focuses on in what community of practice the students *actually* participate, but it does not tell us enough information on what results in the actual community of practice. Thus, if we complement the constructivist perspective by the sociocultural one, a misconception can be regarded as a result of the encapsulation of an *inappropriate* process. Hence, if we can identify what the textbook writings may encourage students to do, then such encouraged process will be a possible factor of the corresponding conception.

Now, we can pose the more specific research questions: (i) What process may students experience when they read the Japanese textbook writings about functions? (ii) What is the difference between the predicted conceptions and the intended function concept?

METHODOLOGY

All the Japanese textbooks are published by several private companies, and authorized by MEXT. In case of public junior high schools, the adopted textbooks depend on the districts divided by each Prefectural Board of Education. In case of high schools, the adopted textbooks depend on each school. Generally speaking, certain students use the same company's textbook series at junior high school, while they may use the different company's textbook series at high school. Thus, the word *function* is defined twice, that is, at junior high school and at high school.

We selected the two textbooks. One is Keirinkan (2012), which seems to be representative of Japanese junior high school textbooks. This is the current most adopted textbook (41.6% share) (Jiji Press, 2011). Another is Suken Shuppan (2011), which seems to be representative of Japanese high school textbooks. All of the current three most adopted textbook for high school in Japan are published by Suken Shuppan (the total 42.4% share) (Jiji Press, 2014). Suken Shuppan (2011) is one of them (13.2% share).

The target sentences in textbooks were the definition of the word *function*, the description of the examples of functions, and the questions of whether something is a function or not. The followings were not analysed: the definitions of the other words related with the function concept, the description of the examples of them, and the other questions.

In this paper, we basically followed Thomson's (2000) *conceptual analysis* of depicting "an unfolding image of person X comprehending the statement" (pp. 309-310). This is one of the constructivist ways of discussing what conceptions can arise. Although he has not provided the well-organized description of the procedural steps of the conceptual analysis, this method can be explained as the following steps: (1) parsing the target sentences into the sets of the partial words, (2) numbering all the words according to the order in the sentence, and (3) interpreting what one can imagine when reading the sentences from the first word to the *n*th word for each positive integer *n*. Through this procedure, we interpreted what the textbook writings might implicitly encourage students to do.

RESULTS

The writings in the textbooks and the interpretations of them are shown in Table 1 and 2. Since not all the interpretations from the conceptual analyses were essential, the tables show only the interpretations related to what they might implicitly encourage students to do. For the convenience of the discussions, the rows of the tables are numbered.

As a result, Keirinkan (2012) seemed to encourage students both to formularize some relationships between x and y (No. 1 & 5 in Table 1) and to repeat to fix x and calculate y (No. 2, 3, 4, & 6 in Table 1). On the other hand, Suken Shuppan (2011) seemed to encourage students only to repeat to fix x and calculate y (No. 1, 2, & 3 in Table 2). The formularizing process was not seemingly provided in Suken Shuppan (2011).

DISCUSSION AND CONCLUSION

Based on the theoretical stance of this paper, we found the two distinct conceptions related to the function concept. One is a *formularizable function conception*, in short, a FFC, which comes from the encapsulation of the process of formularizing. Another is a *calculable function conception*, in short, a CFC, which comes from the encapsulation of the process of fixing x and calculating y. Keirinkan (2012) seemed to provide both of them while Suken Shuppan (2011) seemed to provide only the latter. Although a FFC and a CFC may sometimes work well, a general function must not have such properties. Hence, the most important implication from the above results is that both of the textbooks did not have enough information for students to form the complete function concept.

It is also important that each conception works at a different timing. A FFC seems to be one of SARCs, which works when the students tries to construct a function (e.g., Vinner & Dreyfus, 1989). On the other hand, a CFC does not directly lead to a SARC because it may accept a multiple algebraic rules function as long as the function is calculable. The CFC works only when the function has already been formularized. Besides, not all the students tend to unify the two conceptions because they can manage to solve problems in the textbooks.

If the students can ignore the particular properties of a FFC or a CFC, they can form the general function concept. However, this seems to be a rare case. There are two reasons. The first one is that both conceptions are viable as mentioned above. For the students, there is no reason to ignore the particular properties of them. The second reason is that the processes of formularizing and of calculating seem to prevent the students from shifting "attention." Mason (1989) pointed out that mathematical abstraction results from "a delicate shift of attention from seeing an expression *as* an expression of generality, to seeing the expression *as* an object or property" (p. 2, italics in the original). In the present case, however many times the processes of formularizing or calculating are repeated, it cannot necessarily produce non-formularizable or non-calculable functions. Thus, the students may implicitly see the functions in the textbooks not as the examples randomly chosen from the set of all functions, but rather as the examples randomly chosen from the set of all formularizable or calculable functions. That is, the examples are too biased to see the set of all functions. We will call this bias the insufficiency of *subjective randomness*. Even if the general definition of a function is given under insufficiency of subjective randomness, it will be seen just an expression as an expression of generality, not as an object or property. This will lead to limited conceptions.

Simply speaking, we must find a way of improving the sufficiency of subjective randomness for the future textbook development in the international context. We will show one possible examples seemingly randomly chosen from the set of all functions: the relationship between the opposite x and the hypotenuse y in a right-angled triangle with the given adjacent. If this example is treated without measuring the lengths, it has a potential to make the students intuitively grasp only the relationship between x and y before formularizing the algebraic rule.

In conclusion, we found that Japanese representative mathematics textbooks did not provide sufficient information to see the definition of a function as an object or property. We need to analyse the case of the other textbooks, and to discuss what examples the students need.

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Table 1: Interpretations of Keirinkan (2012) ["//" means a paragraph break]

	Writings in the textbook (translated by the author)	Interpretation
1	[Question] On what quantity the following quantities depend? // (1) the length of the horizontal sides of the squares whose area is 24cm2, // (2) the total weight of the bucket and the water in it, where the weight of the bucket is 700g, // (3) the distance you have walked in case you walk 70m per minute.	The question encourages students to formularize each relationship (1), (2), and (3).
2	For example, in the above question (1), the length of the horizontal sides changes according that of the vertical sides. If the length of the vertical sides is determined, then that of the horizontal sides is uniquely determined.	The example encourages students to fix the length of the vertical sides and to calculate that of the horizontal sides.
3	[Example 1: the opened area of a window] We open the window whose horizontal sides have 90cm. The opened area of the window changes according to the length we slide the window. If the length is determined, then the area is uniquely determined. // In the above Example 1, let x cm be the length we slide the window, y cm2 be its opened area. x and y change according to each other, and they can take various values.	The example encourages students to fix the slid length, and to calculate the opened area.
4	[Definition] … if there are two variables x and y which change according to each other, and if when we determine the value of x, the value of y is uniquely determined according to the value of x, // then we say that y is a function of x.	The writings determine the way of judging whether something is a function or not.
5	In Example 1, there is the relationship y = 90x between x and y. // Like this, if y is a function of x, there are cases where the relationship can be represented by the formula.	The writings encourage students to reflect on the formalizing process.
6	[Question 1] Which is the case where y is a function of x? // (1) You go from the city A to the city B, which is 30km far. The reached distance x km, and the remaining distance y km. // (2) You pour water into a tank, 4L per minute. The amount of water y L per x minutes. // (3) A person's age x and he or her height y cm. // (4) The radius of a circle x cm and its area y cm2.	The writings encourage students to try to formalize each relationship. If they succeed in formalizing, then they will fix *x* and calculate *y*, and notice the relationship is a function. If they fail to formalize, then they notice it is not a function

Table 2: Interpretations of Suken Shuppan (2011)

	Writings in the textbook (translated by the author)	Interpretation
1	[Definition] For two variables x and y, if when we determine the value of x, the value of y is uniquely determined, then we say that y is a *function* of x.	The writings determine the way of judging whether something is a function or not.
2	[Example 1] Let y cm be the perimeter of the square whose sides have x cm. Then, $y = 4x$, and y is a function of x, where $x > 0$.	The writings encourage students to fix the value of x and to calculate the value of $4x$.
3	[Example 2] Let y cm^2 be the area of the square whose sides have x cm. Then, $y = x^2$, and y is a function of x, where $x > 0$.	The writings encourage students to fix the value of x and to calculate the value of x^2.

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TRANSFORMATIONS IN U.S. COMMERCIAL HIGH SCHOOL GEOMETRY TEXTBOOKS SINCE 1960: A BRIEF REPORT

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In the United States, the formal study of Euclidean geometry, including significant attention to congruent and similar figures, has been traditionally concentrated in a single one-year high school course. The Common Core State Standards for Mathematics (CCSSM, 2010), which have been approved by well over 80% of the states, call for congruence and similarity in this course to be understood in terms of geometric transformations, and for the traditional criteria for two triangles to be congruent or similar to be developed through these transformations. However, it seems that many teachers of geometry in the United States have little or no idea of what this means. This study examined 64 textbooks those teachers might have used in their high school experience to see what they could have learned about transformations when they were students. Herein is a brief summary of results.

Keywords: geometry, geometric transformations, Core State Standards for Mathematics (CCSSM), USA

INTRODUCTION

This paper reports on the treatment of transformations of the plane or of figures within it. A *geometric transformation* is a 1-1 function that maps points of a set F onto points of a set G. F and G may be the same plane, 3-dimensional space, or individual figures (we speak of transforming one figure into another). Euclidean geometry can be characterized as the study of those properties of figures that are preserved by transformations that preserve distance (*isometries* or *congruence transformations*) or transformations that preserve ratios of distances (*similarities* or *similarity transformations*). As the various names suggest, two figures are congruent if and only if one is mapped onto the other by an isometry, and two figures are similar if and only if one is mapped onto the other by a similarity.

In the 1960s, curricula in the U.S. started including some study of geometric transformations. Some of the work was with young students, some with slower students starting in the 7th grade, and some work was with better high school students. In the late 1960s, this researcher co-authored a geometry text for average students in which transformations were central to the mathematical development. Very soon thereafter, articles appeared that made it appear as if approaching the study of high school geometry through transformations or through vectors were viable alternatives to the traditional approach. In 1983 the University of Chicago School Mathematics Project began, and this researcher has directed the development of texts for all the grades 7-12 through three editions.

In 1989, the National Council of Teachers of Mathematics (1989) produced a set of guidelines intended to be a vision moving mathematics teaching forward. These “NCTM Standards” recommended that students “deduce properties of figures using transformations;

identify congruent and similar figures using transformations; analyze properties of Euclidean transformations and relate translations to vectors. Additionally, the standards called for college-intending students to apply transformations in problem solving. In NCTM's update of its standards a decade later, all students were expected to "understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices; use various representations to help understand the effects of simple transformations and their compositions."

In 2008, spurred by the lacklustre performance of the United States on the international TIMSS and PISA assessments, and believing a 1996 TIMSS report that placed some of the blame on the lack of a national consensus on what students should learn, the governors of the 50 states undertook to create the Common Core State Standards in Mathematics (CCSSM), with recommendations for mathematics from kindergarten to grade 12 (Common Core State Standards, 2010). There are 43 high school geometry standards in the CCSSM, divided into 6 domains: congruence; similarity, right triangles and trigonometry; circles; expressing geometric properties with equations; geometric measurement and dimension; and modeling with geometry. Each domain is divided into clusters, and each cluster is described by individual standards which themselves might have several parts. Three clusters specifically mention transformations (see Table 3 below). Within these three clusters, 10 standards specifically mention transformations, constituting about one-quarter of all geometry standards.

These 10 particular standards have caused consternation amongst many teachers because some teachers never encountered transformations in their study of geometry as high school students and many others would not be familiar with all the content mentioned in these standards. The question in our minds was the extent of the opportunities teachers might have had to encounter this subject matter as high school students.

THE RAW DATA

This researcher has a moderately large collection of textbooks for grades 7-12 amassed from individuals, schools, and publisher. In 2014, the collection contained 64 geometry texts published commercially in the United States with copyright dates from 1961 to 2015. Because of the way the books were amassed, the collection tends to favor books that have been more widely used. Thus, while the collection was neither systematically collected nor random, there is no reason to believe it is not fairly representative of all geometry texts used in schools in this time period. The one exception to this is that four of the texts are those in which this researcher is an author (see Footnote 1).

RESULTS AND DISCUSSION

Four analyses of the data were undertaken.

First analysis: Extent of opportunities to encounter geometric transformations

To determine the extent of opportunities to encounter geometric transformations as high school students, this researcher examined all 64 texts. Two texts were identified as experimental and not meant for wide use. This left 62 geometry texts to be studied. An overall measure of attention to transformations (independent of the substance of the attention) in these 62 texts is given in Table 1. This table shows that 18 of these texts, all published before the 1990s, contained no material on transformations, and 4 others contained only one lesson

(4 pages or less). This left 40 texts with enough discussion of transformations to enable them to be analyzed further.[1]

Table 1: Treatment of geometric transformations in 62 U.S. geometry texts, 1961-2015

	1960s	1970s	1980s	1990s	2000s	2010s	Totals
No treatment	11	5	1	1	0	0	18
One lesson only	0	2	1	1	0	0	4
One chapter at end of book	1	5	5	3	2	0	16
One chapter in middle, unconnected	0	4	0	2	1	1	8
In various places, unconnected	1	1	2	0	1	4	9
Treatment throughout	1	1	0	2	2	1	7
Totals	14	18	9	9	6	6	62

Discussion of first analysis

A typical book contains from 12 to 15 chapters. In 16 of these 40 texts, the study of transformations is concentrated in the last or second-to-last chapter of the book. This placement assures no use of transformations in the mathematical development of the basic geometric concepts of congruence and similarity during the course year. Moreover, because all these textbooks in the United States contain more material than a typical class can cover in a school year, it is unlikely that a class using these books discussed transformations. In all, this means that students using 38 of the 62 books were likely not to have studied any geometric transformations. Furthermore, of the 52 geometry texts written by authors other than this researcher before the year 2010, only 7 gave transformations what we have called "major treatment".

In 8 of the 62 books, the study of transformations is concentrated into one chapter not at or near the back of the text. As a result, it is possible that a teacher could avoid teaching transformations without having to worry about any applications in later chapters. We have no way of knowing whether this happened, or how often, nor do we have any way of knowing whether teachers of the 38 books mentioned above taught transformations to their students even though the material was in the last chapter of their books or not there at all. Nevertheless, the youngest teachers of today are more likely to have encountered transformations in their own geometry study, since all texts since 2000 have devoted at least one chapter to them.

Second analysis: Scope and language of transformations

Of the 40 texts that discuss transformations, five texts do not define "transformation". The other 35 define transformations as a correspondence (11 texts), mapping (8), function (6), movement or change (4), operation (3), set of ordered pairs (1), matching (1) and process (1). Eight of the 35 definitions do not require the correspondence, mapping, etc., to be 1-1. Euler's f( ) notation is found in 12 of the 35 texts.

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[1] This researcher's interest in this study is reflected in that, of the 7 texts identified as "treatment throughout", 4 (1 in the 1970s, 2 in the 1990s, and 1 in the 2000s) include him as one of the authors. This presents an obvious source of possible bias in all analyses that follow.

When not described in words, the operation of following one transformation by another is universally called “composition”. Of the 40 texts, 37 discuss this operation. The result of this operation, that is the transformation that maps the original preimage onto the final image is called the “composition” (that is, the same name as the operation) in 17 of the texts, “composite” (9 texts), “product” (6), “combination” (2), “resultant” (1), or given no special name (3). (One text used two names.)

Of the four types of isometries (reflections, translations, rotations, and glide reflections), 22 texts mention all four types; 18 mention all but glide reflections. Distance-multiplying transformations are almost always termed similarities and the specific transformation that fixes a single point C and for which the image of P is the point P’ on ray CD so that CP’ = CP is almost universally called a “dilation” (other names: dilatation, size change, size transformation, expansion). The specific name given to a dilation with k > 1 may is variously called an expansion, an enlargement, or a stretch, and when 0 < k < 1, is termed a contraction, a reduction, or a shrink.

There is varying attention to transformations on the coordinate plane. It is common to give equations for images of points under translations, for reflections over the x-axis, the y-axis and the line x = y, and for dilations. It is much less common to give images of points under rotations of 90°, 180°, and 270° about the origin. In a number of books, definitions for both translations and dilations are given in terms of coordinates.

In sum, in all analyses of language and scope, the treatments of transformations in the books that mention them are quite varied.

Discussion of second analysis

In some cases, the language struck this researcher as misleading or confusing. The use of “composition” as a name for both an operation and its result confuses many students. Similarly, a few books called the result of the reflection of a figure the “reflection”, rather than “reflection image”, employing the non-mathematical use of the word, namely that a person sees his reflection in a mirror.

A glide reflection is almost universally defined as the composite of a translation and a reflection over a line parallel to the direction of the translation. Perhaps swayed by this definition, some books convey the impression that a glide reflection is not itself a single transformation. For instance, they may ask students to describe “the isometries that map” one triangle onto a second congruent triangle, expecting a composite of transformations, and never mention when the mapping could be done by one glide reflection.

Because we all are very much influenced by the ways and language that are used when a concept is introduced, the variety of different terms is a cause of concern.

Third analysis: Basic reasons for studying transformations

This researcher identified six reasons for studying transformations in school geometry. (1) They enable single definitions of congruence and similarity to apply to all figures, thus unifying and enlarging the study of these ideas and providing a powerful tool for the study of graphs of functions and relations. (2) They are functions, so function notation and composition of functions can be introduced or reinforced with their study. (3) They provide a mathematical explanation of symmetry. (4) They provide mathematical models of actions in the real world. (5) They provide solutions to problems that might be more difficult to solve without them. (6) There are interesting and important algebraic relations among them beyond the composites of two reflections.

Reason (1) is discussed below in the next analysis. Reason (2) was examined in the second analysis above; it was seen that few books explicitly mention that transformations are functions, though a greater number use function notation and most discuss composition. Table 2 shows the number of texts that handled reasons (3)-(6).

Table 2: Numbers of texts discussing various aspects of transformations.

One chapter at end of book (n = 16)	One chapter in middle (n = 8)	Various places, unconnected (n = 9)	Treatment throughout (n = 7)
Symmetry is explained using transformations.			
11	6	8	7
Transformations model the real world.			
7	5	3	7
Shortest distance problem solved using reflections.			
5	0	4	5
Algebra of transformations is discussed.			
1	1	0	5

Fourth analysis: Attention to specific activities mentioned in the Common Core

A fourth analysis of the texts was undertaken to determine whether the Common Core State Standards dealing with transformations are represented in these texts. For this analysis, we identified 11 activities from the 10 Common Core standards that mention transformations. The activities are identified in Table 3 by cluster.

Table 3: Treatment of Selected Activities Mentioned in the CCSSM

Activity	Degree of attention in 40 texts
Cluster: Experiment with transformations in the plane.	
1. Give a definition of “transformation” as a function from points of the plane into points of the plane.	Few specifically mention functions.
2. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	Coverage rare
3. Describe the rotations and reflections that map rectangles, parallelograms, trapezoids, and regular polygons onto themselves.	8 texts mention all the symmetries.
4. Give definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	In almost all texts.

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software.	In almost all texts.
6. Given two congruent figures, determine a transformation or sequence of transformations that will map one figure onto another.	In 18 texts.
Cluster: Understand congruence in terms of rigid motions.	
7. Use geometric descriptions of rigid motions to transform figures.	In almost all texts.
8. Deduce ASA, SAS, and SSS congruence through isometries.	In 9 texts.
Cluster: Understand similarity in terms of similarity transformations.	
9. Draw to show that a line not containing the center and its dilation image are parallel and to show that a line containing the center and its dilation image coincide.	Difficult to determine; drawings in almost all texts that discuss dilations.
10. Draw to show that the ratio of the length of a dilation image to the length of its preimage equals the scale factor of the dilation.	
11. Deduce AA similarity through similarity transformations.	In 8 texts.

SUMMARY

Of the six books published since the CCSSM were announced, only three seem to have closely followed the standards for geometric transformations. Given the lack of experience of high school teachers with this content when they were in school, a significant amount of professional development is likely to be necessary before this content becomes part of the standard repertoire of geometry teachers.

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CONTEMPORARY STUDY OF 5TH GRADE TEXTBOOKS: TASKS ON WHOLE NUMBERS AND THEIR COMPLIANCE WITH MATHEMATICS OLYMPIAD CONTENT

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Relevant characteristic of mathematics education quality is students' achievements on Mathematics Olympiads. The challenge is that Olympiad content differs from ordinary tasks offered in school. This reflects on the young grade students' relatively low performance in the Olympiads. The presented paper reports on comparative study of presentation of problems on whole numbers in Olympiad problem sets and in mathematics textbooks of 5th grade. The following research questions are stated:
1) What types of problems about whole numbers are included in Olympiad problems? What mathematics knowledge is necessary at the solutions of these problems?
2) How mathematics textbooks for 5th grade support the knowledge necessary for participation at mathematics Olympiads?

Keywords: whole numbers, primary school, Mathematics Olympiad

INTRODUCTION

Latvia actively participates in various international education evaluation projects (TIMSS, PISA), education mobility programmes (Erasmus, Tempus), as well as international research processes. School and university students as well have opportunities to take part in international conferences, and school subject Olympiads.

The participation in education research projects allows comparing Latvian school students' achievements internationally. The most recent data collected in Latvia demonstrate that students' performance in mathematics has improved but not statistically significantly different from the OECD average (PISA 2012 report, 2013). Additionally, the rate of high performance is below the average score. This indicates new challenges in the realisation of education policy. The excellence of education quality is reflected in the students' achievements in International Mathematics Olympiads (IMO). Looking at the IMO ranking table of countries (IMO, 2013) the position of Latvia is low (66th position in 2013).

The students included in IMO team started their competition experience from young grades. Children can participate in the preparatory Olympiad (PMO) and in the Regional Mathematics Olympiad (RMO) starting from the 5th grade. Very popular are the sessions of the Open Mathematics Olympiad (OMO), where any student in Latvia can participate. OMO is conducted regularly since 1974. In recent years, the number of participants at OMO is approximately 3000 every year.

RESEARCH OBJECTIVES

The traditional contest set is well-balanced and includes problems of elementary mathematics that cover number theory, algebra, geometry, and combinatory theory. There have to be among them the problems of algorithmic type, problems that could be solved by deductive method or different methods of reasoning (France, Andzans, 2008). One of the most important themes here is the elementary number theory. Hence the basic knowledge of whole numbers needs to be created already in the early grades.

In the scope of the presented research are the problems about whole numbers (PWN). The reason for this is the relatively low scores of 5th grade students in OMO. Although the knowledge which is tested in Olympiad problems (OLP) fully corresponds to the curriculum, their content and solution methods differ rather significantly from the standard problems given in textbooks. The jury of Olympiads has noticed that students experience difficulties both with the mathematical language and with their insufficient experience in solving Olympiad problems.

The following research questions are stated:

1) What types of problems about whole numbers are included in problem sets of PMO, RMO, and OMO for 5th grade (2003 – 2014)? What knowledge about whole numbers is necessary in the solutions of these problems?
2) Which types of tasks about whole numbers are included in mathematics textbooks for 5th grade? How these books support the knowledge necessary for participation in mathematics Olympiads?

SOURCE OF TASKS

For comparison purposes, 62 tasks on whole numbers from 5th grade Olympiads problem sets were selected. Taking into consideration the fact that some problems consist of two similarly formulated sub-problems, whose solutions may differ significantly, each was coded separately. Thus a total of 73 problem units were examined.

Five mathematics textbooks (TB) were selected for comparison (see Table 1) - experimental textbook (Mencis), two popular TB (Lude and Mencis, Mencis).

Table 1: Sources, number of PWN and their percentage of total tasks

Authors	Code	Period	Items	Total	Percentage
J.Mencis	M	1999	90	238	37.8%
I.Lude	L	2002	209	459	45.5%
J.Mencis, J.Mencis	MM	2008	290	626	46.3%
I.France, G.Lace	FL	2013	165	272	60.7%
Cornelsen Verlag	G	2010	269	472	57%
Olympiad problems	OL	2003- 2014	62	165	37.6%

The textbook “Fokus Mathematik” (Cornelsen Verlag, 2003) is a translation of an internationally approved textbook. It differs in contents, as it includes chapters on integers, but does not examine fractions. The newest textbook (France, Lace) corresponds to the last amendments of the mathematics standard in 2013. The tasks selected in TB are those from corresponding chapters precisely on whole numbers, without examining the tasks on their application or interpretations. The number of selected tasks is indicated in Table 1 (under Items). The total number of problems in these chapters is also given in Table 1 (under Total).

RESEARCH FRAMEWORK

Considering the importance of mathematics TB, many studies of their tasks have been carried out. An extensive review of such studies has recently been published in the ZDM journal (Fan, Zhu, Miao, 2013), where an analytic classification of scientific papers is provided. Inter alia, the studies on the didactic goals set in mathematics textbooks, representation of mathematical themes, and development of students’ mathematical skills are noted.

The set of problems examined in this paper is analysed in order to produce a general impression on the tasks included in TB, with an emphasis on those tasks which could ensure higher students’ results in mathematics Olympiads. As one of the parameters for coding the tasks, the base of mathematical knowledge useful for solving problems was selected (see table 2). The coding was done based on the topic examined in the problem, and the contents and context of the problem.

Table 2: Themes of mathematical knowledge, their codes and brief descriptions

Topics	Code	Description
Terminology	TM	Learning, understanding and applying new terms
Mathematical operations	MO	Conducting mathematical operations and applying their properties, transformation of numerical expressions
Natural and whole numbers, integers	WH	Application of properties of numbers and number sequences; estimation, comparison and rounding of numbers
Standard form of natural number	SF	Construction of numbers, estimation of properties of number records
Divisibility	DV	Application of divisibility and even/odd properties, finding GCD and LCM, division with a remainder
Factorisation	FC	Integer factorisation, prime number detection, prime decomposition
Integer partition	PT	Partitions of integers, calculation of the total sum of a set of numbers
Equations	EQ	Solving and formulating linear and non-linear equations, transformation of algebraic expressions

For students to successfully participate in mathematics Olympiads, they have to master their mathematical knowledge and skills. Kilpatric, Swafford, and Findell (2001) propose that mathematical proficiency includes such inter-related parts as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. In order to be able to estimate the opportunities for learning mathematical skills, the indications in task formulations which characterise the didactical goals of TB authors were analysed. The indications were classified in semantically similar groups, dividing those in three levels (see Table 3). These levels to a certain extent correspond to the proficiencies mentioned above.

Table 3: Indications provided by authors and their classification into didactical levels

Didactical levels	Description	Indications
1. Basic level	Builds understanding of mathematical concepts and the skills of applying mathematical operations	Calculate (calculate in mind); Determine (name, read, select); Check (ensure, correct)
2. Developing level	Stimulates rational and logical thinking, develops problem-solving skills	Estimate (compare, approximate); Note (remember, guess, be careful); Create (order, group, construct); Transform (change the order of operations, simplify, round)
3. Deepened level	Perfects mathematical thinking and application of mathematical language	Examine (forecast, experiment); Find (figure out, formulate a hypothesis); Justify (explain, continue the thought, formulate, prove)

OLP are formulated differently – here the problem solving indications or conditions mostly correspond to the third didactical level, and they must be derived from the context.

RESULTS OBTAINED

The comparison of the Olympiad problems with tasks given in textbooks has been done in a rather general form, by comparing the knowledge useful in solving these problems and by estimating the didactic levels of the tasks given in textbooks.

From the data in Table 4 we can see that in TB the tasks on mathematical operations are dominant, while the themes covered in OLP are rather balanced. If we examine only those problems which correspond to the themes given in the table, from WH to EQ, we can calculate the percentage deviation of OLP from the average of TB tasks (see Figure 1). The configuration shown in Figure 1 indicates the thematic difference of the OL problem set from TB tasks. Textbooks contain comparatively fewer tasks of topics WH, DV, FC, PT.

Table 4: Percentage distribution of knowledge applied by the source of problems

Topics/ Source	OL	M	L	MM	G	FL
TM	0%	10.1%	10.5%	14.2%	14.4%	7.6%
MO	11.3%	39.5%	44.7%	44.6%	54.9%	40.4%
WH	29.6%	16.8%	8.6%	8.9%	8.3%	17.2%
SF	9.6%	12.6%	9.7%	6.2%	10.7%	16.8%
DV	20%	0.8%	1.9%	6.5%	1.8%	2.4%
FC	13%	2.5%	0%	3.5%	2.5%	4.8%
PT	13.9%	2.5%	1.2%	1.6%	0.3%	6.4%
EQ	2.6%	15.2%	23.3%	14.5%	7.1%	4.4%

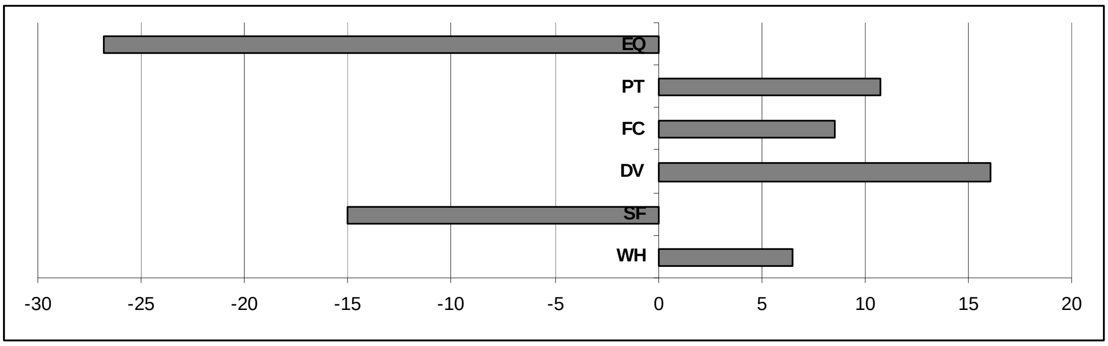

Figure 1. Deviation of OLP from the average TB tasks in percent

Summary of data on the classification of TB tasks into didactic levels is given in Figure 2. We can clearly see here that the TB authors of L and of MM have paid most attention to strengthening the students' basic skills. In turn, the authors of G and of FL have created problems which facilitate the students' deeper understanding of mathematical relationships. More than a quarter of the tasks given in the books MM, G and FL are formulated in words, without providing the algorithm of solution text tasks, while others TB contain fewer of them.

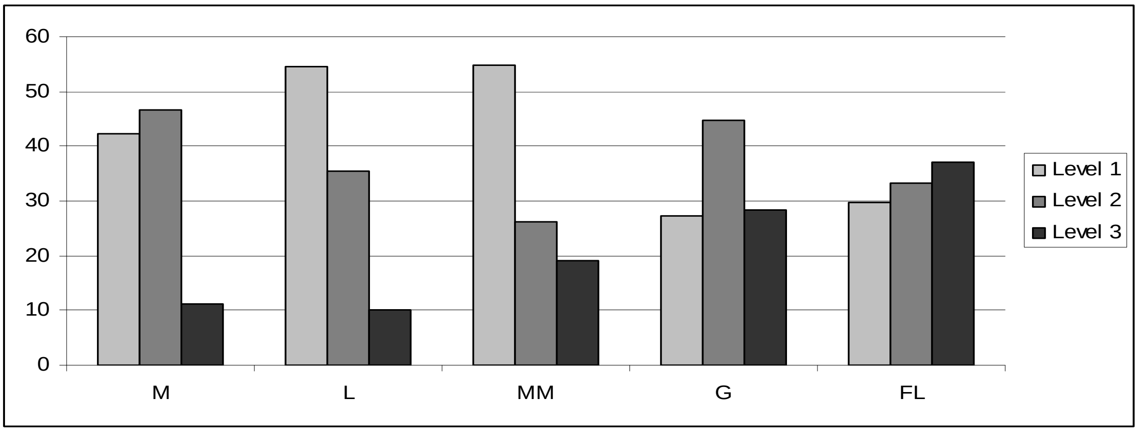

Figure 2. The percentage distribution of tasks by didactic level in each of the sources

CONCLUSIONS

Despite the fact that this article presents rather general data on the problems about whole numbers included in Olympiads and textbooks, they do give a preliminary impression on several of the currently available 5th grade textbooks. It would be possible to obtain a deeper evaluation by expanding the structure of didactical levels and supplementing it with parameters of problem complexity, similarly to the one represented in Son and Senk (2011) paper describing the framework of cognitive expectation.

The results summarised in this paper could be useful to the specialists who are involved in preparing students for mathematical Olympiads. The FL book most corresponds to contemporary study methods and is recommended as a starting position in preparing school students for Olympiad problems. It can be predicted that teachers who routinely use L or MM books must invest greater effort in their extracurricular classes. The G book can serve as excellent supplementary material to instigate students' interest in mathematical problems. Considering the specifics of this textbook, it is recommended for both 5th and 6th grades.

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AN ANALYSIS OF THE PRESENTATION OF THE EQUALS SIGN IN GRADE 1 GREEK TEXTBOOKS

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Young children often develop a partial, operational understanding of the equals sign that refers to completing an action, such as getting the answer to an addition or multiplication question, and fail to develop a relational understanding of the equals sign as a symbol that denotes equivalence. A partial view of the equals sign as an operator can be the result of primary-age pupils' overexposure to canonical equations such as a+b=c. This paper presents a preliminary analysis of the different syntaxes and formats used to present equality statements in the Grade 1 textbooks in Greece. The quantitative analysis reveals an overemphasis on presenting the equals sign within canonical equations. However, qualitative analysis reveals that the equals sign is first introduced in a context that conveys the idea of equivalence relation and is presented within an interesting mix of symbolic and non-symbolic contexts which may minimise the tendency to interpret the equals sign exclusively as an operator.

Keyword: equals sign, equations, understanding, primary mathematics, primary school, Greece

INTRODUCTION

A flexible understanding of mathematical symbols or 'symbol sense' is essential for doing mathematics and thinking about mathematics in a meaningful way (Tall, 1995). Using symbols without having developed a conceptual understanding of their different possible meanings may lead to mechanic and rigid symbol manipulation that may in turn hinder mathematics performance (Gray and Tall, 1992). Research that has explored young learners' development of understanding and use of the equals sign has shown that young children move from an initially 'operational' understanding of equals associated with completing an action or arithmetical operation to a more sophisticated, 'relational' understanding which allows a view of the equals sign as a symbol that denotes equivalence and has the meaning of 'the same as' (Baroody and Ginsburg, 1983; Behr, Erlwanger and Nichols, 1980). Jones and Pratt (2006) and Jones et al. (2013) have further refined this definition by arguing that a relational understanding includes the notion of sameness as well as the notion of substitution which entails understanding that equals also means "can be substituted for" (Jones et al., 2013). Research evidence has indicated that some primary school children do not move beyond an operator view of the equals sign. This can lead to difficulties in understanding the concept of equations in more advanced mathematics and may have a negative impact on children's understanding of algebraic principles (Seo and Ginsburg, 2003). It has been suggested that persistence in viewing the equal sign as an operator only, does not reflect developmental constraints rather, it is the result of primary-age pupils' overexposure to canonical equations such as a+b=c. Intervention studies have demonstrated that young children are capable of grasping the relational meaning of equals at a very early age (e.g. Warren, 2007) and have emphasised that developing a more sophisticated understanding of equals does not have to do with replacing an operational view with a relational view, but with complementing operator notions with relational conceptions (Jones et al., 2013). Analyses of mathematics textbooks in the U.S.A and China have revealed that the equals sign is mostly presented within arithmetic statements of the canonical form and do not support the development of an equivalence relation view (Li et al. 2008; Seo and Ginsburg, 2003). In light of the above this paper presents a preliminary exploration of the different syntaxes and formats used to present equality statements in the Grade 1 textbooks in Greece.

PRESENTING THE EQUALS SIGN: SYNTAXES AND CONTEXTS

Jones and Pratt (2006) discuss the following different syntaxes of equality statements and the kinds of understanding that have been associated with these. Canonical equations are equations of the syntactical form a + b = c (e.g. 5 + 3 = 8, 12 – 7 = 5) where expression = number. Non-canonical equations include different syntaxes such as: c = a + b (e.g. 9=5+4), a = a (e.g. 8 = 8), a + b = c + d (e.g. 3 + 5 = 6 + 2) where number = expression, number = number and expression = expression respectively (Jones and Pratt, 2006, p. 302).

When asked to make judgements about the correctness of equations, children who have a predominantly operator view of the equals sign as a symbol that denotes ‘find the answer’ or ‘do the operation’ recognise only canonical equations as correct and find it difficult to assign a meaning to non-canonical equations (e.g. Kieran, 1981). Seo and Ginsburg (2003) argue that this is because formal instruction introduces the equals sign in the context of addition sentences such as: + b = ?. They point out that children have very few opportunities to view the equals sign in other formats (e.g. 5 + 5 = 7 + ?) or in non-arithmetic contexts. For example, a non-arithmetic context would involve using the equals sign to denote that 1 metre = 100 centimetres or that 1 hour = 60 minutes. The question that this paper seeks to address is: What are the different syntaxes and contexts in which the equals sign is presented in the Grade 1 Greek textbooks?

BRIEF DESCRIPTION OF THE GREEK GRADE 1 MATHEMATICS TEXTBOOK

The Greek primary mathematics textbook has been authored and reviewed by a team consisting of University academics with expertise in mathematics education and teachers. It is published by the Pedagogical Institute of the Department of Education. The textbook consists of two core volumes and is accompanied by four workbooks. The table below presents a brief overview of the content and structure of the two core volumes which are the focus of this paper (translation into English by the author).

The analysis presented further below is informed by the types of syntaxes of equality statements that Jones and Pratt (2006) and Seo and Ginsburg (2003) have described.

Table 1: The structure of the Grade 1 mathematics textbook

<table>
<tr><td colspan="3">Volume 1</td></tr>
<tr><td colspan="3"><u>Part A:</u> Numbers: Numbers up to 20; Operations: Addition with numbers to 10; Geometry: Space orientation, Shapes; Measurement: Magnitude comparison, Coins up to 10.</td></tr>
<tr><td>Unit 1
Numbers up to 5; Space and Shapes.</td><td>Unit 2
Addition, Analysis of numbers up to 5.</td><td>Unit 3
Numbers up to 20, Sums up to 10, Coins.</td></tr>
<tr><td colspan="3"><u>Part B:</u> Numbers: Numbers up to 50, Place value, Subtraction with numbers up to 10; Operations: Sums with multiple terms, Addition bridging over ten; Geometry: Drawing of lines, Position and movement on squared paper, Shapes; Measurement: Patterns, Time.</td></tr>
<tr><td colspan="3">Unit 4
Subtraction, Drawing of lines, Patterns.</td></tr>
<tr><td colspan="3">Volume 2</td></tr>
<tr><td colspan="3">Part B continues</td></tr>
<tr><td colspan="2">Unit 5
Numbers up to 50, Units and Tens, Squared paper.</td><td>Unit 6
Units and Tens, Shapes, Time.</td></tr>
<tr><td colspan="3"><u>Part C:</u> Numbers: Numbers up to 100; Operations: Addition and Subtraction of two-digit and on-digit numbers, Addition bridging over ten, Multiplication; Geometry: Drawing lines, Puzzles, Tessellations, Shapes, Symmetry; Measurement: Measurement of continuous properties, Weight, Coins.</td></tr>
<tr><td>Unit 7
Drawing of lines, Puzzles, Addition and Subtraction, Bridging over 10.</td><td>Unit 8
Numbers up to 70, Operations, Measurement, Symmetry.</td><td>Unit 9
Numbers up to 100, Operations, Weight, Shapes.</td></tr>
</table>

SYNTAXES AND CONTEXTS IN WHICH THE EQUALS SIGN IS PRESENTED IN THE GRADE 1 GREEK TEXTBOOKS

The notion of equals and the equals sign is first introduced in Unit 2, in a chapter titled: Number Comparison, The Symbols =, > and <. Children are presented with pictures of two different-in-size groups of fish. Following a complete example, they are asked to determine the quantity and write the relevant number under each set. Then they are asked to compare the two numbers and place the appropriate symbol (=, > or <) between them to denote whether they are equal or whether one is larger or smaller than the other. Subsequently an image similar to the one below is presented and children are asked to place the appropriate symbol (=, > or <) between pairs of numbers such as: 4…3, 2…6, 8…8, 6….9, 5…5.

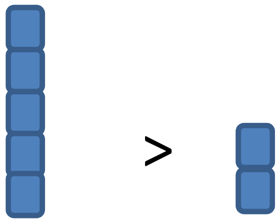
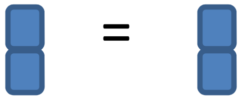
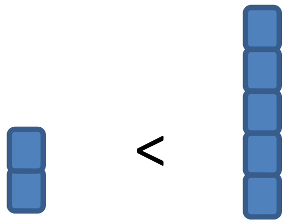
Larger than	Equal to	Smaller than
		
5 > 2	2 = 2	2 < 5

Figure 1. A reproduction of one of the pictorial representations used in the introduction of the notion of equals and the equals sign.

It is noteworthy that contrary to what previous textbook research has indicated, the introduction of the notion of equality and the equals sign occurs here within the context of magnitude and number comparison and not in the context of performing operations (Seo and Ginsburg, 2003). The equals sign is not presented with the plus or minus sign, rather, it is presented alongside symbols that denote strict inequality. On this basis it could be argued that the equals sign is introduced in the Grade 1 Greek textbook within a context that aims to convey the equivalence relation and not an operator view of the symbol. The table below presents a quantitative view of the presentation of the equals sign in the textbook. It shows the number of instances on which the equals sign is presented in canoninal or non-canonical equations and the number of instances that children are encouraged to use the equals sign in a canonical or non-canonical format.

The category ‘Presentation/canonical’ includes equations of the format: 3 + 1 = 4, … + … = …, 6 + 2 = ?, 3 – 2 = ?, 10 + 10 + 4 = ?, 2 +… = , 10 - … = ?

The category ‘Presentation/non-canonical’ includes presentation of the equals sign in formats and contexts such as: 2 = 2, 6 + 2 = 2 + 6, 7 = 5 + 2, 9 = 5 + 4, 68 = sixty eight, 68 = 60 + 8, 9 + 4 = 9 +1 +3 and also between the pictures of two children having the same height.

The category ‘Use/canonical’ includes instances where the children are encouraged to write equations and use the equals sign in a canonical format after being presented with a canonical example. For example, children are shown two sets of objects being added together to create a new set. The relevant pictorial representation is accompanied by a number sentence of the type: 2 + 6 = 8. The children are asked to write similar sentences underneath other pictorial representations showing the addition of two sets.

The category ‘Use/non-canonical’ includes instances where the children are asked to select and use the appropriate symbol to denote the relationship between two sets or two numbers, for example: 8 … 8.

Table 2. Number of instances where the equals sign is presented and used.

	Presentation		Use	
Volume 1	Canonical	Non-canonical	Canonical	Non-canonical
	65	5	11	5
Total	86			
Volume 2	Canonical	Non-canonical	Canonical	Non-canonical
	140	40	6	0
Total	186			

Table 2 shows that the equals sign is predominantly presented within canonical equations. The instances where children are encouraged to use the equals sign are limited and it is rather surprising that there are less such instances in volume two in which the equals sign appears almost twice as much as in volume one. Even though there are fewer opportunities to view and use the equals sign in non-canonical formats, when these opportunities occur, a variety of different non-canonical formats is used as described further above. Non-arithmetic contexts do not occur (apart from the height comparison example), however the equals sign is used in a variety of symbolic and non-symbolic contexts (i.e. comparing numbers as well as sets of objects or dots).

CONCLUSIONS

The quantitative view of instances on which the equals sign is presented and used in the Grade 1 Greek textbook indicates that the equals sign is presented in relation to performing operations mainly. This is in line with findings of previous textbook research. However, qualitative analysis shows that, contrary to what other textbook analysis has indicated, the equals sign is first introduced in a context designed to convey the idea of equivalence relation and not in relation to operations. Even though the number of non-canonical equations is smaller, these are presented in a variety of different formats which according to Seo and Ginsburgh (2003) can minimise the tendency to interpret the equals sign exclusively as an operator. Conclusions related to the kind of understanding that children might develop as a result of using this textbook are beyond the scope of this paper as this would require analysis of teacher practices and the learning environment within the textbook is used. It can be concluded however that the Grade 1 Greek textbook indicates efforts to develop the relational meaning of equals alongside the predominantly operational view of the symbol.

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A COMPARATIVE STUDY OF STATISTICS IN JUNIOR HIGH SCHOOLS BASED ON MATHEMATICS TEXTBOOKS OF CHINA, THE UNITED STATES AND AUSTRALIA

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The content of statistics in current mathematics textbooks used in junior high schools of China, the United States and Australia, are chosen as the objects in this study. The paper conducts a comparative study in light of content, content proportion, presentation, and content width and depth. The study analyzes characteristics of the content of statistics in the textbooks of these three countries and provides enlightenment for the statistical content in China's textbooks.

Keywords: statistics, secondary school, comparative study, Australia, China, USA

RESEARCH BACKGROUND

Statistics is gaining more weight in mathematical courses. In *Full-Time Compulsory Education Mathematics Curriculum Standards (the Experimental Version)* (MoE, 2001) promulgated by the Ministry of Education of China in 2001, "statistics and probability" is granted higher proportion and identified as a content area in mathematical courses in various stages of compulsory education, just as "numbers and algebra", "space and graphics" and "practice and comprehensive application". *The Compulsory Education Mathematics Curriculum Standards* (2011 Editionlists "data analyzing sense" as one of the important aims of mathematical courses, data analysis is specified as the core of statistics (MoE, 2012). Many countries incorporate statistics as an important part of mathematical courses in junior high schools (Cao, 2012).

The content of statistics in mathematics textbooks used in junior high schools of China, the United States and Australia, are chosen as the objects in this study. The paper conducts a comparative study in light of content, content proportion, presentation, and content width and depth, so as to provide enlightenment for the statistical content in China's textbooks.

RESEARCH DESIGN

Selection of mathematics textbooks

The paper studies textbooks issued or recommended by educational administrative bodies in three countries, which are introduced in detail as follows.

China: *Mathematics（for Junior High Schools）*, which are written according to *The Compulsory Education Mathematics Curriculum Standards (the 2011 Edition)* and were published by Beijing Normal University Publishing Group in 2012. Designed for students of grade seven to nine, the textbooks comprise of six books, and two books are used every year.

The United States: *IMPACT Mathematics*, which were published by McGraw-Hill Companies in 2009. Designed for students of grade six to eight, the textbooks include three books, namely *IMPACT Mathematics（Course 1）*, *IMPACT Mathematics（Course 2）* and *IMPACT Mathematics（Course 3）*.

Australia: There are no universal mathematics textbooks in Australia, so the paper studies *Heinemann Maths Zone 7 (8, 9, 10) VELS Enhanced* which were published by Pearson Publishing Group and are widely used in Victoria. The textbooks consist of four books.

The number of semesters for junior high schools varies from country to country, for instance, the junior high schools in China comprise of three grades for three years namely grade seven to nine, while junior high schools in Australia comprise of four grades for four years namely grade seven to ten. In the United States, grade six, seven and eight are equivalent to grade seven, eight and nine in China and Australia. Considering this, the textbooks for grade seven, eight and nine in China and Australia, and the textbooks for grade six, seven and eight in the United States are studied.

Research methods

The methods of quantitative and qualitative description are combined together.

Comparison dimensions

Content Layout: The paper analyzes tables of contents in mathematics textbooks used in junior high schools of these three countries and compares the chapters and sections concerning statistics.

Content Proportion: Through comparing the number of pages concerning statistics (mainly the main body, excluding the preface, the table of contents, the postscript), the paper conducts a quantitative description of the proportion of statistics in these three countries' junior high school mathematical textbooks.

Mode of Presentation: The study compares the style, columns and statistical activities of these three countries' junior high school mathematical textbooks.

Content Width and Depth: Content width refers to the coverage of textbooks. Considering that the paper studies statistics in these three countries' junior high school mathematical textbooks, it represents content width by knowledge points of statistics. Content depth is mainly concerned with the presentation mode of knowledge points' concepts. There are three ways of presenting concepts, namely direct description, induction and deduction, and they are granted of the value 1, 2 and 3 respectively.

Direct description: Only one case is introduced to present relevant concepts.

Induction: A general result is derived through making inductions out of at least two cases.

Deduction: A method of logical reasoning, which adopts common mathematical theoretical knowledge to explore particular or irregular mathematical concepts.

The content depth of each knowledge point is represented by the formula $S_i = \frac{1\times A + 2\times B + 3\times C}{A+B+C}$. In the formula, i refers to knowledge point, A refers to the sum of i's concepts presented through “direct description”, B refers to the sum of i’s concepts presented through “induction”, and C refers to the sum of i’s concepts presented through “deduction”.

Content depth is represented by the formula $S = \frac{\sum_{i=1}^{n} S_i}{n}$. In the formula, n refers to the number of knowledge points, and S_i refers to the content width of the knowledge point i.

RESEARCH RESULTS AND ANALYSIS

Content layout

By studying the content layout of statistics in junior high school mathematical textbooks used in China, the United States and Australia，we can conclude that statistics in China’s textbooks is introduced in the order of “data collecting, assorting and analyzing”, including two chapters, eight sections and three “combination and practice”. In textbooks of the United States, statistics is introduced along the line of “investigation”, including three chapters, five sections and eighteen “investigation”. In textbooks of Australia, it is introduced along the line of knowledge, including three chapters, eighteen sections, four“investigation”and three “design task”.

Content proportion

An international study comparing content proportion in junior high school mathematical textbooks in ten countries finds that in average, the proportions of “numbers and algebra”, “measurement and geometry” and “statistics and probability” are 56%, 32% and 12% respectively, showing that the share of “statistics and probability” is the lowest (Wu & Cao, 2013). In the mathematics curriculum standards of Australia, statistics and probability accounts for a relatively small part, yet relevant titles appeared 58 times, demonstrating its importance. In addition, as the grades increase, no visible change of these contents’ proportion is found in textbooks of various grades (Kang, 2011).

The study calculates the page number of statistics in each grade’s mathematical junior high school textbooks used in China, the United States and Australia, and the total page number of each grade, then it divides the total page number by the page number of statistics, and the result is shown in the Table 1.

	China	USA	Australia
Grade 7	37/370=0.1	39/657=0.0594	49/513=0.0955
Grade 8	26/370=0.07	36/555=0.0648	50/592=0.0845
Grade 9	4/304=0.0132	20/657=0.0304	56/560=0.1
Total	67/1044=0.0642	95/1869=0.0508	155/1665=0.0931

Table 1: total page number divided by the page number of statistics

Table 1 shows that with a proportion below 10%, statistics accounts for a relatively small part in textbooks of these three countries. The share of statistics in textbooks of all grades is fairly even in Australia and the total share is the highest among these three countries. Difference in the share of statistics is the most obvious in China's textbooks of all grades. The total share of statistics in textbooks of the United States is the lowest among these three countries.

Mode of presentation

Textbooks in three countries all follow the order of chapters and sections, with theme pictures, introductions and study aims in each chapter's beginning, review exercises in each chapter's end, and samples and exercises in sections. In textbooks of China and the United States, each section starts with problem situation or problem investigation, and the content of statistics is introduced by question strings or investigation activities which prompt students to think. There are columns like "let's do it" "let's think" "let's discuss" in textbooks of China, columns like "think & discuss" "develop & understand" "share & summarize" in textbooks of the United States, while in textbooks of Australia important statistics knowledge and concepts are presented with direct description, analogies, induction, etc. In China's textbooks, there is a "review and reflection" column in the beginning of review exercises, so that students can review what they have learned and develop a knowledge structure which suits their own cognitive stage. In textbooks of Australia, there is a "prep zone" at the start of chapters, so as to help students review all the knowledge relevant to this chapter.

A comparison of the statistics activities in these countries' textbooks shows that in textbooks of the United States, the knowledge of statistics is integrated into statistical activities, and investigation activities lead students to experience the process of discussion, comprehension and summary. In textbooks of Australia, two statistical activities named "investigation" and "design task" are designed parallel with sections. These statistical activities prompt students to think and design in the form of questions and tasks respectively, and help students apply statistical knowledge and methods into the process of solving problems and completing tasks. In China's textbooks, a column named "combination and practice" is designed parallel with chapters, enabling students to experience the whole process of "starting from specific problems, raising valuable statistical questions, collecting data, assorting data, analyzing data and making reasonable induction".

Content width and depth

Content width is represented by the number of knowledge points in these countries' junior high school mathematical textbooks. Using the formula introduced in previous texts, the content depth of statistics in these three countries' textbooks can be calculated and listed in the Table 2.

	China	USA	Australia
Content width	16	22	26
Content depth	1.1875	1.8636	1.0769

Table 2: content depth of statistics in these three countries' textbooks

From Table 2 it can be inferred that the number of knowledge points concerning statistics are 16, 22 and 26 respectively in textbooks of China, the United States and Australia. In terms of content width, textbooks of Australia ranks first, followed by that of the United States and

China. In terms of content depth, textbooks of the United States ranks first, followed by that of China and Australia.

FURTHER FINDINGS AND ENLIGHTENMENT

The content of statistics in these three countries' textbooks is distinct with their own edges

American textbooks emphasize developing students' problem awareness by integrating statistical knowledge into statistical activities and the process of problem-solving. Through inviting students to think, to discuss and to comprehend problem situations and guiding students to exchange ideas and make summaries, textbooks of the United States help students grasp relevant statistical knowledge and develop statistics awareness in the process of problem-solving. By comparison among these three countries, the content of statistics in American textbooks ranks third in terms of share in the whole book, second in width and first in depth. American textbooks raise high demands on students' thinking capability and the content is typically "precise and deep".

Textbooks of Australia emphasize the concepts and skills concerning statistics. They present important statistical knowledge and concepts through direct description, analogies, induction and so on, and enforce students' understanding with many examples, while giving exemplary steps and methods of solving general problems. Besides, there are many exercises for review and enforcement in Australian textbooks. By comparison among these three countries, the content of statistics in Australian textbooks ranks first in terms of its share in the whole book, first in width and last in depth, and coupled with a relatively low demand on students' thinking capability, statistics in Australian textbooks is typically "wide-ranging and plain".

China's textbooks follow the process of "data collecting, assorting and analyzing" in overall designing, and emphasize grasping basic statistical methods in the process of data analyzing, so that students can comprehend the role of statistics in decision-making and develop statistics awareness. By comparison, the content of statistics in China's textbooks ranks second in terms of its share in the whole book, third in width and second in depth, and the demand on students' thinking capability is lower than textbooks of the United States and higher than textbooks of Australia.

Textbooks of China can increase content width by increasing knowledge points accordingly

Statistics studies data analyzing in essence, including analyzing whether a method is good, analyzing applicable conditions of methods, creating new methods, etc.

According to the research findings and analysis listed above, it can be found that by comparison, the coverage of China's textbooks is the lowest with the smallest number of knowledge points. For instance, apart from circle graphs, bar graphs, line graphs and histograms which appeared in textbooks of all these countries, there are double bar graphs, stem-and-leaf plots and box-and-whisker plots in textbooks of the United States and Australia. It is advised that China's textbooks increase relevant knowledge points accordingly, so that students can know more statistical knowledge and methods. Naturally, the content should not be presented simply with definitions of statistics, the making of statistics graphs and calculations of numerical characteristics; rather, it should be presented with comparing each method in terms of deriving information, so as to let students know their applicable conditions and help students select the right method accordingly. In this way, the basics of statistics are embodied in the acquiring of statistical knowledge and method.

Textbooks of China can enhance statistical activities design and optimize the presentation of textbooks

The statistical process should be given particular importance to, that is, to let students experience the whole process of "data collecting, data assorting, data analyzing and making inductions". That's because the thinking mode for statistical problems is different from traditional mathematical problems in that the latter emphasizes deduction, while the former emphasizes induction since statistics concerns generalizing concrete items. To help students build statistics awareness, the most effective way would be to let them actually engage in statistical activities.

CONCLUSION

Statistical activities are integrated with statistical knowledge in textbooks of the United States, while statistical activities and knowledge are interwoven in textbooks of Australia. In textbooks of China, statistical activities and subjects concerning "combination and practice" are designed as a whole, so that after learning statistical knowledge of all chapters, students can experience a relatively whole statistical process. It is advised that while retaining the characteristics of their own, China's textbooks can learn from textbooks of the United States or Australia by designing some relatively easy statistical activities while presenting statistical knowledge and methods. Limited by prior knowledge, maybe students can only experience part of the statistical activities, but letting students gradually experience the whole statistical process in various stages of statistics learning and giving particular emphasis on each stage are already improvements. These improvements would play a better role in helping students grasp the method of using statistics to solve problems and develop the sense of data analysis.

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UNDERSTANDING OF LINEAR FUNCTION: A COMPARISON OF SELECTED MATHEMATICS TEXTBOOKS FROM ENGLAND AND SHANGHAI

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This study describes a comparison of English and Shanghai textbooks in terms of understanding the concept of linear function. Selected textbooks from both countries were analysed in a number of different ways. First, the extent of coverage was compared as well as the background context of this topic. Second, the examples and exercises for pure mathematics knowledge used in each country were analysed with regards to the extent of understanding required. 'Understanding' was defined by employing a combination of the most prominent theories from the existing literature and this included 'understanding' as both pure mathematics knowledge and application in the real world. Particularly for application, how selected textbooks presented questions and their solutions were compared. Findings suggested that this topic was equally important in selected textbooks for both countries from the coverage perspective. In general, the Shanghai textbook required more abstract understanding than English textbooks. However, the English textbooks tended to introduce linear function examples with an emphasis on a graphical approach while the Shanghai textbook had more of a focus on a symbolic/algebraic approach. In terms of applying the concept of linear function into a real life situation, the English textbooks emphasized algebraic solutions while the Shanghai textbook emphasized both algebraic and graphical solutions. These findings are discussed in terms of how the two countries under different cultural contexts view the teaching and learning of linear function. The discussion explores possible influences of those textbooks on students' performance and what could be learned from each other in the teaching and learning this topic.

Key words: understanding, comparative analysis, textbook analysis, England, Shanghai

INTRODUCTION

Recently, the results of PISA 2012 which included 65 participating countries have aroused wide concern within academia as well as with the public. Shanghai students not only maintained their highest position in the league table compared to PISA 2009, but also 'have the equivalent of nearly 3 years of (extra) schooling' above the international average around which the performance of United Kingdom students were located (OECD, 2013). In order to ascertain how this gap exists, this study tries to find out the differences in textbooks between Shanghai and England with regard to what the textbooks look like and what the characteristics of these selected textbooks are.

Many see function as a key mathematical topic at secondary school level (Brenner et al., 1997; Llinares, 2000). Therefore, one particular type of function, linear function, is chosen to embody the initial systematic learning of function.

A MODEL OF UNDERSTANDING FUNCTION

Four current models of understanding function were combined for the purposes of this study. Each model is proposed from different perspectives: different representations (Hitt, 1998), how representations act within a two-dimensional structure (DeMarois & Tall, 1996), symbolic view (Sajka, 2003) and transferring from an operational view to a structural one (Doorman, Drijvers, Gravemeijer, Boon, & Reed, 2012). We combined these four into a more general model of understanding function in order to provide a broader perspective on understanding within function. There are six levels within this combined model of understanding function: Level 1 Variable perspective, Level 2 Dependent relationship, Level 3 Translating representations, Level 4 Property noticing, Level 5 Object analysis and Level 6 Inventising. At Level 1 students would review their perspective about quantity from a constant to a variable. Level 2 comprises of three main representations (algebraic expression, tabular one and graphical one). Level 3 indicates the ability to connect different representation together, translating between the three representations. Some properties such as gradient and intercepts could be understood at Level 4. At Level 5 students can regard function as a ‘whole’ in terms of understanding. In the last level Inventising, the learner can also link to other mathematical knowledge.

RESEARCH QUESTIONS

Based on the aim of the research given above, the following research questions are put forward:

1. Does this topic have similar importance within the two countries’ textbooks?
2. What is the background context for linear function for the two countries’ textbooks?
3. What emphasis do selected textbooks place in terms of the model of understanding function, in particular which understanding level(s) do the textbooks offer to students with regard to pure mathematical knowledge?
4. What approach do selected textbooks use in terms of application, e.g. how do selected textbooks present the application problems as well as their solutions?

METHODS

An analytical framework

To examine how textbooks facilitate students' understanding, we analyse this topic at the macro level to answer research questions 1 and 2, and at the micro level to answer research questions 3 and 4. At the macro level, we use two elements of the conceptual framework from Li, Chen, and An (2009), whose research analysed the similarities and differences on conceptualizing and organizing fraction division among Chinese, Japanese and US mathematics textbooks. At the micro level, the framework of analysis for the basic knowledge parts of the text book came from the above model of understanding function. The analysis is summarised as follows:

A. At the macro stage:

1. General arrangement of the topic in terms of percentage of pages allocated;
2. Background context of this topic.

B. At the micro stage:
   1. Understanding levels in examples on basic knowledge;
   2. Application would be analysed by two aspects: (a) how problems are presented, and (b) how to resolve them using the three main representations of linear function/graph (algebraic expression/equation, graphic representation, tabular representation).

Selection of textbooks

In England, schools have great autonomy to choose the appropriate series of textbooks for their students. The English textbook series were developed for two types of ability students, Foundation level and Higher level, but both of them followed the same syllabus/national guidance for England. Three secondary schools located in the North East of England agreed to take part in this study. According to the national league table for mathematics based on the end of secondary schooling qualification results in 2012, students' performance in these three schools were within the top 30% of all England secondary state schools in England. Of the textbooks selected for this study from these schools, four of these came from Collins and the other two were from Oxford. Thus, although the selected textbooks in these schools seemed typical, we acknowledge that they are a convenience and not a representative sample.

On the contrary, the textbook is mandatory during compulsory education stage in Shanghai. Each term in the school year has one separate mathematics textbook. Linear function is introduced in the second term of Grade 8 (approx. age 14), and therefore the one appropriate Shanghai text book was included in the present study.

Data analysis

At the macro level, we examined the format of the textbook content, i.e. (1) general arrangement of the topic in terms of page use, also examining if there was a separate chapter on linear function; if not, how many sections were allocated; and (2) background context of linear function. In addition, the percentage of pages used for linear function was also used to illustrate the importance of this topic in both areas.

At the micro level, we examined two aspects: the understanding levels for linear function conveyed in the textbooks, and expected application approaches. In addition to levels of understanding, we also examined how examples were presented and the expectation of how they would be resolved. As for how application problems were expected to be resolved, the expected answer was initially coded according to the three types of representations that were possibly involved. In fact, tabular representations could not be found in any of the selected textbooks, and the Shanghai textbook required mixed-methods (algebraic expressions and graphs). The approaches were therefore re-coded as using graphic representation, equation/algebraic expression, and mixed-method.

RESULTS

Linear function at the macro level

We calculated that the percentage of page usage was the proportion of the pages devoted to linear function divided by the total number of pages in all the textbooks used at the secondary school stage. It was found that the average percentage of page usage in six Foundation/Higher level textbooks from the England was broadly similar to that in the Shanghai textbook, at around 2%, indicating that linear function has the same importance at the macro level in both Shanghai and England.

In terms of the expected previous background knowledge of function/graph, results showed that Shanghai students would have had more experience in algebraic approaches while their counterparts in England have had more experience in relating the concepts to real world situations.

Linear function at the micro level

The examples from selected Foundation level textbooks evidently placed emphasis on Level 4, without presenting Level 5 or Level 6 concepts (see Table 1). Both the Shanghai textbook and selected Higher level textbooks covered Levels 3 to 6 of the understanding model. For the more abstract understanding levels, namely Level 5 and Level 6, the Shanghai textbook provides double the percentage of examples compared to the English Higher level textbooks, with particular emphasis on Level 5.

Table 1 The percentage of examples at each level

	Level 3: Translating representations	Level 4: Property noticing	Level 5: Object analysis	Level 6: Inventising
Higher level textbooks in England	13.60%	45.40%	36.40%	4.50%
Foundation level textbooks in England	22.20%	77.80%	0%	0%
Shanghai textbook	18.20%	9.10%	63.60%	9.10%

In terms of how selected textbooks present real world problems, it seems that selected Higher level textbooks in England only use the graphical representation to find a formulae in application. The Shanghai textbook puts the emphasis on pure word problems to generate the formulae. On the other hand, how problems are expected to be resolved is quite opposite in approach. The selected Higher level textbooks in England required a single type of solution (algebraic expression/equation), while the Shanghai textbook required students to use two kinds of representation (algebraic expression/equation and graph) to solve the problem. In conclusion, the Higher level English textbooks tended to be presented in a graphical way and solved as algebraic expressions. In contrast, the Shanghai textbook mainly used pure word problems and required two kinds of representations, algebraic and graphic approach, within solutions if possible. Shanghai students were therefore given more opportunities for using different representations in terms of application.

DISCUSSION AND CONCLUSION

Our analysis from the macro perspective suggests that the concept of linear function is viewed as equally important by both countries' mathematics textbooks with similar percentages of pages used. However, there are important differences. First, previous knowledge of this concept or the approach to function are opposite in the two countries' textbooks. In England, the concept of linear function is based on the real world and then expressed mathematically, while for their counterpart in Shanghai the abstract algebraic approach is the starting point. This finding is consistent with the characteristic of their own curricula in that the English curriculum emphasizes the enrichment of contexts while the Chinese curriculum pays more attention to mathematical knowledge structures and systems (Bao, 2002). Within this finding of course, we also acknowledge that the approach to mathematics knowledge would have historical and cultural influences.

Second, the model of understanding function used in our research demonstrates that the understanding requirement of linear function is indeed higher in the Shanghai textbook than that of English textbooks. However, we could still find that the Shanghai textbook had a weaker stage at Level 4 Property noticing, especially for the meaning of gradient in the system of Cartesian graphing, though this will subsequently be introduced in the Grade 11 (approx. age 17).

Thirdly, the application of linear function is given the same important emphasis in Higher level English textbooks as it is in Shanghai textbooks. The differences in how problems are presented are consistent with how each country approaches function, which in England is a preference for a graphical approach while Shanghai tends to an algebraic approach. There is a debate concerning which approach or representation would be most effective in terms of students' learning outcomes. It seems with respect to learning outcomes, the symbolic or algebraic approach is better than the graphic or tabular one (Cox & Brna, 1994; Gagatsis & Shiakalli, 2004). However, Abdullah (2010) argued that students who operate superficially with the symbols showed difficulties using the Cartesian graph. Ideally, textbooks should use a variety of representations to promote understanding (Gagatsis & Shiakalli, 2004) and as a result to improve students' academic performance. Selected English textbooks from this study displayed more graphical features while the Shanghai textbooks emphasised more abstract and symbolic features. Further research should include systematic examination of textbooks and causal relationships between textbooks and students' learning outcomes, which could make sense of why a certain way of presenting is better than others (Fan, 2013).

In terms of future research, textbook analysis provides a glimpse into possible differences within classroom instruction. Therefore, possible cross-system differences in teachers' use of textbooks for developing classroom instruction will be investigated by means of teachers' interviews.

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THE STUDY OF GEOMETRIC CONTENTS IN THE MIDDLE GRADE MATHEMATICS TEXTBOOKS IN SINGAPORE, TAIWAN, AND U.S.A.

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The purpose of this study was to compare the differences of geometric contents in the middle grade mathematics textbooks among the KH (Kang Hsuan) in Taiwan, the CMP (Connected Mathematics Program) in U.S.A., and the MSN (New Syllabus Mathematics) in Singapore. The quantitative and qualitative methods were used to examine the differences on: (1) the total geometric problems, (2) the topic of geometry among the three textbooks. The results showed that there is a difference on the total numbers of geometric problems among the three textbooks. Data also showed that there are similarities and dissimilarities on the themes in geometry among three versions. They all highly focus on the theme Triangle. The ranked second themes are different among the three versions. The CMP focuses on Polygon in geometry materials. The KH textbooks put more emphasis on Circle. The NSM focuses on Solid geometry. Implications for possible curriculum revision and future research studies are discussed.

Keywords: geometry, middle grade mathematics, Singapore, Taiwan, USA

INTRODUCTION

The findings of mathematics textbooks related studies indicated that mathematics textbooks play an important role in mathematics learning and teaching (Cai & Ni, 2011; Fan, Zhu, & Miao, 2013; Stein, Remillard, & Smith, 2007; Reys, Reys, & Rubenstein, 2010; Tarr, Chavez, Reys, & Reys, 2006). The quality of mathematics textbooks will influence students' learning outcomes and the teachers' teaching efficiency (Reys & Reys, 2006; Stein et al., 2007; Tarr et al., 2006; Törnroos, 2005). Middle grade students in Singapore have high achieving in mathematics achievement, and New Syllabus Mathematics (NSM) has the highest market share (80%) in Singapore. On the other hand, since Taiwan middle schools can choice textbook edition freely, and there are three main publishers. Kang Hsuan's market (KH) share is the highest and about 38.6% (ETTV, 2007). The Connected Mathematics Program (CMP), which was funded by National Science Foundation to improve American mathematics education which are widely used in U.S.A. The research of USA plays a key role in mathematics education community all over the world. Therefore, this study focused on comparing the similarities and dissimilarities of geometry topics in America CMP, Singapore NSM, and Taiwan KH Mathematics textbooks. The purpose of this study was to examine the differences on the topic of geometry among America, Singapore, and Taiwanese middle grade mathematics textbooks. Accordingly, the research questions are as following:

1. Are there any differences on the total items on the topic of geometry among CMP, NSM, and KH textbooks?
2. What are the similarities and differences on the topic of geometry introduced among CMP, NSM and KH textbooks?

METHOD

Sample

The CMP textbooks are designed for grade 6-8 students. It highlights the learning of mathematics concepts via problem solving and encourages discussion and exploration (Herbel- Eisenmann, & Wagner, 2005). Six units related to the topic of geometry included in this study (two units “Shapes and Designs” and “Covering and Surrounding” in grade 6; two units “Stretching and Shrinking” and “Filling and Wrapping” in grade 7; and two units “Looking for Pythagoras” and “Kaleidoscopes, Hubcaps, and Mirrors” grade 8).

The KH textbooks are designed for grade 7-9 students. It highlights to foster students’ ability on mathematical thinking and reasoning (KH, 2010). Seven units related to the topic of geometry included in this study (four units “Square root and Pythagorean Theorem”, Geometric figures and ruler and compass constructions”, “Basic properties of triangle”, and “Parallel” in grade 8; three units “similarity”, “Circle”, and “Geometry and Proof” in grade 9). The NSM are designed for grade 7-10 students based on the Guideline of Singapore Ministry of Education (Singaporemath. Com. Inc, 2010). This study only considered 16 units related to the topic of geometry from grade 7-9. Six units, including “Estimation and Approximation”, “Perimeter and Area of Simple Geometrical Figures”, “Volume and Surface Area”, “Basic Geometrical Concepts and Properties”, “Angle Properties of Polygon”, and “Geometrical Constructions” are in grade 7. Three units, including “Congruence and Similarity”, “Pythagoras’ Theorem”, “Volume and Surface Area” are in grade 8. Seven units, including “Coordinate Geometry”, “Congruent and Similar Triangle”, “Area and Volume of Similar Figures and Solids”, “Trigonometrical Ratios”, “Further Trigonometry”, “Measurement–Arc Length, Sector, Area, Radian Measure”, “Geometrical Properties of Circles” are in grade

Analysis Framework

“One item” was used as an analysis unit. For example, A rectangular field is 13 m long and 10 m wide. It has a cement path m wide around it. What is the area of the cement path? (Teh & Loh, 2007a, p172). This item only included one question. It was defined as “one item”.

Based on the mathematics guidelines of geometry of America, Taiwan, and Singapore, 7 main categories were defined as “Parallel”, “Simple Geometric Figures”, ”Triangle”, ”Polygon”, “Circle”, “Coordinate geometry”, and “Solid geometry” (KH publication Inc., 2010; Singaporemath. Com. Inc., 2010; Teh & Loh, 2007b).

Reliability

The reliability of this study based on ‘inter-rater reliability’. The author and two inter-raters who are experienced mathematics teachers coded the textbooks independently. The test result of reliability is 0.93.

RESULTS

Results showed that the total items related to geometry in CMP had 766 items. The total items of geometry are highly over the total items of geometry in KH (286 items) and NSM (324 items). This finding is consistent to the earlier studies (Fan & Zhu, 200; Insook, 2008; Reys & Reys, 2006; Yang, Reys, & Wu, 2010) that American mathematics textbooks covered too many topics.

Table 1: Total items on the topic of geometry among CMP, NSM, and KH textbooks

	KH in Taiwan	CMP in America	NSM in Singapore
Total items	286	766	324

Table 2 reports the distributions of items on the 7 main categories among the three countries' textbooks. Data showed that all three textbooks highly focus on the topic of Triangle (KH: 37.8%; CMP: 29.2%; NSM 40%). The NSM in Singaporean has the highest percentages of problems on the topic "Triangle" (40%) than KH (37.8%) and CMP (29.2%) textbooks.

Table 2 reports the distributions of items on the 7 main categories among the three textbooks

KH	Triangle	Circle	Polygon	Simple Geometric Figures	Coordinate geometry	Solid geometry	Parallel
	37.8%	20.6%	15.0%	13.6%	5.2%	4.9%	2.8%
CMP	Triangle	Polygon	Simple Geometric Figures	Solid geometry	Coordinate geometry	Circle	Parallel
	29.2%	24.5%	16.8%	13.6%	12.9%	2.1%	0.8%
NSM	Triangle	Solid geometry	Circle	Polygon	Coordinate geometry	Parallel	Simple Geometric Figures
	40.0%	19.1%	18.9%	11.7%	4.0%	3.1%	1.2%

Three sets of textbooks in the main category sorting on second geometric subject materials is quite different. The sort of the second topic in KH textbooks is "Circle" (20.6%); The CMP is "Polygon" (24.5%); while the sort of the second in NSM is "Solid geometry" (19.1%). Three sets of textbooks in the third geometric subject materials also have different sorting. The KH sort of a Polygon third (15.0%); The CMP sorted of Simple geometric figures third (16.8%); while the NSM is the Circle (18.9%). Three sets of textbooks in the least important themes are also different. The topic of "Simple geometric figures" in NSM has the lowest proportion (1.2%), but in the CMP and KH respectively share of 13.6% and 16.8%. Meanwhile, the lowest proportion in KH and NSM is the topic Parallel (KH: 2.8%; NSM: 0.8%). On the distribution of the three materials, the three sets of textbooks before the two topics that occupy the sort of materials over half, showing three versions of the sort of attention in the first two themes. In addition, three sets of textbooks in the topic of “parallel” arrangement on the theme is spontaneously low.

DISCUSSION AND CONCLUSION

Comparing to the two Asian countries, the CMP textbooks in U.S.A. include double and triple more problems than the NSM in Singapore and the HK in Taiwan. This result is consistent with the earlier studies (Reys, Reys, & Chavez, 2004; Zhu & Fan, 2006). The more problems included in the textbooks, whether or not students should have better performance on mathematics? This probably needs further studies to confirm it. In addition, data also showed that three textbooks all highly focus on the geometric theme-Triangle. This indicates that the topic of triangle has been considered to be the most important topic in geometry in the middle grade level. Results also showed that it is different in the main category sorting on second geometric subject materials among the three versions. The ranked second themes in geometry for CMP, HK, and NSM are Polygon, Circle, and Solid geometry, respectively. Why they all highly focus on the topic of Triangle, and the ranking second topic is different? This needs further studies to explore it.

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A COMPARISON OF FUNCTIONS IN MIDDLE SCHOOL TEXTBOOKS AMONG FINLAND, SINGAPORE AND TAIWAN

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The purpose of this study was to examine the differences on the topic of functions among Finnish, Singapore, and Taiwanese middle grade mathematics textbooks. This study adopted a content analysis method to examine how the textbooks introduced functions and the problem types presented in the middle grade textbooks. Results show that the three textbooks use different ways to introduce the concept of functions. The Finnish textbooks use function machines with input and output tables to introduce the concept of function. The textbooks in Taiwan use verbal and visual representation to introduce the concept of function. However, the Singaporean textbooks use verbal representation. Regarding representation forms in problems, the majority of problems in all three countries are provided in the purely mathematical form. However, the distribution of the representation forms in the Finnish textbooks is more balanced. The problems in the Finnish textbooks included a fair number of visual form and verbal form whereas the problems in the Taiwanese and Singaporean textbooks inclined to use purely mathematical form. About 98% of function problems in the middle grade textbooks in Taiwan and Finland and 86% in Singapore are close-ended problems. The Singaporean textbook has more open-ended problems than Finnish and Taiwanese textbooks. Comparing to the two Asian countries, the Finnish textbooks have more problems but these problems are more straightforward in terms of complexity than the Asian countries.

Keywords: middle grade mathematics, functions, Finland, Singapore, Taiwan

INTRODUCTION

Mathematics textbooks are generally agreed as an important resource in support of mathematics teaching and learning (Cai, Nie, & Moyer, 2010; Fan, Zhu, & Miao, 2013). A fair number of researchers have shown their interest in comparing mathematics textbooks and discussing how textbooks affect mathematics teaching and learning (Senk, Thompson, & Wernet, 2014). Particularly, the Third International Mathematics and Science Study (TIMSS) studies have analyzed hundreds of textbooks over 50 countries (Fan & Zhu, 2007). This shows the importance of comparing the differences among different countries.

The Finnish, Singaporean and Taiwanese textbooks were selected in this study because these three countries were usually thought as high performing countries in the international comparison tests, in particular in PISA). In addition, there has already much research about textbooks comparison in East Asian countries and US (e.g., Hong & Choi, 2014) but there is relatively less research in East Asian and European countries. We believe that cultural differences between these two regions will reflect on their textbooks and this will finally affect their mathematics teaching and learning.

From above discussion, the purpose of this study was to examine the differences on the topic of functions among Finnish, Singapore, and Taiwanese middle grade mathematics textbooks. Accordingly, the research questions were as following:

1. How was the topic of functions introduced among Finnish, Singaporean and Taiwanese textbooks?
2. What representations forms (pure mathematical, verbal, visual or combined) were used in problems by Finnish, Singaporean and Taiwanese textbooks
3. What context types (application or non-application) were used in problems by Finnish, Singaporean and Taiwanese textbooks
4. What response types (open-ended or close-ended) were used in problems by Finnish, Singaporean and Taiwanese textbooks
5. What cognitive demand types (memorization, procedures without connections, procedures with connections or doing mathematics) were used in problems by Finnish, Singaporean and Taiwanese textbooks

METHOD

This study adopted a content analysis method to analyze the selected middle grade textbooks (7^{th}-9^{th} grade). *Laskutaito mathematics textbooks* (WSOY, 2009) in Finland, *New Syllabus mathematics textbooks* (Teh & Loh, 2011) in Singapore and *Kung Hsuan mathematics textbooks* (Kang Hsuan Educational Published Group, 2012) in Taiwan were selected as representative textbooks for each country. These textbooks were most popular textbooks in each country which were adopted by many schools in each country. We coded all of the problems in the student textbooks, including worked examples (with solutions), exercises (with no solutions) and summary test problems provided in the end of the textbooks chapter.

Analysis framework

This study applied vertical analysis (Charalambous, Delaney, Hsu, & Mesa, 2010) which offered in-depth understanding of mathematical content. According to this approach, the problems in the textbooks were analyzed by its (1) content sequence, (2) representation forms, (3) context types, (4) response types and (5) cognitive demand types. (1) The content sequence implied how each topic was introduced and developed through the three textbooks (Hong & Choi, 2014). That is, how a lesson began and what sequence it followed. (2) Four types of representation forms (Zhu & Fan, 2006) were defined as: (2.1) purely mathematical form meant mainly mathematical expressions was included in a problem; (2.2) verbal form meant mainly using written words described a problem. (2.3) visual form: A problem was solved mainly by using its graph, chart, table, figure or any other visual objects. (2.4) combined form was a problem contained two or three of those forms above and there was no clearly distinction about which one was used. (3) An application problem was posed under the context of the real-life situation; a non-application problem was a situation unrelated to any practical background in everyday life (Zhu & Fan, 2006). (4) An open-ended problem is a problem with more than one correct answers and a close-ended problem is a problem having only one correct answer even though there are many approaches to the correct answer (Zhu & Fan, 2006). (5) Stein, Smith, Henningsen and Silver's (2000) classification of cognitive demand of problems were adapted in this study. There are four levels of the cognitive demand types: (1) Memorization means that problems asking students to recall the basic facts, rules, formula etc. There is no connection to concepts (2) Procedures without connections are usually step-by-step procedural calculations without connection to concepts. No explanation is needed in this type of problems. (3) Procedures with connections are problems that require students not only to do calculation procedures but also to make purposeful connections to meaning or relevant mathematical ideas. (4) Doing mathematics means that problems require students' complex thinking with exploring and understanding concepts. The solving process of this type of problems is usually not predictable or rehearsed.

Reliability

First, three coders individually coded 20% of all the problems in each textbook. The inter-rater reliabilities (Cohen's kappa) ranged from .72 to .88. All the differences were then reconciled. After that, each coder was distributed to code all the remaining problems in one country's textbooks.

RESULTS

The concept of functions are introduced in different approaches among the Finnish, Singaporean and Taiwanese textbooks though the Singaporean and Taiwanese textbooks use a more similar way. The Finnish textbooks initially introduce functions by the function machine with an input and output table attached while the Singaporean and Taiwanese textbooks use situational problems and ask students explore the relationships between quantities in those problems. However, the way how Singaporean and Taiwanese textbooks use the situational problems were somewhat different. The Singaporean textbooks explicitly indicate relationships between quantities in those problems but the Taiwanese textbooks ask students to use tables to explore relationships by themselves before giving an explicit explanation.

Table 1 shows the distributions of problems in the representation form among three countries. The majority of problems in all three countries are provided in the purely mathematical form (FI: 37.1%, SG: 67.8%, TW: 56.1%). However, the distribution of the representation forms in the Finnish textbooks is more balanced. The Finnish textbooks included a fair number of visual form (30.6%) and verbal form (23.0%) problems whereas the Taiwanese and Singaporean textbooks inclined to use purely mathematical form problems.

Table 1: Distributions of problems in the representation form among three countries

Representation	Finland		Singapore		Taiwan	
Form	N (526)	%	N (121)	%	N (171)	%
Purely math	195	37.1%	82	67.8%	96	56.1%
Visual	161	30.6%	25	20.7%	17	9.9%
Verbal	121	23.0%	13	10.7%	30	17.5%
Combined	49	9.3%	1	0.8%	28	16.4%

Note. ( ), the total number of problems

Table 2 shows the distributions of problems in the context types among three countries' textbooks. It can be found that the Taiwanese textbooks have higher percentages in application problems (23.4%) while the percentages in Finnish and Singaporean textbooks are around 10%. The results in the Taiwanese textbooks is somewhat different from the earlier studies which show that textbooks in the East Asian countries usually have less application problems in comparing to the non-East Asian countries (Zhu & Fan, 2006).

Table 2: Distributions of problems in the representation form among three countries

Context	Finland		Singapore		Taiwan	
Types	N (526)	%	N (121)	%	N (171)	%
Application	70	13.3%	14	11.6%	40	23.4%
Non-Application	456	86.7%	107	88.4%	131	76.6%

Note. ( ), the total number of problems

Table 3 shows the distributions of problems in the response types among three countries' textbooks. It is found that about 98% of problems in the textbooks in Taiwan and Finland and 84% in Singapore are close-ended problems. The Singaporean textbook has higher percentages of open-ended problems (14%) than Finnish (1.7%) and Taiwanese (2.3%) textbooks.

Table 3: Distributions of problems in the response types among three countries.

Response	Finland		Singapore		Taiwan	
Types	N (526)	%	N (121)	%	N (171)	%
Open-ended	9	1.7%	17	14.0%	4	2.3%
Close-ended	517	98.3%	104	86.0%	167	97.7%

Note. ( ), the total number of problems

Table 4: Distributions of problems in the cognitive demand among three countries

Representation	Finland		Singapore		Taiwan	
Form	N (526)	%	N (121)	%	N (171)	%
Memorization	74	14.1%	0	0.0%	4	1.9%
Procedures without connections	256	48.7%	56	46.3%	59	34.2%
Procedures with connections	193	36.7%	48	39.7%	86	50.9%
Doing mathematics	3	0.6%	17	14.0%	22	13.0%

Note. ( ), the total number of problems

Table 4 shows the distributions of problems in the cognitive demand among three countries. It is found that the Taiwanese and Singaporean textbooks have more percentages in the high level of cognitive demand problems (procedures with connections and doing mathematics) whereas the Finnish textbooks have more percentages in the low level of cognitive demand problems (memorization and procedures without connections).

CONCLUSION

Comparing to the two East Asian countries, the Finnish textbooks have more problems but these problems are more straightforward in terms of complexity than the East Asian countries. The function machine has been recognized as a better way to introduce functions (Tall, McGowen, & DeMarois, 2000) and visual presentations benefit students' problem solving performance (Cai, 1995). In addition, various types of problems presented in a more balanced way may help students learn a concept more coherently (Zhu & Fan, 2006). From above discussion, we suggest that the Taiwanese and Singaporean textbooks should consider including more visual or verbal representations in their textbooks. In contrast, the Finnish textbooks may increase the complexity of their problems.

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MATHEMATICS TEXTBOOK RESEARCH AND DEVELOPMENT FOR THE PROMOTION OF INDEPENDENT LEARNING AND INQUIRY LEARNING

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As the most important resource of students' learning, textbook should not only show the knowledge of the subject, but also show the occurrence and development process of the knowledge, and embody the learning process of the students. As the main carrier of students' learning, textbook should be kept some space for the students to operate directly in it, such as record and list systematically. Inquiry learning activities should be designed in textbooks, and important results of the activities should be showed appropriately to facilitate the students' learning after class. But the results should not be arranged immediately after the inquiry activities. We need to make innovation of the style of textbook and show the results of inquiry learning by means of learning links and so on. In order that the students can find out their own learning situations, we suggest the addition of learning objective and learning assessment. In order to improve the students' independent learning capability, we suggest showing moderately and summarizing the learning structure of the related knowledge; showing the guidance process of the learning methods, such as reflection after solving problems; and providing moderate help by means of prompt and so on at the place where we estimate the students have difficulties. Considering the difference of students' capability, help should include several levels, such as first showing the general help and then the more specific one. Thus all students with different learning capability can obtain help and necessary inquiry space from the textbooks.

Keywords: independent learning, inquiry content, textbook form, China

RESEARCH BACKGROUND

There is a Chinese proverb: teach a man how to fish rather than give him a fish. This is the eternal pursuit of education, especially in the information age of knowledge explosion. Therefore we should try to improve the students' independent learning and inquiry learning capability. Since textbooks are important carriers of students' learning, their style should lead the direction of teaching reform, so as to promote students' independent learning and inquiry learning.

Before the 21st century, most Chinese researchers believe that textbooks and teaching methods are detached. So textbooks are only designed to show the knowledge clearly, without considering the students' learning style and the classroom teaching mode. The result of such design is that students obtain the knowledge only by their independent reading or teachers' explaining. Based on such textbooks, the students' learning is often a process of 'understanding and acceptance'.

Since the 21st century, a new round of curriculum reform has been carried out in China. Many useful attempts are made in textbooks, such as paying attention to the real life, so as to excite the student' learning interest; paying attention to the students' activity process, so as to promote the accumulating of their activity experiences; paying attention to the students' self-inquiry, so as to encourage their discovery learning and so on. But on dealing with the results of inquiry activities, the following dilemma often occurs: one approach is to hide the activity results, which will be revealed in the classroom exchange after the students' activities; another one is that the activity results are presented immediately after the students' activities. The former ensures that the inquiry activities must be carried out, since no inquiry activities lead to no subsequent knowledge. But it can also bring some troubles, such as how do those students with poor learning ability do if they cannot obtain the conclusions, and how do the students who haven't participated the classroom learning carry on the self-learning. Moreover, the lack of some intermediate links requires that the teachers decompose and refine the contents once again when they use the textbooks. The latter can help the students to learn by themselves after class with the activity results as a reference. But in most textbooks, activity results are arranged immediately after the activity contents, thus the students' eyes will unconsciously "discovery" the answers or conclusions. Such inquiry is easily reduced to a false one, which cannot embody the value of the activities scheduled.

In recent 10 years, some schools in China have launched some teaching reforms with guidance cases as the carriers and advocating independent inquiry, cooperation and exchange. But many guidance cases cannot follow the students' cognition laws well, or cannot embody the functions of guidance well.

RESEARCH WORK AND CONCLUSIONS

From January 2011 to June 2012, we unite the teachers of Teaching Research Centre of Chengdu Longquanyi District, the sixth middle school of Yichang and other five middle schools to start the design research of guidance cases, integrate the guidance cases and textbooks, and work out some textbooks suitable for independent learning and inquiry learning.

In the course of the research, we reached the following consensus:

Textbook should pay attention to the development of the students' inquiry capability; hence textbook should show the formation process of mathematics knowledge rather than show simply the mathematics knowledge.

The use object of textbook are students, so textbook should follow the laws of learning knowledge and expand in order rather than rely on the teachers' processing once again.

As the carrier of the students' learning, textbook should be easy for the students to use. Students can directly learn, reflect and list systematically in textbook. Textbook should be a good helper of the growth record of students' learning and review.

Textbooks are learning materials rather than teaching materials.

Based on the above understanding, we work out a redesign of textbook. The specific measures are listed as the following.

Expanding content according to the learning process

Learning materials should follow the learning laws of subject knowledge. The learning process in accordance with most students' cognition should be designed and showed specifically in textbook. According to such learning process, most students can basically reach the learning objectives with the help of their classmates and teachers. Of course, the learning ability has difference. Therefore, based on following the basic learning process, the specific links are required to have certain elasticity, so that the students with different learning ability can make suitable choice. For specific cases, see the Figures below.

Adding the learning objective and evaluation links

We suggest the addition of learning objectives. The students will have correct direction in their learning if they grasp the learning objectives. Certainly, they can also compare with the learning objectives after they have learned the textbook, and evaluate the learning effect by themselves. The learning objectives are written for the students, so we should use those terms which can be easily accepted by the students, rather than use terminology. If there's any terminology, we try to simplify it. For example, "comprehend the squared difference formula" is inferior to "can tell the structure characteristic of the squared difference formula and apply it to simple calculation".

We suggest the addition of learning evaluation links, and the deployment of related reference answer and assessment scale, so that the students can detect by themselves, find out their own learning situation and learning evaluation.

Keeping space for students to operate directly

Since textbook is aim to be learning material, students can learn directly on it. Therefore, we should arrange appropriate learning activity for the students to consolidate and practise, reflect and exchange, and we should keep space in textbook for students to write answers.

The capacity of textbook may increase with the added practice, and some people may worry about this will increase the economic burden of the students and the related departments. In fact, the added practice in textbook can substitute the workbook bought by students, and thus reduce the fees for buying workbooks. Moreover, such textbook is easier for the students to use, and avoids the interchange between workbooks and textbooks. The added practice can also effectively overcome the confusion and mixed quality of supplementary material at the present stage. Taking into account of the fees and format etc., it is also feasible to work out some workbooks independently.

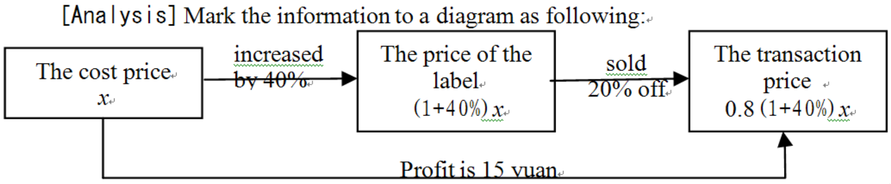
Making innovation of the style and highlight the probe space

Textbook is expected to encourage students' independent learning, make students to solve those problems that can be done by themselves, and provide the chance for the students to inquiry independently. If students encounter some difficulties when they solve problems, or if some important conclusions are expected to be reflected in textbook to arouse the students' attention, then students can choose learning links and other similar manners; for example, Figure 1.

[Example · demonstration]

A shop marked the price of a kind of clothing by improving 40% of the cost price, and sold 20% off. The profit is still 15 yuan for each. What is the cost price of this kind of clothing?

[Analysis] Mark the information to a diagram as following:



(2) According to the diagram, try to describe a certain amount with algebraic expression containing x from different angles.

[Solution]

Learning links

Figure1. Case 2: The application of the linear equation with one unknown

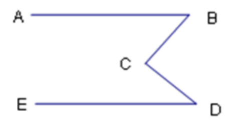
Helping students timely and moderately, emphasizing learning guidance

Students' learning is inseparable from the help and guidance of textbook. Therefore, textbook should show that it can help students and promote students' independent learning. This can be showed as the following:

(1). Prompting timely

Case 3 in Figure 2 show that appropriate hint is necessary, because the students of Grade 7 have difficulties in solving this problem after they have just learned the properties of parallel lines and the sum of three interior angles of a triangle.

Known: AB//ED, prove: ∠ABC+∠CDE=∠BCD.
(six lines left below for students to write the proof)

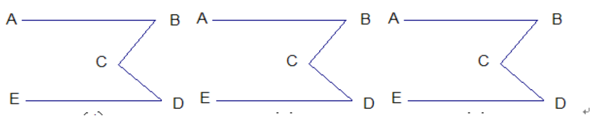


Hint 1: In general, what conclusions can be obtained from the condition of "two straight line parallel "? What is the difficulty of using the condition?

Hint 2: Conclusion is "∠ABC+∠CDE=∠BCD", so we may consider to split ∠BCD into two smaller angles, one equals to ∠ABC, and the other equals to ∠CDE. Which theorem involves one angle equals to two angles?

Reflection 1: What is your difficulty in solving this exercise? How do you overcome the difficulty?

Reflection 2: Do you have other methods? Give some annotation to show your opinions in the diagram.



Variance 1: Exchanged the condition and conclusion, does the proposition still holds?
When ∠ABC+∠CDE=∠BCD, is AB parallel to ED?

*Variance 2 : If the point C is not between AB and ED, what are the conclusions? Can you make some new guess?

Figure 2.Case 3: The solving and changing of a problem

The help should be temperate and not by one step, which may be too specific and cannot be transplanted to other situations; the students who are often helped by one step may gradually lose the habit and ability of thinking, because of the loss of thinking space and chance. We suggest providing multi-lever help so that students with different lever have their own thinking space and obtain the help that they need.

In Case 3, if the condition "two lines are parallel" in Hint 1 is omitted, then Hint 1 become "In general, what conclusions can be obtained from the condition? What is the difficulty of using the condition?" Such hint is universal and can be easily transplanted. There are also some similar hints, such as "What is the characteristic of the conclusions?" "How do you connect the conditions with the conclusions?" In one word, Hint 1 is a kind of hint with thinking and habit of problem-solving. If the students still cannot solve the problem with Hint 1, then Hint 2 can provide a little more specific help. If there are still some students with poor learning ability cannot solve the problem, then an answer linking may be workable.

(2). Increasing the guidance of learning methods

The guidance of learning methods is especially important to promote the students' independent learning; hence textbook should show these learning methods. There is much specific guidance of learning methods, such as reading guidance, reflection guidance and so on. The following two paragraphs only explain the significance of reflection guidance and how to show reflection guidance in textbook.

Though many students may be good at problem-solving, they cannot excavate the thinking methods, or accumulate the consciousness and initiative of mathematics activity. So it is necessary that teachers deliberately guide the students to think about the problem-solving process, to consciously externalization the activity experiences in the problem-solving process; remind the students to expand and extend the problems, to think about the relationship between problems, and to think about problems from the height of structures; remind the students to think about the methods of thinking and transplant the methods to other context. In fact, some outstanding teachers already have the fine tradition on these aspects, such as the so called multiple solutions for one problem, multiple changes for one problem, one solution for multiple problems and so on. Textbook should show clearly these processes to promote that the students experience these activity processes, feel such learning methods, and form such habit and ability.

In Case 3, Reflection 1 lead the students to think about the difficulties and methods of problem-solving; while Reflection 2 lead the students to think about the problems from different perspective, and develop the students' diversity conscious of thinking about problems. Variance 1 and Variance 2 guide the students to obtain the variance of the original problem. Variance is a highlight of mathematics teaching in Chinese, and many teachers can organize the teaching by applying variance skilfully. But we need to penetrate the method of variance to the students, and guide the students to master variance. Here Variance 1 and Variance 2 point out clearly the methods of variance. The students can obtain the methods and ability of variance by long-time such training. Thus, one problem turns into a series of problems. After the students grasp variance, they will naturally jump out from the sea of problems.

(3). Showing moderately the learning structures

Mathematics learning has some certain laws, for example, when one learn a new concept, he will experience a process of "perception of the reality prototype, abstract of the concept, identification of the concept and the quest for the reality prototype, the context application of the concept". Another example is the inquiry learning of propositions. One may also experience a process of "the introduction of context, inquiry and discovery, test and verification, comprehension, consolidation and application". If the students are guided to perceive such learning structures, then they can transplant the proposition to the learning of subsequent related content, and cultivate the ability of constructing independently the learning system. Therefore, textbook can lead the students to perceive and summarize such learning structures by virtue of typeface, font size, word crude or appropriate language guidance.

Extract some certain words from Case 1, as shown in Figure 3, we can easily see the learning structure of the formula. In fact, such learning structure can be entirely transplanted to other similar formulas.

Moreover, since the reading object of textbook are students, sentences in textbook should be simple and clear, easy to read and comprehend, and facilitate the students to perceive the objectives, requirements, mode of every link, avoiding the obstacle of understanding and wasting time to comprehend the unintelligible words. Use more modal of communication with the students, and apply vivid language as far as possible. Such cordial process of communication can excite the learning interest of the students.

Activity 1: Review
Activity 2: Explore the formula
[Reflection · communication]
Believe you have felt that the results of the above 4 exercises have some characteristics.
[Induction · generalization]
[Reasoning · argumentation]
Activity 3: Comprehend the formula
Activity 4: Use the formula
[Example · demonstration]
[Reflection · communication]
what are the steps?
What should pay attention to?
[Using · consolidation]
Use the formula directly
Use the changed forms of the formula
Use the formula in turn
Continuously use the formula
[Reflection · communication]

Figure 3. Case 4: A structure of the formula learning

SUMMARY

In order that the students can find out their own learning situations, we suggest the addition of learning objective and learning assessment. In order to improve the students' independent learning capability, we suggest showing moderately and summarizing the learning structure of the related knowledge; showing the guidance process of the learning methods, such as reflection after solving problems; and providing moderate help by means of prompt and so on at the place where we estimate the students have difficulties. Considering the difference of students' capability, help should include several levels, such as first showing the general help and then the more specific one. Thus all students with different learning capability can obtain help and necessary inquiry space from the textbooks.

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AN INTRODUCTION TO MATHEMATICS TEXTBOOKS IN CHINESE PRIMARY AND SECONDARY SCHOOL AND THE RELATED MAKING-POLICY

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At this present stage, mathematics textbooks in Chinese primary and secondary school is becoming diversity with a variety of versions. It is directly related to the management policies on the process of their formation and use, but also determined by the economic level and the democratization of China. Since the new reform of curriculum, the writing style of Chinese mathematics textbooks for primary and secondary schools has undergone significant changes, which depends primarily on 'mathematics curriculum standards' issued by the Chinese government. However, there also exist lots of unsatisfactory 'convergence' phenomena despite with different versions. Hence, it still has a long way to go in the process to achieve the diversity and scientific management of Chinese mathematics textbooks.

Keywords: textbook policy, primary school, secondary school, China

INTRODUCTION

With the improvement of economy and the acceleration of process of democratization, at present, the mathematics textbooks for primary and secondary school in China has undergone a great refinement and improvement. It is becoming diversified with a variety of versions.

CURRENT MATHEMATICS TEXTBOOKS AND POLICY IN CHINA

Evolution of policy for mathematics textbook

Since the founding of New China, the policy for Chinese textbook is mostly issued by our government and is responsible for its implementation. In the early years of New China, Chinese government has attached great importance to the formation of mathematics textbooks, and a so-called 'one outline, one book' policy is presented, that is, the mathematics textbooks in whole China has only one outline and a set of textbooks for China's primary and secondary school. Due to lack of specified personnel in textbook writing, the China government set up the People's Education Press, as a sign of the establishment of Chinese professional writing and editorial team for textbooks until the beginning of China Cultural Revolution in 1966. In the period of the Culture Revolution, the management of Chinese textbooks comes into a chaotic situation. Till September 1986, the 'National Committee of Textbook Review for Primary and Secondary School' is established, and in October 1987, the State Education Commission promulgated the 'Regulations and Rules of National Examination and Approval Commission on primary and secondary school textbooks', which indicated that an authoritative textbook review system has been established. From then on, the

Chinese mathematics textbooks on primary and secondary school were moving from 'Unified Issue System' towards to 'Approval System'.

With the implementation of the textbook 'Approval System' of national policy, Beijing, Shanghai and some other developed regions started writing and popularizing their own book mathematics textbooks in each jurisdiction. This ended the 30-years history of 'one outline, one book'. But then there are still areas of China as much as 80 percentages using a version of the book.

In 2001, an unprecedented curriculum reform has been launched, which was initiated by the State Council as a government action. With an approval by the State Council, the Ministry of Education published the 'Outline of Basic Education Curriculum Reform' (hereinafter referred to as the 'Outline'). The 'Outline' clearly states it should improve the management system for the basic education curriculum reform to achieve high-quality and diversity of textbooks. From then on, the proceeding of diversity of mathematics textbooks started.

The writing of textbooks and policy demands

In addition to the 'Outline' claiming the provisions of the written materials of textbook, the Ministry of Education issued the 'Approval management approach on the primary and secondary school textbooks', which provides for the establishment of approved textbooks channels requiring textbook editors should report to the People's Republic of China in accordance with relevant provisions. The textbooks could be written only when the related materials are examined qualified. The separation between writing and reviewing of the textbooks is implemented according to related regulations, so members of review committee should not participate in the preparation of any version of the textbook. Textbook validation work is carried out by both the administrative department for education of the State Council and the province. The publication and distribution of textbooks adopt public bidding, and textbook price is limited strictly. The textbooks for students in compulsory education are purchased by the government, and in some minority areas, such as in Tibet, the textbooks for the senior high school students are also free, which are bought and distributed by the China government.

As the same as the come out of 'Outline', a large number of experts and scholars organized by the Ministry of Education started to develop standards of compulsory curriculum for each subject. The 'standards of compulsory mathematics curriculum' (trial version) (hereinafter referred to as 'standard 1') was issued by the Ministry of Education in July, 2001, which became the basis for the writing of trial textbooks and programmatic guidance naturally. The construction of mathematics textbooks has relation with the policy of writing and using of the national textbook directly. Related policies and even the content of mathematics textbooks are a major social concern. Powerful publishers have organized teams bidding to participate in the preparation of the work of editing and publishing of textbooks one after another. In each press, professors from universities and fellows from Chinese Academy of Sciences are invited as the editor in chief. The editors are consists of scholars as well as many frontline teachers of special grade, etc. The main members of the writing group generally are about 10 people. Mathematic textbook editing teams of China have a large increase since 2001.

According to incomplete statistics, there are more than 200 full-time or part-time compilers now, which is more than 15 times of that of the early days of foundation of China. The edited textbooks are reviewed by the Ministry of Education Examination Committee to decide whether the textbooks should be used. For example, eight versions of textbooks for Grade 7-9 and four versions textbooks for the regular high school have passed the censors of the Ministry of Education until now, which named the national standard textbooks and were put into the bags of students in different regions of the country.

Generally, the current mathematic textbooks have undergone large changes than that of the past decades during the stage of the compulsory education. The possible reasons are listed in the following. Firstly, the mathematic content in each part (e.g. Number and algebra, Graphics and geometry, Probability and statistics, Comprehensive practice) has been mix edited. Secondly, various versions of textbooks are committed to changing the learning style of the students. Thirdly, the current mathematic textbooks display the relations of the mathematic and life. Last but not least, it paid attention to the realization of the objectives, namely, knowledge and skills, process and method, accumulation of the mathematical method of thought as well as experience of the basic activity. The changes of mathematic textbooks of regular high school are similar as the several characteristics as that of the compulsory education. While the difference is that the content is divided into two parts, that is, elective course and compulsory course, respectively, which is effected by the requirements of the standards of the mathematic curriculum of regular high school issued by Ministry of Education. In addition, the content and structure design of the singular high school also experienced much more changes than the past.

The selection policy of the textbooks

With the diversity, China has been changing the textbooks-selection policy from the ‘designation system’ in the past to the ‘selection system’ nowadays. Generally, what kind of textbooks should be used is voted by the textbook selection committee, which is consists of the administrative department of education at all levels (General Department of Education) of province, prefecture and local, experts, special grade teachers, representatives of principals, students, parents as well as research staff, etc. The results are open and transparent. As the regulation of the Ministry of Education, principally in the prefecture-level city, the versions of textbooks during the compulsory education are no less than three while the versions of the textbooks in the high school are no less than two. Textbook published by the press belongs to the local province should not exceed the 60% of the total amount of materials, and the selected textbook of each subject could not be replaced by the other one during the process.

THE FACTORS AFFECT THE POLICY-MAKING OF THE TEXTBOOKS

The policy of textbooks reflects the will of the state policy, which is the rule of action for the construction and implementation of the textbooks. The main factors affecting the development of textbooks are shown in the following.

First, changes in the whole management system of the country, such as after the ruling of Chairman Xi, the process of democratic and the acceleration of the civilization would bring civilization and democracy of the education inevitably. The impacts of a series of management policies including writing, reviewing and approval, using of the textbooks would be more democracy. Second, the changes of requirements for talents in national economy and technology development (e.g. all walks of life in urgent need of creative talents) would certainly affect the adjustments of the policy for mathematic curriculums as thinking subjects. Thirdly, because the territory of China is vast and the economic development is imbalanced, the state would promote the diversification of textbooks with great effects. Fourthly, the reform of national examination and evaluation system would affect the reform of textbook selection and validation policy directly. Fifthly, the demand of majority of schools, teachers and students in the trial area is an important factor affecting textbook policy adjustments. Finally, the conflicts of interest of the presses would have influence on the textbook policy possibly.

FURTHER DEVELOPMENT OF THE POLICY OF MATHEMATIC TEXTBOOK

a) In the future, the policy and management of textbooks of the country would tend to stable, such as the standards of mathematics curriculum would be revised once a decade, then the revised approval would be also revised correspondingly. It would become a regulation.
b) With the unique school and personalized of mathematics education are concerned, the textbook with specialty is expected to be approved to be written and used.
c) With the rapid development of Internet technology, electronic textbooks would emerge as the time require. Hence, the Ministry of Education could develop a series of related policies.
d) With the acceleration of the process of social civilization, textbook selection will be more standardized.
e) Evaluation of the existing use of the various versions of textbooks, there would be a more scientific basis for evaluation and management policies available.

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INTEGRATED EDUCATION AT PRIMARY SCHOOLS IN LITHUANIA

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In this article we present a concept covering the full cycle of primary education and its realisation in a set of primary-school textbooks called "Vaiva" (the original Lithuanian title is "Vaivorykštė") that connects the following disciplines: introduction to the world, native language (Lithuanian), mathematics, arts and crafts, music and dance, and ethics. The Paris school of semiotics was used as the methodological background for this concept. The material for each year is divided into abstract topics over nine months. Students have one book containing all the topics for each month. Each month's topic is divided into three or four specific subtopics. This means that over four years of primary education about 130 topics are covered. One specific subtopic – time calculation – is considered here in more detail. Benefits of integrated teaching are also mentioned.

Keywords: primary education, semiotics, time calculation, integrated teaching, Lithuania

INTRODUCTION

The changing aims of primary education (with increasing focus on the development of skills and problem solving rather than on the transfer of skills) will inevitably change the traditionally perceived disciplinary interaction and other methodological aspects of primary education. Children still have to be taught to read, write, count, draw, sing, etc. However, today's global challenges mean that the autonomous approach to the educational content of each discipline must be changed in favour of providing possible interdisciplinary connections. The integration of disciplines in education has been discussed for a very long time. Thousands of examples of integrated lessons can be found in the methodical literature and on the internet, although a more systematic integration of disciplines is not yet widespread. Lithuania also deals with an integrated approach to education to a large extent, but it is usually restricted to the combination of a couple of disciplines and episodic applications of this methodics. We offer a concept covering the full cycle of primary education and its realisation in a set of primary-school textbooks called "Vaiva" that connects the following disciplines: introduction to the world, native language, mathematics, arts and crafts, music and dance, and ethics. At the moment, textbooks for grades I-III have been published and are being used in Lithuanian schools, and a set of textbooks for grade IV are being developed. Our textbooks are based on the integration of thematic affinities, but we are not limited to this; in the set for grades III-IV the principle of processing integration is invoked – problem solving and other patterns are used in different disciplines.

The transition from a distribution of cognitive disciplines, traditionally used in schools, to multidisciplinary integration and a holistic understanding of the world requires theories that declare a more universal way of perceiving the world. One of these is semiotics, which becomes an invisible background to our integration concept.

INTEGRATION IN "VAIVA" TEXTBOOKS

Semiotic point of view

Our concept is based on the Paris school of semiotics, where the world is seen as a meaningful whole. It is interested in value formation and its functioning in a particular cultural context and focuses on discourse, rather than on the analysis of signs (Greimas (1966), Greimas & Courtes (1979)). Traditional school subjects can be considered as ways of looking at the world, the means of understanding and describing it. Semiotics sees these as "languages" in the broad sense. All languages have something in common, for example, key units (in casual speech it is the words with their meanings) and rules of switching these units (syntactic structure of everyday language). Certain content can be translated from one language to another. Translation can be called a transformation or conversion (Duval (2006)), while transcoding is a more common word in semiotics. Mathematicians, apparently, are interested in the accuracy of content conversion. Let's look at an example from early education at the game where one child "writes" a number on the back of another child with his finger and this child has to guess the number written, (let's stop and think about how many procedures of conversion are performed by the child who "reads out" this simple tactile message). However, semiotics, when talking about transcoding, highlights the increase of knowledge, acquired during the translation from one language to another. A. J. Greimas, the founder of the Paris school of semiotics, generally argues that the values appear during the transcoding (Greimas (1970)). It is about the whole range of possibilities: reading a literary or other artistic work with critical comments; rendering a visual in musical language; graphic recording of spoken text, etc. Similarly, one can talk about the integration of school subjects: the discussion of any object or phenomenon in the introduction to the world classes, literary description in native language lessons, its mathematical description, drawing and the like.

Thematic integration

Let us look at a simple example of a transcode. The primary education programme requires teaching about trees. In the knowledge of the world lessons, including knowledge about nature, students look at: what tree it is, where it grows, what conditions are needed for its growth, why leaves drop, how people use it, the structure of a tree; from the aspect of history: age of the tree, growth conditions in the past; if it is an exclusive tree – stories related to it; from the mathematical aspect: wood thickness, height; width and depth of the roots, the amount of fruit, growth rate and fruit yield predictions; from the aspect of native language: stories about a tree, the symbol of a World Tree; from the aspect of art: wood as an aesthetic object; from the aspect of music: songs about a tree; from the aspect of ethics: environmental problems, wood and forest relations and so on. There is no doubt that such a grouping of the different disciplines into one theme shape a tree as a multidimensional object of understanding. This all leads to a holistic understanding of the world being formed in the minds of children. In addition to this, there are other methodical benefits to presenting the material this way. After all, in modern schools the boundaries between lessons are blurred, thus a teacher can switch between disciplines or different approaches to the subject matter more easily. For example, if the teacher sees that students are already tired of counting, they move to drawing, reading or writing. All of these transitions, changes of aspects are not noticeable, because the same tree is still being discussed. Children no longer know what lesson it is – science, art, mathematics, native language, and so on. This rotation of activities is certainly more motivating for students and it is far more coherent. In traditional teaching, trees would also be discussed, but in different classes, and certainly not at the same time. Practice shows that in integrated teaching students learn more in a broader context, and in greater depth.

Way of developing the Vaiva textbooks

Our set of textbooks is based on the principle of thematic connection. We have divided the existing primary education curriculum and organised the parts in a new way by creating topics. In Lithuania, elementary school children learn for nine months in one classroom, so we divided the teaching materials dictated by the introduction to the world into nine topics of more abstract months. These themes, depending on the amount of school weeks, were divided into three or four more specific topics accordingly. For example, the title of the September volume in the first class is "Me and Others" (from the point of view of semiotics, this is leaving one's own space to reach a foreign one; it is hard to realise how much dramatic tension lies in such a transition). The main theme is divided into specific weekly topics: "Let's Introduce", "My Family", "How to Cross the Street", "My School" (Žukas and others (2009 vol. 1).

These topics are interrelated in a logical sequence: you come to an unfamiliar environment –you get to know it; after introducing yourselves, you tell each other about your family. We touch the most complicated thing for a first grade student during the third week: learning to cross the road; during the fourth week, we become more familiar with the new space – a school that becomes "my school". During a school year, we cover about 30 similar themes; during the four years of primary education we cover up to 130 themes. These topics cover the full curriculum of the introduction to the world and together they make a clear framework as the framework includes: the child's self-perception and self-expression; human relations; comparison of past and present; nature; technology; healthy lifestyle. Each of these specific topics, in turn, becomes the pivot around which all the disciplines of formal education revolve. So our goal was to combine the two grids: the introduction to the world dictated the topics of conversation, and each discipline, consistent with this topic, sought their own methodical purposes – consistent learning to count, read, write etc. The most complex task for the authors of the textbook and which required the most creativity was the combination of the methodical objectives of each discipline with weekly topics.

Time calculation

We will present one topic from January for the first grade students, namely, time and timing (Žukas and others (2009 vol. 5)). In the methodical advice to the teacher we suggest starting the subject with the video clip in a railway station; this is provided in the electronic supplement of the textbook. In a few seconds of video material we see and hear information conveyed in different ways: information on electronic noticeboard, an analogue clock, an electronic clock, and scrolling text line with verbal information matching the announcements in several languages (in Vilnius railway station it is provided in Lithuanian, English and Russian). In this short video clip, we recognise different methods of communication, or languages, e.g. time display on analogue and electronic clock, written and audio announcements in several languages. The goal of this information overload is to help passengers who have to find and catch the train in time. But it is not possible to catch the train if you cannot count time. This motivation starts a conversation with students about timing. Combining an everyday situation with the teaching process is reconstructed to the cognitive aspects or disciplines, and in the end – and this is where the lessons in ethics are particularly suitable – again summarised.

- **Introduction to the world**. The principles of timing (seconds, minutes, hours, days, etc.), the history of clocks, types of clocks, elementary antique clocks structure (sun, sand, mechanical and other clocks) are discussed.
- **Native language**. How to say, how to ask the time.
- **Mathematics**. How to count seconds, minutes, hours, how to add, subtract.
- **Design and Technology**. What clocks can look like; students create a paper clock and decorate it.
- **Music**. By comparing the rhythm of different time units and different musical rhythms, the pulse of the music is analysed.
- **Dance**. The rhythm of different time units, as discussed in the music lessons, becomes the basis for creating a dance with three rhythms.
- **Ethics**. What a schedule means, why it is inappropriate to be late, how you feel if you are late.

Having learnt how to read the time and say it, and having understood the essence of a schedule, we can go back to the plot of the video and ask students to suggest other real life situations that require knowledge of time.

In the discussion of the transfer of information at the train station, we saw that the information senders want to transmit the same information in different ways, in different "languages", including different sensory channels (visual, auditory). Meanwhile, when we talk about the transition of different rhythms of the clock (seconds, minutes and hours) to the rhythm of music or dance, we already have a clear increase of knowledge – the perception of time helps to understand the different nature of music and dance rhythms. This convergence of different contexts, which are combined after finding a basis of comparison, a common denominator, is the foundation principle for any creative thinking. In a song, real seconds, minutes, hours and rhythm no longer exist: the idea of different time pulses – slow, medium and fast – is shifted to music. Transferred to song, the timing principle becomes more abstract, but a new musical form retains its original variety.

Teaching to understand the flow of time, count time is associated with ambiguity, which is not easy for children to understand. At first, counting is learned in the decimal system, while time is calculated in the sixty decimal, though the same numbers are added and subtracted. Things get more complicated during sports events, for example, swimming, when time is counted in a mixed system, i.e. both in minutes, seconds and in tenths and hundredths of a second. Thus, even in such elementary mathematics such as timing students must learn transcoding procedures, in other words, they have to know various computing languages and be able to combine them.

System of topics

The abovementioned 130 topics which are embraced within 4 years of teaching are not only a horizontal system when it comes to the most important issues of the person and the surrounding world, but have a vertical dimension as well. For example, the human body is discussed in all four grades. In the first grade the senses are introduced, talking in general terms about the person. In the second grade we invite students to take a closer look in the mirror and we discuss parts of the body. In the third grade, we focus on the growth of the body, new increasing commitments and responsibilities of the child. In the fourth grade we continue the conversation about the structure of the human body and discuss the senses in greater detail (the difference between hearing and listening, seeing and watching, etc.). This spiral spans and complicates the presentation of other topics. We should also mention that in continuing the conversation about the links of educational content to the daily lives of students, it should be noted that in the first grade math class, when the human body is the general topic, students measure their height, weight, waist, the length of their feet. They do this very enthusiastically by helping one another, filling in tables, calculating charts, performing tasks (for example, seeing if the two lightest students are heavier than the heaviest one, and so on). Students compare their results with the sizes of shoes and clothes provided in the textbook. This transcoding explains the principle of setting the size of clothes and shoes.

Problematical reasoning

Currently, the education process is increasingly focused on problematical reasoning and problem-solving skills. We take a look at this area of education methodics from the point of view of multidisciplinary integration. Real life problems presented to students will be complex and syncretic in nature, while different disciplinary skills must be used to find the solutions. One of the key principles of problematic reasoning is the understanding that there are several possible solutions to any one problem. How to create this corpus of opportunities? Typically, this problem-solving method involves brainstorming. It can be enriched with integrated educational opportunities. Students could be encouraged to look at alternate ways of perceiving the same object or phenomena in various disciplines.

INTEGRATION IS THE BASIS FOR CREATIVE THINKING

Finally, we would like to emphasise that interchanging different points of view or cognitive strategies is highly productive in a creative sense. After all, creative thinking in any activity is based on the juxtaposition of different contexts and on the invention of the denominator or common denominator connecting those contexts. In our case, the basis of the connector is the common discussion topic, allowing combining or even merging different areas of knowledge, or school disciplinary contexts, while moving from one context to another we can talk about the increase of value as a result of transcoding.

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SHORT PAPERS

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ANALYSIS OF INTEGRAL AND DIFFERENTIAL CALCULUS TEXTBOOKS AND MATHEMATICAL MODELLING ACTIVITIES IN THE LIGHT OF THE DIDACTIC TRANSPOSITION THEORY

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This paper presents the results of research in which we analyzed the didactic transposition of the Integral content in Differential and Integral Calculus textbooks, as well as mathematical modelling activities developed by higher education students. Our argument is intended to highlight the potentiality of such articulation for the teaching of integral, in relation to the characterized didactic transposition attributes.

Keywords: calculus, didactic transposition theory, mathematical modelling

SUMMARY

This paper presents the results of research in which we analyzed the didactic transposition of the Integral content in Differential and Integral Calculus textbooks, as well as mathematical modelling activities developed by higher education students. Initially, we were based upon Yves Chevallard’s Didactic Transposition Theory (Chevallard, 1988; 1992), and identified the attributes of didactic transposition from scholarly knowledge to knowledge to be taught, regarded to be fundamental in the teaching of Integral, concerning the objectives and interests in the learning of such subject.

Once the knowledge adaptation, elucidation of knowledge and operationalization of knowledge attributes were defined, we analyzed two textbooks on Integral and Differential Calculus largely used, especially in Brazilian universities. The results from this analysis indicate that, in such books, the defined attributes are partially met.

We then moved on to looking at the development of four Mathematical Modelling Activities, in the way as they are characterized by Mathematics Education, in which the integral content can be highlighted. The relation between the analysis of these activities and the results from the textbooks analysis leads us to defend the articulation between the use of textbooks and the development of mathematical modelling activities.

Our argument is intended to highlight the potentiality of such articulation for the teaching of integral, in relation to the characterized didactic transposition attributes.

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"A FOUNDATION FOR UNDERSTANDING THE WORLD…": SCHOOL MATHEMATICS AND ITS UTILITY

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Mathematics education is often constituted as being for something else, particularly for examinations. This paper takes a set of mathematics textbooks widely-used in England and subjects these to an interrogation that draws on and seeks to develop Dowling's (2009, 2013) Social Activity Method (SAM). The analysis looks at the composition of GCSE mathematics and the ways in which the public domain is recruited and re-contextualised in order to develop marketing of and, potentially, induction into, the content. The findings are that there is little reference to non-mathematics practices for the purposes of explanation.

Keywords: social activity method (SAM), GCSE mathematics, England

SUMMARY

As one aspect of a broader sociological study, I am interested in the way mathematics education is so often constituted as being for something else (its myths of participation; see Dowling,1998), particularly in relation to the GCSE examination: a national test sat by pupils at around the age of 16 years in England. The study takes the widely used Pearson/Edexcel GCSE mathematics textbooks (Pledger et al. 2010) and subjects these to an interrogation that draws on and seeks to develop Dowling's (2009, 2013) Social Activity Method (SAM). The result is a social-semiotic analysis of emergent strategies of alliance and opposition in the two textbooks concerned. The analysis looks at the composition of esoteric domain GCSE mathematics and the ways in which the public domain is recruited and re-contextualised in order to develop marketing of and, potentially, induction into, the content.

The argument is put forward that, at both foundation and higher tiers, non-discursive resources (eg visual exemplars and instrumentation) are privileged over the discursive. Further, relatively little use is made of analogy; that is, there is little reference to non-mathematics practices for the purposes of explanation. Nevertheless, myths of participation are established strongly throughout both texts. There is then a marketing claim that school mathematics makes for optimal involvement in everyday life; but little content that could lead to its being so operationalised.

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ENHANCING A TEACHER'S FUNDAMENTAL INTERACTION WITH THE TEXTBOOK THROUGH A SCHOOL-BASED MATHEMATICS TEACHER RESEARCH GROUP ACTIVITY IN SHANGHAI

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Chinese teachers' knowledge of mathematical instructional content is mainly attained through intensive studies of textbooks via a "supportive" system. In this paper, we explore a teacher's interaction with the reformed textbook under the support of one national leading mathematics educator and one regional leading primary teaching specialist in a school-based teacher research group (TRG) activity in Shanghai (SH). The lesson topic is of division with remainder at grade 2. Results show that in the TRG, the mathematics educator highlighted that the goal of inquiry-based activity embedded in the textbook is to cultivate students' questioning ability in doing mathematics. The teacher reflected that she understood more sufficiently the teacher's role in establishing the connection between the to-be-learned knowledge in the textbook and her students' existing knowledge, and the way to develop students' independent thinking skills in mathematics.

Keywords: teacher knowledge, mathematical content, Shanghai, China

SUMMARY

Previous research (e.g. Ding et al., 2013; 2014) indicates that Chinese teachers' knowledge of mathematical instructional content is mainly attained through intensive studies of textbooks via a "supportive" system. In this paper, we explore a teacher's interaction with the reformed textbook under the support of one national leading mathematics educator and one regional leading primary teaching specialist in a school-based teacher research group (TRG) activity in Shanghai (SH). The lesson topic is of division with remainder at grade 2. Our research question is why the ways of treatment of this specific topic from the perspectives of the mathematics educator and specialist is considered more helpful by the teacher than that of the textbook.

We take Fan's (2013) conceptual framework of textbooks as an intermediate variable in the context of education. We further analyse the different treatments to the lesson topic according to three components of Gravemeijer's (2004) local instruction theories of the relationship of the learning goals, the selection and use of tasks and tools, and a conjectured learning process.

The data analysed are the SH textbook, the teacher's initial lesson plan and lesson video transcript, the video transcript of the TRG discussion, and the teacher's reflection note after the TRG.

Results show that in the TRG, the mathematics educator highlighted that the goal of inquiry-based activity embedded in the textbook is to cultivate students' questioning ability in doing mathematics. The specialist emphasized the relationship between the connection of mathematical knowledge in the textbook and the cognitive conflict and process of students in their learning. The teacher reflected that she understood more sufficiently the teacher's role in establishing the connection between the to-be-learned knowledge in the textbook and her students' existing knowledge, and the way to develop students' independent thinking skills in mathematics.

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THE POTENTIAL OF HANDWRITING RECOGNITION FOR INTERACTIVE MATHEMATICS TEXTBOOKS

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Web-based interactive textbooks, unlike traditional textbooks allow learners the autonomy to select their own learning sequence, thus facilitating personal construction of knowledge. This paper outlines my PhD work completed to date on the development of innovation handwriting recognition software specifically for mathematics education – called MathPen. I also discuss my pilot study which has provided some indications of its potential to support the flow of mathematical thinking during online interaction with educational resources as well as peers.

Keywords: e-textbook, technology, MathPen

SUMMARY

Exploring mathematical problems, making mathematical conjectures and proving & disproving with standard mathematical arguments play an important role in the development of sound mathematical reasoning (Schoenfeld, 1989; Davison and Kroll, 1991; Hoyles & Noss, 2003). Web-based interactive textbook, unlike traditional textbooks where there is a "planned sequence to be followed in a specific order" (Yerushalmy, 2009, pp.101), allows learners the autonomy to select their own learning sequence, thus facilitating personal construction of knowledge

However, it has been noted that "the development of e-learning in the sciences in general, and mathematics in particular, has not met the general expectation" (Ahmed 2008, pp.1089), which may be in part due to "practical and intuitive mathematics input for users is still under investigation" (Mikusa et al, 2005, pp. 621). Research on what constitutes 'practical and intuitive' is very limited, and research with a specific focus for mathematics education is sparser still.

In my presentation I outlined my PhD work completed on the development of handwriting recognition software specifically for mathematics education. I also discussed my pilot study which has provided some indications of its potential to support the flow of mathematical thinking during online interaction with educational resources as well as peers.

A preliminary version of the product is available online:

http://www.mathpen.co.uk/WhatsOn

In order to encourage discussions and feedback, tablet PCs were available on the day for hands-on experience.

For more information, see: Lo et al (2013a; 2013b).

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THE CHARACTERISTICS OF NEW MATHEMATICS TEXTBOOKS FOR JUNIOR SECONDARY SCHOOL IN CHINA: A CASE STUDY

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This paper presents a case study analysis of the new version junior secondary school mathematics textbooks published by Beijing Normal University, China. A prominent feature of these textbooks is to pay special attention to students' holistic development. As a result, the new mathematics textbooks aims to 'four' knowledge basics and 'four'537 problem-solving capabilities. In terms of structure, mathematics textbooks are focused on integration of number and algebra, shape and space, and statistics and probability.

Keywords: secondary school, technology, basic knowledge, mathematical problem-solving, China

SUMMARY

Based on the case study of the new version junior secondary school mathematics textbooks published by Beijing Normal University, this article tries to analyse the characteristics and features of reform-oriented middle school mathematics textbooks. An illustration of one of the textbooks is provided in Figure 1.

Figure 1. A textbook from the junior secondary school mathematics textbook series

A prominent feature of these textbooks is to pay special attention to students' holistic development. As a result, the new mathematics textbooks aims to 'four' knowledge basics and 'four' problem-solving capabilities. In the content of mathematics textbooks, with an emphasis on the acquired knowledge and experience, teaching materials must be real, interesting and challenging.

In terms of structure, mathematics textbooks are focused on integration of number and algebra, shape and space, and statistics and probability. Special attention is given to an application of various teaching and learning methods.

In the last, this paper shows some challenges for the new textbooks.

FOREWARNED IS FOREARMED: A MATHEMATICS TEXTBOOK

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This paper reports on a study in The Bahamas where there has recently been an increased focus on the use of a uniform mathematics textbook in all public secondary schools. This study uses action research to investigate the impact of a mathematics book designed to help students to "Avoid error!" by making them more aware of some of the common dangers and allowing them to engage the powerful adage "Forewarned is forearmed". The main research tool was a questionnaire and mathematical competency tests completed by 200 college preparatory students during a 14 week semester. The implications of this study may impact a wide range of professionals in education and the Bahamian education system.

Keywords: mathematical errors, secondary school, The Bahamas

SUMMARY

This paper reports on a study in The Bahamas where there has recently been an increased focus on the use of a uniform mathematics textbook in all public secondary schools. The primary requirement to tackle debt involves a simple choice between the actions "Earn more" or "Spend less". A similar approach can be applied when students attempt to answer a mathematics problem i.e. "Get it right!" or "Avoid error!" Naturally, students will strive to "Get it right!" in attempting to use their knowledge and experience when confronted by problems on a wide ranging syllabus. In some cases, they may fail unnecessarily. These students are the most vulnerable to the subtle traps which can go unheeded in any problem. Like a speedster oblivious to the presence of hidden cameras, they may be lured unsuspectingly into using wrong approaches which cause marks to be forfeited and valuable time to be wasted.

This study uses action research to investigate the impact of a mathematics book designed to help students to "Avoid error!" by making them more aware of some of the common dangers and allowing them to engage the powerful adage "Forewarned is forearmed". Promoting awareness of potential dangers provides the major purpose of this intervention. The study uses primarily quantitative methods, but includes a small qualitative component in the form of interviews and a researcher's journal to enhance the findings. The main research tool was a questionnaire and mathematical competency tests completed by 200 college preparatory students during a 14 week semester.

The implications of this study may impact a wide range of professionals in education and the Bahamian education system. It may inform pre-service and in-service teachers, school administrators, mathematics education instructors, textbook designers, and education policy makers, particularly as they work to design mathematics education experiences to foster the use of a culturally rich mathematical application book, designed to highlight common dangers, in the future.

SITUATIONAL AUTHENTICITIES IN LOWER SECONDARY SCHOOL MATHEMATICS PROBLEMS: REASONS FOR CALCULATION AND ORIGIN OF QUANTITATIVE INFORMATION

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This pilot documental research proposes to analyse a number of trigonometry and geometry problems from lower secondary school Irish mathematics textbooks. In this paper we present the findings from an initial analysis of a random sample of 24 geometry and trigonometry (12 of each) problems selected from three lower secondary school mathematics textbooks. The aim of this analysis is to evaluate two specific aspects of problems' situational authenticities and it is based on Palm's (2002) theory of authentic problems. The initial results show that in majority of the cases the textbook authors do not give students the reasons why the calculation, or other mathematical procedure, they are supposed to carry out is situationally meaningful. In the cases when the reasons are given, they appear superficial and limited.

Keywords: secondary school, geometry, trigonometry, Ireland

SUMMARY

This pilot documental research proposes to analyse a number of trigonometry and geometry problems from lower secondary school Irish mathematics textbooks, in order to evaluate their *situational autenthicities* (Palm, 2002; 2008). Situational authenthicity is a key area of concern, particularly in the context of the new national curriculum in Ireland which sees a move towards an emphasis on mathematical understanding and real-life applications and problems.

In this paper we present the findings from an initial analysis of a random sample of 24 geometry and trigonometry (12 of each) problems selected from three lower secondary school mathematics textbooks. The aim of this analysis is to evaluate two specific aspects of problems' situational authenticities and it is based on Palm's (2002) theory of authentic problems.

Palm's (2002; 2008) theory of authentic problems provides a useful framework for such an analysis, enabling one to establish how adequate these problem formulations are. Palm suggests that a whole authenticity-oriented evaluation of the situation, problem formulation and expected students' performances should take into account the following aspects of school mathematics tasks: (A) Event; (B) Question; (C) Information/data; (D) Presentation; (E) Solution strategies; (F) Circumstances; (G) Solution requirements, and (H) Purpose in the figurative context. In this initial study special attention is paid to two particular aspects: (1) the reason for the asked calculation or other mathematical procedure (related to A and B) and (2) the origin of the quantitative information (related to C). These selected aspects resonant well with the historical fact that both geometry and trigonometry have their roots in solving meaningful practical problems by using specified measuring instrument and measurement procedures; a fact which is reflected in the etymology of their names.

The initial results show that in majority of the cases the textbook authors do not give students the reasons why the calculation, or other mathematical procedure, they are supposed to carry out is situationally meaningful. In the cases when the reasons are given, they appear superficial and limited. The origin of quantitative information is generally unknown. Measurement-based origin of data is mentioned in four problems, but the type of measuring instrument used is only specified (clinometer) in one problem. The implications of both findings on formation of students' beliefs regarding school mathematics and its learning were briefly discussed.

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IF NOT TEXTBOOKS, THEN WHAT? ENGLISH MATHEMATICS TEACHERS' USE OF ALTERNATIVE CURRICULAR RESOURCES

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This paper reports findings from an interview study with thirty secondary mathematics teachers in England. Designed to determine what specific curricular resources individual teachers use, with curricular resources defined quite broadly, we found that the teachers were largely free to decide whether and how much to use textbooks and any supplementary or substitute resources. The study confirms the perception that English teachers tend to make quite limited use of textbooks even when textbooks are available, and revealed a range of alternative curricular resources that seem to influence English teachers' work.

Keywords: teaching resources, textbook use, England

SUMMARY

Although in several countries, such as Japan and the United States, mathematics teachers tend to use textbooks rather heavily in planning and delivering their lessons, in England the situation seems to be different (Mullis et al., 2012; Ruthven, 2013). This situation deserves attention especially in light of the body of research showing or elaborating on the important role that (well-designed) textbooks can play in supporting classroom work and in promoting teachers' understanding alongside student learning (e.g., Davis, Palincsar, & Arias, 2014; Stylianides, 2007).

In this paper, we report findings from an interview study with thirty secondary mathematics teachers, from six schools in two counties in England, designed to determine what specific curricular resources individual teachers use (with the term 'curricular resources' used broadly, including but not limited to textbooks and other print material, digital resources, video, tools, and people). None of the schools had a policy about the use of textbooks; teachers were largely free to decide whether and how much to use textbooks and any supplementary or substitute resources.

The study confirms the perception that English teachers tend to make quite limited use of textbooks even when textbooks are available, and revealed a range of alternative curricular resources that seem to influence English teachers' work. Teachers' personal collections of preferred resources, which may or may not include textbooks, develop for a variety of reasons. Use of even the most frequently reported or most familiar resources is varied. In the presentation, we elaborated on these findings and discussed a couple of the issues that emerge from English teachers' use of alternative curricular resources.

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WORKSHOPS

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AUTHORING YOUR OWN CREATIVE, ELECTRONIC BOOK FOR MATHEMATICS: THE MC-SQUARED PROJECT

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The EU-funded ‘MC-squared’ project is working with a number of European communities to develop digital, interactive, creative, mathematics ‘textbooks’ that the project calls ‘cBooks’. The cBooks are authored in a Digital Mathematics Environment in which participants can construct books with various interactive ‘widgets’. This paper provides an outline of the MC-squared project illustrating an interactive storyboard of the Digital Mathematics Environment architecture. This includes examples of how authoring by cBook designers of interactive ‘widgets’ is possible. The workshop that relates to this paper is augmented, of course, by suitable ‘hands-on’ materials aimed at two possible cBooks: one focusing on aspects of geometric and spatial thinking using building blocks, the other on aspects of number and fractions.

Keywords: e-textbook, digital textbook, creative mathematical thinking, creativity

INTRODUCTION

The MC squared project (http://www.mc2-project.eu) aims to design and develop a new genre of authorable e-book, which we call ‘the *cBook*’ (c for creative), by extending e-book technologies to include:

- diverse interactive components [often called dynamic widget];
- interoperability [meaning that the *cBook* should work on any contemporary computer or tablet],
- collective design [including publishers, developers, researchers, school educators]
- contextual pedagogical cues to augment the learning activities

To develop the concepts and community knowledge for the design of authentic creative learning activities a collaborative design-based research methodology inspired by ‘communities of practice’ (Wenger, 1998) and ‘communities of interest’ (Fischer, 2001) is being used. For more background to this, see Bokhove, et al. (2014). Here we briefly outline some of the features of the DME, the environment used for authoring the first cBooks for the project. The workshop that relates to this paper is supplemented, of course, by suitable ‘hands-on’ materials.

DESCRIPTION OF THE C-BOOK ENVIRONMENT

A Digital Mathematical Environment (DME) provides the starting point for authoring *cBooks*. The functionality planned for the first version was:

- Text, graphics, and media files; these can be added using a WYSIWYG editor.
- Pre-made interactive ‘widgets’; these can range from more closed multi-step equation boxes to very open tools for construction tasks.
- Storing student work.

Figure 1 is a storyboard of the *cBookAuthor* that the MC-squared team envisages to design classroom tasks (the snapshot in Figure 1 is based on the Digital Mathematics Environments, DME, of the Freudenthal Institute).

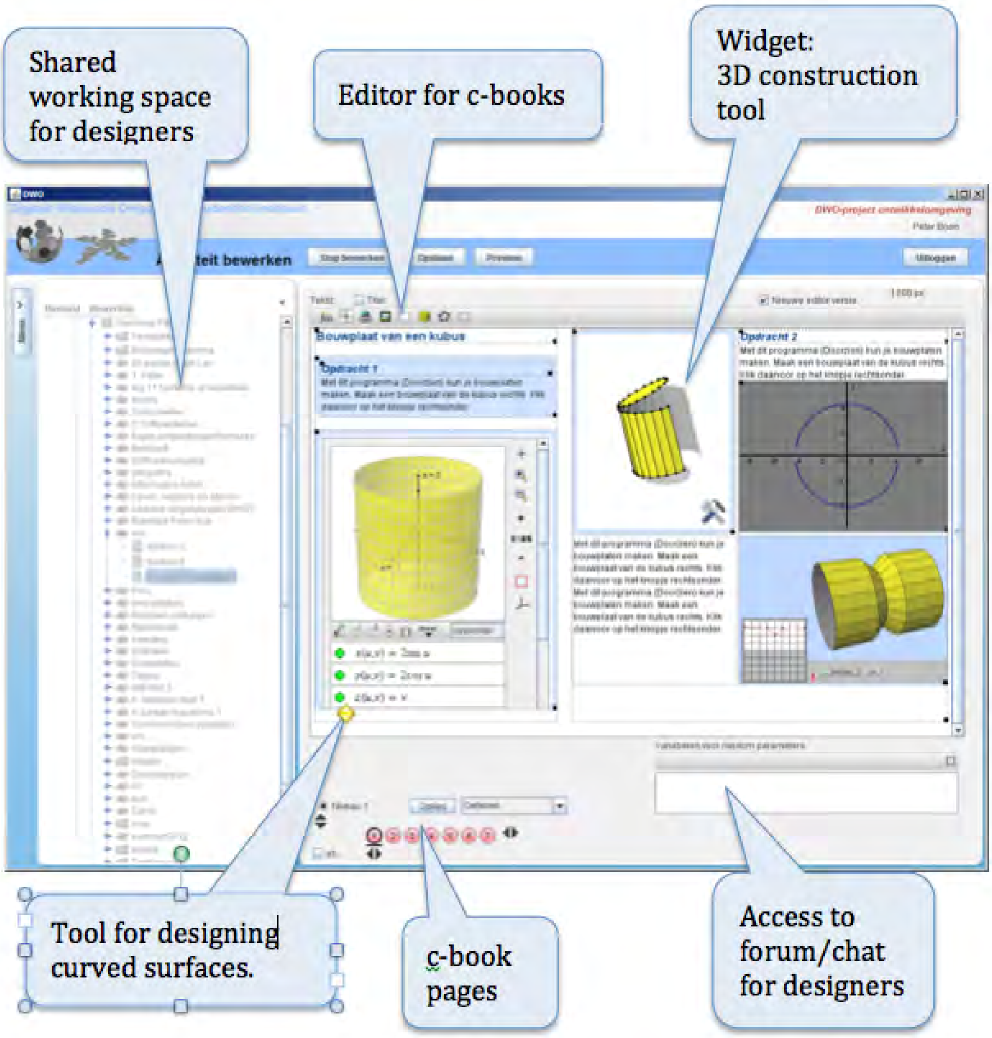


Figure 1: Storyboard of a future authoring a cBook

During the course of the project the features of the authoring environment are being extended to facilitate the creative design process of the *cBooks*. Each of the ‘communities of interest’ are providing feedback on their experience of using some of the DME features, which includes giving advice and guidance on authoring requirements. Future features are planned to include:

- More interactive widgets that can be integrated into a *cBook*.
- Chat and forum function for communication between community members regarding *cBooks* that are being designed, with a shared working place and a repository of earlier designed *cBooks* that can be (partly) re-used, adapted and discussed.
- Collection of appropriate data with respect to student usage, enabling provision of intelligent support in the form of feedback to learners, but also to designers within the authoring environment. Logging and mind-map features for facilitating the creative design process.
- Pedagogical cues set by the designers and/or tutors that provide learning analysis about the students' engagement with learning activities both for student learning development and teacher reflections.

In the first phase of the project, the aim has been to use the authoring environment 'as is' with only a small selection of widgets added to the ones that were already available in the DME. The remainder of this document provides some examples of the current features.

AUTHORING

cBooks are authored in a WYSIWYG environment, as depicted in Figure 2. The main editor allows authors to add and remove pages to a *cBook*, adjust their order, add different types of feedback and scoring, and - most importantly - add a variety of elements to these *cBook* pages, ranging from basic static texts to complex, interactive widgets.

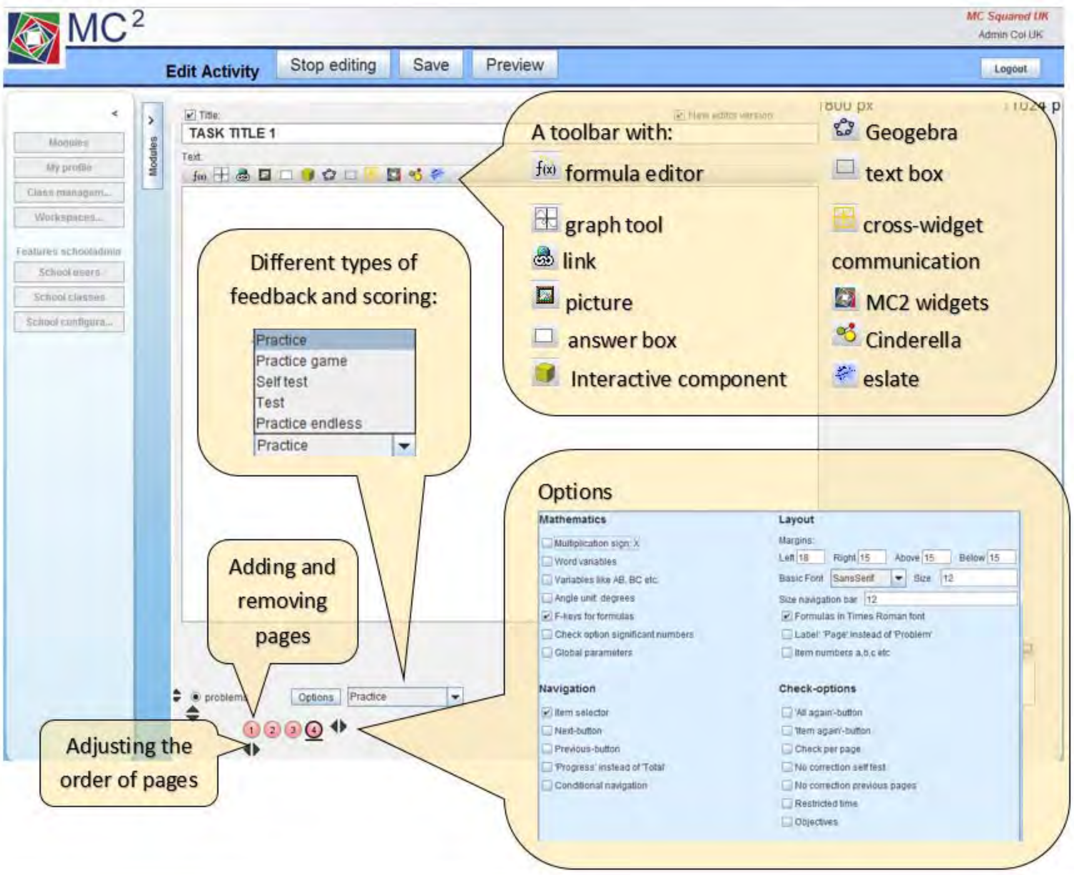
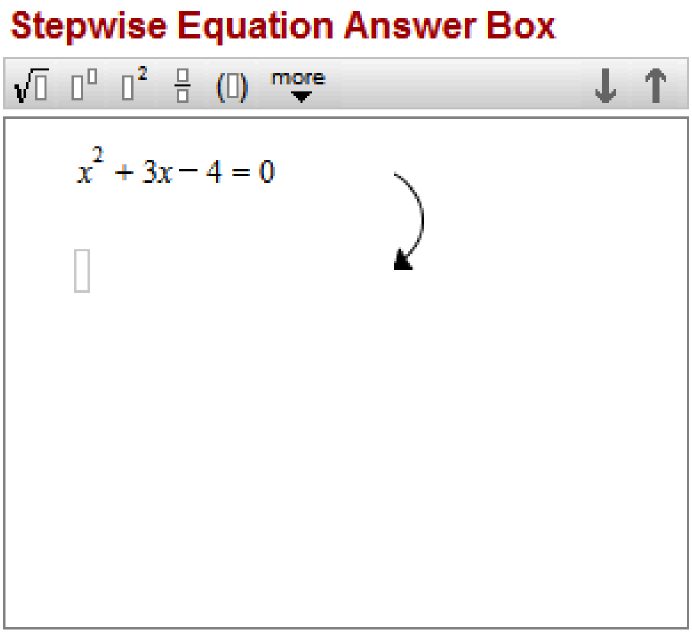
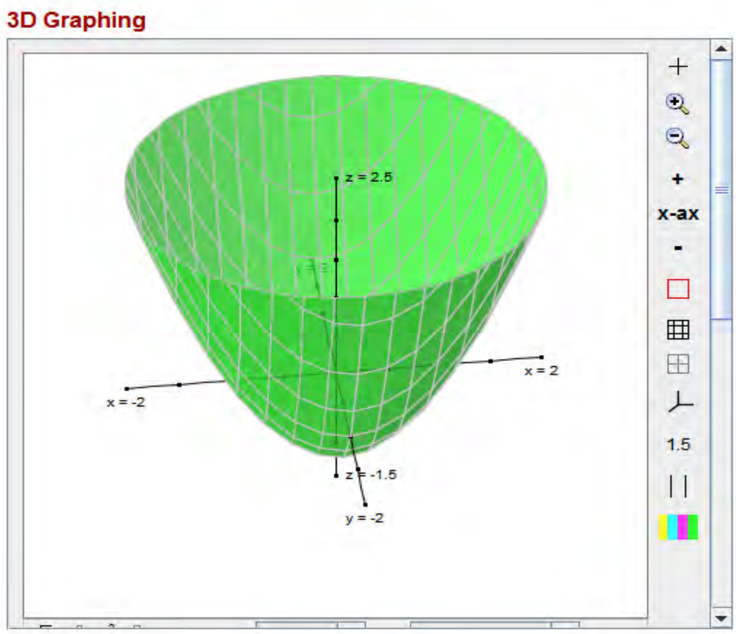
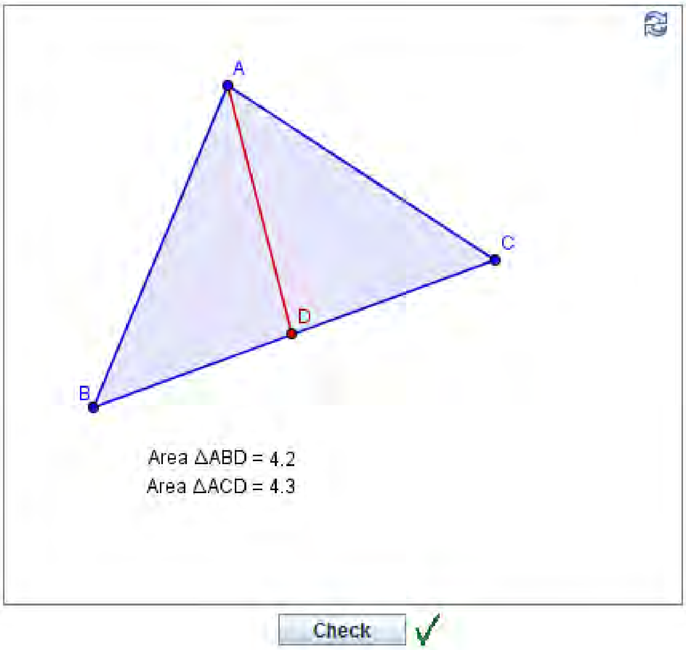


Figure 2: overview of the editing window (adapted from: Abels, Boon & Tacoma, 2013)

EXAMPLE WIDGETS

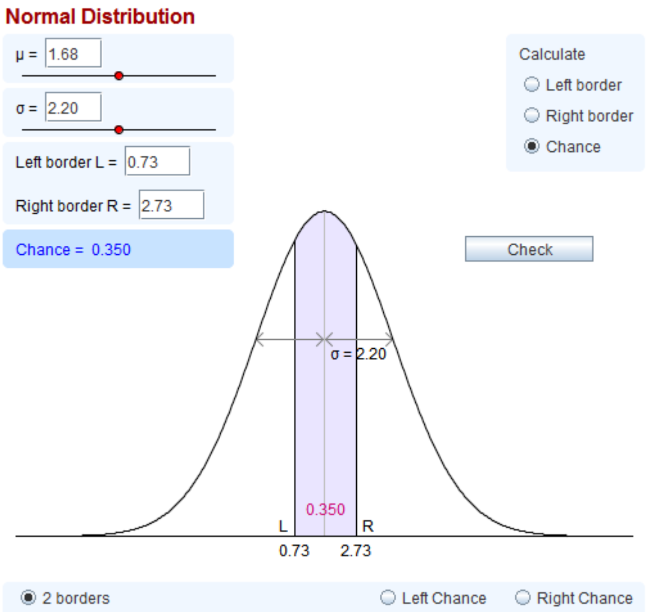
There are several widgets that can be used in authoring. They can be classified into basic widgets, algebra widgets, geometry widgets, statistics and probability widgets, and other widgets. To typify these categories we give an example for every category. One characteristic of the widgets is that they can be customized in such a way that they can be very open, exploratory tools or very closed (even serving as static diagrams).

<table>
<tr>
<td>
</td>
<td>Basic
In a stepwise equation answer box, students can solve equations step by step. After each step they press Enter, to check whether the step is correct and to be able to move to the next line. A variety of options available: custom feedback, strategy functions, scoring, visual aspects and more.</td>
</tr>
<tr>
<td>
</td>
<td>Algebra
With this tool, 3D graphs, surfaces and curves can be created, viewed and manipulated. The tool contains a collection of examples of graphs, surfaces and curves, which can be chosen as base for new figures. There are customizable options for graphing, what manipulations students are allowed to do, and example graphs that can be used.</td>
</tr>
</table>



Geometry

Geogebra can be used for a wide variety of tasks, by providing an empty workspace or a pre-made construction. The task for the Geogebra instance to the right is: Move point D such that the area of the triangles ABD and ACD become the same (a difference of 0.1 is allowed). There are customizable options, for example, what Geogebra buttons are available to the students, and what elements are visible. In addition, constructions can be checked on correctness.



Statistics and probability

This statistical widget represents normal distributions and allows students to see how different values of the relevant variables change the distribution. There are customizable options for what elements are visible but also whether students can manipulate the distribution with sliders. In addition, check options are available.

Other widgets

This applet can be used by students to draw sketches, containing points, lines, rectangles, circles and text. It, for example, allows students to express

TO CONCLUDE

This paper provides an overview of some of the features of the (authoring) environment that is used by the ‘Communities of Interest’ in the MC-squared project. The tool is used to author creative *cBooks* with interactive features for mathematics education. The workshop that relates to this paper is augmented, of course, by suitable ‘hands-on’ materials aimed at two possible cBooks: one focusing on aspects of geometric and spatial thinking using building blocks, the other on aspects of number and fractions.

The first groups within the community of interest have been established. The next step is to develop new *cBooks* and improve the initial versions. This can only be done in close collaboration with different stakeholders: researchers, designers, and teachers. If you would like to join the community or participate in the project or require more information please contact the first author.

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REFLECTIONS ON THE DESIGN OF INQUIRY ACTIVITIES IN CHINESE JUNIOR HIGH SCHOOL MATHEMATICS TEXTBOOKS

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Inquiry learning is not essentially a new way of learning, but it is in line with the discussion of three-dimensional mathematics curriculum standard. We should note the following three areas: establish accurate objectives to prevent the bias phenomenon, select the right content to prevent the generalization phenomenon and present the comprehensive type to prevent the loss phenomenon.

Keywords: inquiry content, task design, middle school, secondary school, China

INTRODUCTION

Mathematical inquiry learning, one of the current approaches to learning that mathematics curriculum reform advocates, has received great attention in mathematics teaching practice. Textbook is the basic clue of student learning activities and also an important resource to achieve the course objectives and implement teaching, therefore well-designed mathematics exploration activities has become an important part of compulsory education materials design. Currently, various versions of the junior high school mathematics textbooks all include exploration activities with distinctive characteristics, but there still exist some problems. According to the literature information, the teaching implementation of inquiry learning has been more discussed while there was little discussion about the inquiry activities design. This paper intends to present several aspects in mathematics exploration activity design which are worth our attention to simply start the discussion.

MATHEMATICAL INQUIRY LEARNING AND ITS SIGNIFICANCE

Meaning of mathematical inquiry learning

What is mathematical inquiry learning? According to pertinent literature, scholars from home and abroad have interpreted inquiry learning in various ways. Some regarded inquiry learning as a learning activity. For instance, Schwab thought that “inquiry learning refers to such an learning activity: Children themselves get involved in the process of obtaining knowledge, developing necessary exploration ability for research; at the same time, to form the basis of knowing nature--- scientific concepts and further develop a positive attitude to explore the world” [1] Some consider learning as a learning method, such as Chai Xiqin, a researcher at Curriculum research institute from People's Education Press. She thinks that “Inquiry teaching is a special teaching method essentially to extend the field of science into the classroom, which provides students with opportunities to understand scientific concepts and the essence of scientific research through a research process similar to scientists' and to develop scientific exploration ability” [2]. From others’ perspective, the inquiry learning is a kind of imitative scientific activity. According to professor Song Naiqing from Southwest China Normal University, “inquiry-based learning, in essence, is a type of scientific research activities” [3]. All these interpretations, despite some differences in statement, have much in common. One is the focus on problem. Inquiry learning is to raise students’ problem consciousness, coming up with challenging and appealing questions. Another is initiative. Inquiry learning places emphasis on students’ initiative. Students always take the initiative in the process the entire time and explore actively, from raising problems, formulating the exploration plan, to gathering information, dealing with materials and drawing conclusions. The third one in common is the highlight of the process, emphasizing the experience students may gain in exploring new knowledge and so on.

The author thinks that inquiry learning, in essence, is not a new way of learning. There are two reasons. First, inquiry learning, independent learning and cooperative learning are all springing up with the new curriculum. Based on the statement and interpretation of the curriculum standards, these new ways of learning are mainly for making some changes to the current situation of our traditional teaching, which gives too much emphasis to reception learning, learning by rote and mechanical training, not to deny the traditional way of learning. Traditional learning methods can also be divided into reception learning and discovery learning. The curriculum standards advocated adding some positive elements or instruments based on the traditional approach to learning. Reception learning can also involve exploration, initiative and cooperation and the same is with discovery learning. The difference just lies in degree and manner. Therefore, such words as “inquiry learning”, “self-learning”, and “cooperative learning” are just for discussion and emphasis.

Second, mathematics learning itself is a process of inquiry. In the learning process, students should be actively involved in thinking, observation, induction, analogy, association, deduction and so on. Inquiry-based learning certainly includes these mathematical forms, and it is a way of learning with emphasis on problem, initiative and process. So it still belongs to reception or discovery learning.

The significance of mathematical inquiry learning

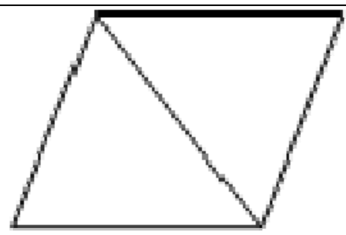
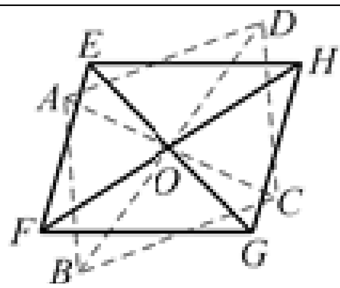
As to the significance of inquiry learning, scholars have given different opinions, but in terms of mathematics learning, the author believes that it can be attributed to three aspects: mathematics inquiry learning enables enable students to explore mathematics through various kinds of activities, allowing students to gain a deep understanding of knowledge; mathematical inquiry learning include a wide range of activities such as observation, questions-raising, speculation, assumptions, literature query, planned investigations or experiments, collecting, analyzing, interpreting data, reasoning, cooperation and exchange, etc. All these activities are effective ways of solving problems, through which students can acquire the research methods; mathematical inquiry learning helps students gain a good understanding of mathematics, develop the scientific attitude and habit and foster the spirits of seeking truth from facts, keeping improving, being modest and prudent, objective and fair, and innovative. Mathematical inquiry learning is in line with the three-dimensional learning objectives stated in the curriculum standards, and perhaps this is one reason why mathematical inquiry learning is favoured

THE DESIGN OF INQUIRY ACTIVITY FOR JUNIOR HIGH SCHOOL

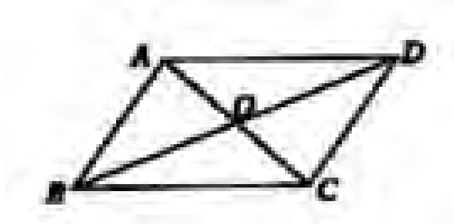
To establish accurate objectives and avoid biases

As mentioned earlier, the objectives of mathematical inquiry learning are more comprehensive, not only including the knowledge, skills, but also the processes and methods, emotional attitudes and values. The three main objectives should be unified to help students construct knowledge in the process of exploration, to allow them to gain experience and insight in exploration activities so as to form scientific attitudes and emotional values.

It is essential to establish accurate objectives in exploration activity design to prevent biases. Currently, the exploration activities in junior high school mathematics textbooks lay too much emphasis on the teaching of knowledge and specific conclusions while relatively neglecting other goals. Specifically, in a exploration activity, there are a series of questions designed to “pave the way” for students with each question pointing to explicit knowledge or conclusions, and students just “explore” under the instruction the textbook presents, in which way students don’t need to select thinking strategies or methods, or they are just limited to hands-on activities, lacking refinement of thinking after activities.

Case: to explore the nature of parallelogram Design one:	
Activity one: Cut a sheet of paper parallelogram along a diagonal line, and you can get two triangular pieces of paper. Will the two completely coincide? What’s the relationship between their edges and angles? Please prove your assumption.	
Activity two: Prepare two pieces of transparent parallelogram paper of the same shape and size. First fold the two pieces of paper respectively to make their two diagonal lines and make them overlap on the desk. Then a pin will be used to fix the point of intersection of the two diagonal lines. Keep the underneath paper still and rotate the above paper 180°round the pin to observe whether the two will coincide. From activity two, can you tell which line segments in a parallelogram are equal? Prove your assumption.	

The intention of inquiry activity is to get students to come up with assumptions, propose various methods to prove it through activities and discussion. That is: what can students get through the activities? How do they get it? How do they prove it? However, the above inquiry activity designed provides more procedural steps with too obvious result directivity. Activity one is designed to present the nature that “the two opposite sides of the parallelogram are equal” and “two diagonals of the parallelogram are equal”. Activity two is to prove “the diagonal of a parallelogram splits it equally”. The two activities designed don’t provide sufficient space and opportunities for students to explore, ignoring students’ independent thinking and method selection. Compared with design one, the following design two is more appropriate.

Case: to explore the nature of parallelogram Design two:	
Design two: Fold a piece of paper, cut out a triangle-shaped folded piece of paper and make sure the two edges are overlapped to obtain a quadrilateral. (1) What does the quadrilateral look like? Discuss with your peers. (2) Is there any equidistant line segment and equal angles in the quadrilateral you obtained? How do you know? Discuss with your peers. (3) In ABCD, the two diagonals AC, BD intersect at point O. Which are congruent triangles? Which line segments are equal? Try to verify your conjecture.	

By means of stitching parallelograms, this design raises several questions. By hands-on practice in question (1), students get a variety of different quadrilaterals, some of which are parallelograms while others are not. In this activity, students get to know more about parallelograms and in the same time set a good foundation for the later conjecture and reasoning. Question (2) provides students with opportunities to explore the nature of parallelograms in various ways, such as measurement, translation, rotation, folding, etc. Question (3) makes the conjecture into inference and demonstrates the characteristic of “fifty-fifty diagonal”.

One of the most important goals of inquiry learning of mathematics is to cultivate the mathematical thinking ability. Without the involving of thinking, the ability of thinking can never be developed. We have first to focus on this rather than the context or any practical use of mathematics problems, or treat “a chain of questions” as inquiry activities. In fact, the experience students gained during the inquiry activities is as important as the knowledge they obtained, as it is said in many documents that “wisdom is shown in the process. In essence, wisdom is not shown in the result of experience nor in the result of thinking, it is shown in the process of experience and thinking" [4] and "In teaching, we should not only focus on whether the inquiry activity obtain the related findings nor satisfy with the solutions of specific questions, we should guide our students to think more and rethink of the former knowledge and cognition to reach a higher level of mathematics study" [5].

PROPER SELECTION OF CONTENT TO AVOID OVERGENERALIZATION

Inquiry learning of mathematics is profound in its value, but not all the topics are needed or worthy to be studied. We should select the most proper ones and avoid overgeneralization, by which the chosen one may cause the inquiry activity shallow or beyond reach.

Questions too difficult or too simple are not suggested. In the view of psychology, the "Zone of proximal development (ZPD)" is most appropriate time for learning. higher or lower difficulty of the questions compared with ZPD may both reduce the efficiency of learning, because students may failed to build connection between the difficult problem and their former knowledge construction and failed to practice their thinking ability with the simple ones. Such as the proof of the theorem "two parallel lines have same corresponding angles". This theorem is first learned in the seventh grade, which is contradictive proofed with the help of the axiom "when the corresponding angles equal, two lines parallel with each other". For middle school students, this is the first time get touch with contradiction. When design teaching materials, teachers may ask questions and let students try to prove it, who may find it really hard with the prior knowledge and experience. Teachers are not suggested to make it into an inquiry question which may only be a waste of time.

In addition, the descriptive, prescriptive and declarative mathematical knowledge, such as the name of the mathematical concepts, mathematical symbols or writing format, are not needy for students to explore. Students can master them through teacher's explanation or their own reading, like the concept of similar terms as described above, also like the parallel and vertical marks, etc.

Students are suggested to explore such knowledge with appropriate difficulty. Of course, the specific teaching objectives should be taken into consideration. If one is teaching content so that students grasp a conclusion, then the teachers' explanation itself is enough. The inquiry study is essential when not only the conclusion but also the process and method is needed for students to understand to gain the experience, or when the diversity of problem-solving strategies is needed to be demonstrated and shown. The value of teaching content itself should also be taken into account when design the appropriate level of inquiry activity , especially when this content is full of thinking which can show the process of the generation of knowledge and contribute to the development of students' capabilities

Case: conditions for congruent triangles

Questions: To draw a triangle exactly the same as the one Xiao Ming draws, what conditions should it has? Do we must know all the sides and angles? Could conditions be as less as possible? Is one condition ok? How about two? Three? Or more?

If the conclusions presented approach is adopted, textbooks can just show "side-side-side" and other conditions to allow students to draw and make sure the appropriate conditions. But it would be difficult for them to go through the process of obtaining conclusions and mathematical reasoning. Why six conditions for congruent triangles become three? Why not “side-side-angle” but “side-side-side”? In fact, these activities play a very important role in students’ further studies. For instance, when they learn the conditions for similar triangles, they are very likely to remember the approaches they use when learn the congruent triangles. It is based on this consideration that the textbooks provide us with materials for these activities and progressive teaching questions. During the activity, students not only get the conditions for congruent triangles but also learn the method of analysis and accumulate experience in mathematical activities

PRESENT FULLY PREVENT THE PHENOMENON OF MISSING

Activities can be explored into different categories according to different criteria, “mathematics exploratory activities in secondary school can be divided into formative inquiry, constructive inquiry and the applied inquiry according to the law of students’ formation and development of cognition” [6]. This classification better reflects the value of the different investigations, thus I quite agree with it.

Formative inquiry mainly makes formation process of knowledge in textbook into issues to be explored so that the students in inquiry activities can autonomously construct mathematical knowledge, such as the abstraction of mathematical concepts and exploration of subject. Students can use two 30° triangular ruler to form equilateral triangle to find theorem themselves: In a right triangle, if one acute angle is 30, its opposite size is equal to half the size of the hypotenuse. Such activities can let the students naturally draw the conclusion and lay a good foundation for subsequent rigorous proof. Applied inquiry intends to make mathematical knowledge and the application of law (mathematics application and practical applications) into issues to be explored such as the inquiry of open questions, integrated questions and the resolution of applied questions. Constructive inquiry makes knowledge systems and networks into explorable questions to guide students to construct knowledge structure and system after the unit of study, the analogy study of before-and-after study knowledge and construction of before-study knowledge system etc to form good cognitive structures for students.

In the current design of the junior high school mathematics textbooks, the inquiry activities are mainly about the process of formation or application of some knowledge which emphasis formative inquiry and applied inquiry and insufficient attention is put on constructive inquiry. The author recently made a survey on mathematics inquiry learning in the national province training class. One of the questions is to examine the choice of the type of inquiry activity which contains four branches. Only 18% select “to make the construction of knowledge systems into explore issues and guide students to build knowledge systems and networks, to form good cognitive structure such as summarize and analogy the learned unit.”

“Mathematics knowledge system can be divided into mathematical content system and research system. mathematical content system concerns connections between the specific mathematical content, such as the concept network systems and proposition network systems; while research system focus on methods and specific process of the research about a concept, proposition or specific topics, such as the study of parallelogram, we need to study definition, nature, identification, and the order to start research among these knowledge, the way to do research" [7]. Some materials at present contain the part of "review and reflection", "summary and reflection" which encourage students to sort out their own learning content, try to find the link between the different content and concerns the construction of mathematical content in the form of questions, but the construction of mathematical research is generally less concerned. In fact the two are inseparable, just as mathematical content and mathematical methods, mathematical content reflects the mathematical methods and mathematical methods use mathematical content as a carrier , therefore constructive inquiry must not only focus on construction of mathematical content system but also on the construction of mathematical research systems.

Case: Special parallelogram

Before learning the specific content of the parallelogram, the students have learned the concept of a parallelogram: definition, nature and determination, various means like direct manipulation, graphical translation, rotation, axis of symmetry, simple reasoning and preliminary reasoning, students gradually explored the nature of parallelogram like the opposite side of the parallelogram, diagonal nature and the common identification method, and accumulated some experience in mathematical activities. Therefore, before learning the special parallelogram content, teachers can raise some questions to enable students to self-construct the content of subsequent knowledge systems.

1) What nature and identification of a parallelogram?
2) From what perspective the nature and identification of the parallelogram is described? (the opposite side, diagonal, diagonal sides)
3) What special parallelogram did we learn in elementary school? (Rectangle, diamond, square) and What are the differences and connections between them ?
4) Based on previous experience in the activities, what do you want to study about special parallelogram? (Definition, nature, identification) how you intend to use research methods?

Such exploration activities enable students to review the previously studied contents and at the same time integrate the subsequent content allowing students to have a clear intention about the content and research methods before learning.

SUMMARY COMMENT

It is a challenging job to design inquiry activities in mathematics textbooks, but it has great significance to change teaching methods. I believe that through the cooperation among textbook designers, educational researchers and classroom teachers, we will certainly solve the problem and seize the essence of inquiry

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ANALYSING MATHEMATICAL TEXTBOOKS WITH PARTS OF GREIMAS' SEMIOTIC THEORY

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In the workshop participants were guided about, and given the opportunity to use, Greimas' semiotics theory in order to analyse short texts from a narrative perspective. Examples of text from textbooks for year 5 on negative numbers were provided and after a common introduction into the theory, analyses was carried out in smaller groups. The textbooks were chosen from Sweden and Lithuania (and were shown translated to English). In the workshop, participants discussed and shared experiences from the interpretations and use of the narrative level of the theory.

Keywords: semiotics, narrative analysis, Lithuania, Sweden

SUMMARY

In the workshop participants were guided about, and given the opportunity to use, Greimas' (1970; 1979; see also Ricœr, 1989) semiotics theory in order to analyse short texts from a narrative perspective. Examples of text from textbooks for year 5 on negative numbers were provided and after a common introduction into the theory, analyses was carried out in smaller groups. The textbooks were chosen from Sweden and Lithuania (and were shown translated to English).

In the guiding introduction we considered the four phases of the narrative level of the theory: manipulation, competence, performance and sanction. Greimas, who lived 1917 to 1992, was the master of the Paris school of semiotics. He investigated what produces meaning in a text. He developed a quite general theory, mainly consisting of the semiotic square, and the narrative grammar, which allows the analysis of texts of any genre. Kudžma (2005; 2013) has applied Greimas' theory for analysis of mathematical texts since 2005. Also master students in Lithuania have used this theory for analysis of mathematical texts.

When a group of Nordic and Baltic mathematics education researchers applied the theory to the introduction of negative number they were surprised to find that in some cases the text did not contain any problem or a question. Additionally the way to narrate the introduction of negative numbers was different in the books with some starting from an equation and others from daily life situations like measuring temperature, water level or from economic terms like debt.

In the workshop, participants discussed and shared experiences from the interpretations and use of the narrative level of the theory.

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A COMPARATIVE STUDY OF ILLUSTRATIONS IN THE OLD AND NEW MIDDLE SCHOOL MATHEMATICS TEXTBOOKS IN CHINA

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As a medium of knowledge learning, textbook illustration has become an integral part of textbooks, especially for mathematics. On the one hand, symbolic-graphic combination is the basic characteristics of mathematics as well as an important kind of mathematical thinking method; on the other hand, mathematics is an abstract and formal subject so illustrations play an essential role in facilitating students' comprehension of abstract numbers, symbols and language. Following the implementation of curriculum reform in China in 2001, great changes have taken place in mathematics textbooks for junior high school, change of illustrations included. Therefore, the purpose of this paper is to provide a theoretical basis for the selection and arrangement of illustrations. This paper analyses differences between the main mathematics textbook in China (PEP edition) using the method of comparative analysis in order to study changes of illustrations. In this research, mathematics textbooks for grade 7 to grade 9 are focused on. Two conclusions can be drawn from the research. First, the number of illustrations changes greatly; second, with functions of decoration, characterization, organization and explanation, the illustrations introduced in new textbooks can facilitate students' learning process.

Keywords: textbook illustrations, curriculum reform, middle school, secondary school, China

INTRODUCTION

Illustrations are the most accessible, effective and explicit learning resources in teaching activities. This paper selects the last edition of junior high school mathematics textbooks developed by People's Education Press before curriculum reform(short for old textbooks hereinafter) and the first edition of junior middle school developed by PEP after the curriculum reform(short for new textbooks hereinafter). Classification, statistics and comparison of illustrations in thirteen textbooks are made, and then illustration distribution, change of illustration forms and influences on teaching activities caused by illustration are analyzed. It is expected that through this study, theoretical basis could be provided to selection and arrangement of illustrations in textbooks and teaching procedure. Meanwhile, the paper also proposes constructive suggestions on how to use textbooks more scientifically and effectively, especially on how to select and set up contents of illustrations.

PREVIOUS STUDY OF ILLUSTRATIONS

Present studies are mainly focused on four aspects, which are effectiveness of illustrations; presentation time of texts and graphics and its effects on learning; property, type and function of illustrations from the perspective of epistemology; application strategies of illustrations and their influences on improving teaching efficiency.

As to effectiveness of textbook illustrations, study of Donald (1983) showed that illustrations facilitated readers' understanding and memorization of reading comprehension and helped to obtain more than one-third of information. Shen Deli (2001) made opticokinetics on junior high school students and the results showed that no matter in reading comprehension index or eye movement index, text with illustrations outstands that without illustrations. As to research on presentation time of texts and illustrations, Mayer et al. (1995) conducted research on different learning effects when texts and illustrations were simultaneously and successively presented. Learners were given the same conditions of time and it turned out that creative solutions were evidently created under the conditions of simultaneous presentation of texts and illustrations. Afterwards, another similar research was conducted by Mayer in 2001, the results came out that simultaneous presentation of texts and illustrations had more positive influences than separate presentation on learning effects. Furthermore, Mayer (2001) divided the functions of illustrations into decorative function, representational function, organization function and interpretation function. Study of Levin, Bender and Lesgold (1976) indicated function of illustration in memory retention is more effective than simple repetition.

According to present foreign and domestic studies, conclusions can be drawn that if form, colour and presentation time are reasonably arranged and illustrations themselves are closely related to texts, students' comprehension of mathematics knowledge could be enhanced.

STATISTICS AND COMPARISON OF ILLUSTRATIONS

In order to illustrate the change in the content of illustration, this paper firstly classifies type of various illustrations, and discusses their functions; then the number of different types of illustrations is counted respectively according to clarification; finally, deeper analysis is made in changes of illustrations.

Types of illustrations and their functions

In this paper, illustrations are classified form the perspective of contents in illustrations, which are respectively context diagram, mathematics model diagram, data figure and experimental operation figure. And then further statistic and comparison analysis in number are made.

Context diagram describes contexts in which mathematics problems are set. It highlights students' thinking process. At the same time, context diagram expands history and mathematical knowledge in reality.

Mathematics model diagram mainly refers to general model of mathematics such as function image, geometric figure, statistical chart and their quick screen capture in textbooks. This type of illustration concisely, intuitively and vividly describes basic mathematics concept and principles.

Data figure refers to the illustrations that provide part or even all the data and information for mathematics problems when the condition is incomplete. This kind of illustration mainly concentrates on examples.

Experimental operation diagram mainly shows thinking process of mathematics activities in the process of operation or in mind. This kind of illustration can not only clearly show students the experimental process, but also stimulate their interest of operating.

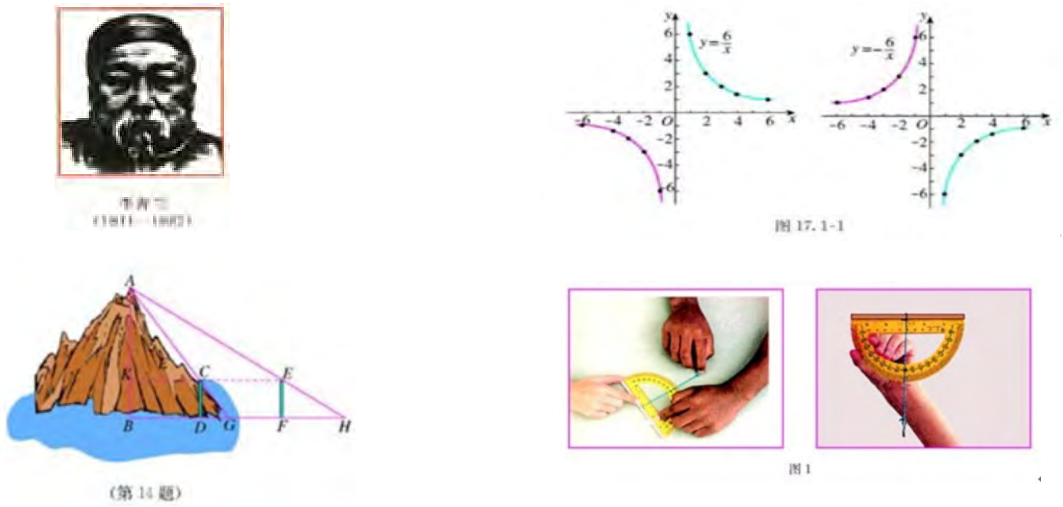

Figure1: Examples of four types of illustrations

Textbook illustrations strengthen and promote learners' comprehension of texts, while the function of strengthen differs from different types of illustrations. When teaching function of illustration in texts is considered, this paper follows research findings of Mayer. In this paper, the function of illustrations is classified into the function of decoration, the function of characterization, the function of organization and the function of explanation. Relationships between contents of illustrations and their functions are shown in Table 1.

Table 1: relationships between content and functions of illustrations

Functions / Content	Function of decoration	Function of characterization	Function of organization	Function of explanation
Context diagram	●			●
Mathematical model diagram		●	●	●
Experimental operation diagram		●	●	
Data figure		●		●

Statistical analysis of the number of illustrations

This paper makes statistical analysis by selecting only parts of illustrations in the parts of function and equation from both new and old textbooks to ensure specification and detail. Illustrations of function and equation in new and old textbooks are statistic analyzed. The bar chart presenting distribution of illustration from the new and old textbooks is shown in Figure

2. Illustrations in the parts of function and equation are made into pie chart in order to compare their proportion. The pie charts are shown in Figure 3.

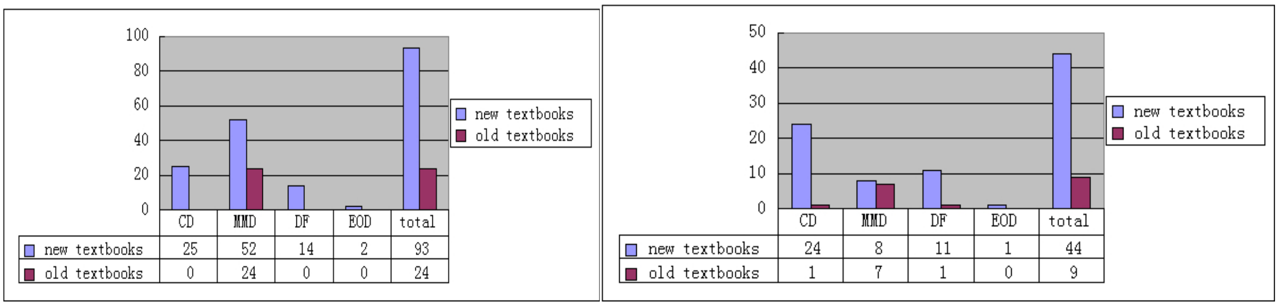

Figure 2: Statistical chart of the number of function and equation illustrations (CD: Context diagram; MMD: Mathematics model diagram; EOD: Experimental operation diagram; DF: Data figure)

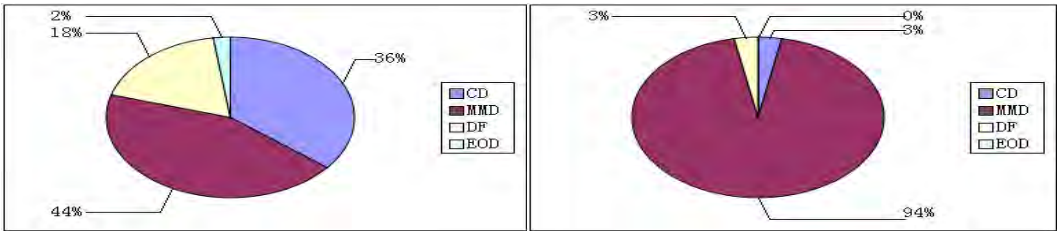

Figure 3: Pie charts of illustration proportion in new textbook (left) and old textbook (right)

It can be seen from the statistics that numbers of illustrations in new textbooks are four times larger than that of old textbooks. The illustrations can not only facilitate students' comprehension of abstract concepts, but also help to permeate mathematics thoughts. Moreover, types of illustrations in new textbooks are more diversified. In old textbooks, illustrations of mathematics model account for 94% of the whole, while in new textbooks, this proportion decreases to 44%. Another feature of illustrations in new textbooks is the combination of more context diagrams, the proportion of which increases to 36% comparing with 3% in old textbooks. This proportion has made context diagrams the second largest number of all illustrations. Besides, this change of new textbooks also embodies idea of New Curriculum Standard. Set up of mathematics situations as well as relationship between mathematics and real life are focused, through which students can experience value of life. Most notably, the experimental operation diagram first appears in new textbooks. It hints that students' practical ability and spirit of scientific research are emphasized in mathematics classroom after curriculum reform.

TYPICAL CASE STUDY

Illustrations from summary of equation properties and function variable are selected as below in order to demonstrate influence of illustration content change on students' comprehension of text through targeted comparison.

Comparison of contextual information graphic in equation properties

In first section of chapter four in old textbook: equation and its properties, contextual information graphics are used to illustrate equation's properties – see Figure 4. In the graphic in Figure 4, a balance is selected as a realistic situation, while the number of blocks in balance is used to explain equation's property. However, though the context follows the old textbook, great adjustment has been made in illustration content. The first section of the second chapter in new textbook selects a pair of contextual information graphics as shown in Figure 5.

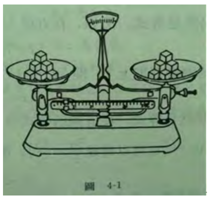

Figure 4: Contextual information graphic in old textbook

Figure 5: Contextual information graphic in new textbook

New textbooks describe two properties of the equation by using two images, and single graphic is also split into two balances, while the arrow pointing is added in order to demonstrate dynamic process of equation calculation. The increase of illustrations and the amount of information shows thinking and derivation process of equation properties, which is conductive to students' comprehension and memorization of equation properties.

Illustration of function variables

The same example is selected when function variable is introduced in both new and old textbooks to explore relationship between area of rectangular and the length of one side of rectangle. The old textbook isolates relationship between function expression and real situation by describing this problem with plain text while in new textbooks, the context diagram is inserted. Afterward, students can smoothly comprehend meaning of problems. Then students can find out relationship between side length and area through observing this, and finally make the expression.

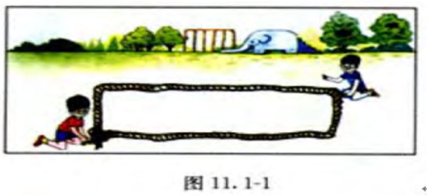

Figure 6: Context diagram in new textbook

CONCLUSIONS AND SUGGESTIONS

According to comparison between numbers, types and contents of illustrations in new and old textbooks, the advantages of new textbooks are as follows:

More abundant type

Types of new textbooks are more abundant with more diverse content. The appearance of context diagram and experimental operation diagram reflect communication, integration and infiltration between mathematics and other objects. Through plenty of illustrations, students experience that mathematics is closely connected to real life. Abstract mathematics knowledge is presented to students explicitly in the mode of “context creation- modeling-explanation- exploration” with the carrier of various illustrations.

Stronger functionality

Functionality of illustrations is enhanced by increasing the amount of information, especially the function of explanation. As are shown in 3.3, illustrations in new textbooks show dynamic effect of mathematics calculation. Proper appearing time of illustrations facilitates students’ comprehension of texts. Increase of students’ interpretation function also deepens students’ understanding of abstract knowledge.

Time features

Modern technology such as satellite image and screenshots of computer program are introduced into context diagrams and mathematics models in new textbooks. The advantage of illustration in solving mathematics with the help of modern information technology is quite significant in nowadays.

SUMMARY

While selecting illustrations, quality of illustrations should be guaranteed while the number and colour of illustrations are paid attention to. When selecting illustrations, functionality of illustrations should be considered of. Secondly, the illustrations should fit the text. Thirdly, theme of illustrations should closely relate to students’ real life. Lastly, selection of illustrations should be accord to students’ age and physical development.

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APPENDICES

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Appendix A: ICMT-2014 conference schedule

Day 1 (Morning), 29 July 2014

08.00–10.00	**Registration** (Day Chair: **Marcus Grace**; Foyer, Level 1, Building 58 (Murray Building))				
10.00–10.30	**Opening Session** (Chair: Dr. Jenny Byrne; Lecture Theatre, 58/1067)				
10.30–11.30	**Plenary Session 1** Speaker: Prof. Jeremy Kilpatrick, University of Georgia, USA ***From Clay Tablet to Computer Tablet: The Evolution of School Mathematics Textbooks*** (Chair: Keith Jones; Lecture Theatre, 58/1067)				
11.30–12.00	**Coffee Break** (Building 40, Garden Court)				
12.00–13.00	Parallel Session 1.1 Room: 58/1007 Chair: K. Ruthven	Parallel Session 1.2 Room: 58/1009 Chair: A. Takahashi	Parallel Session 1.3 Room: 58/1023 Chair: J. Novotná	Parallel Session 1.4 Room: 58/1067 Chair: I. Campton	Workshop 1 Room: 58/1065 Chair: Z. Miao
	A87. The Nordic network for research on mathematics textbooks: Eight years of experience **Barbro Grevholm**, Univ. of Agder, Norway	B12. Providing textbook supports for teaching mathematics through problem solving: An analysis of recent Japanese mathematics textbooks for elementary grades **Akihiko Takahashi**, Tokyo Gakugei Univ., Japan	D6. Impact of changes in teaching strategies on how teachers work with a textbook **Jarmila Novotná**, Charles Univ. in Prague, Czech; **Petr Eisenmann**, Jan Evangelista Purkyně Univ., Czech	D103. The improvement of teachers' interpretation of mathematics textbook **Pi-Jen Lin**, **Wen-Huan Tsai**, National Hsinchu Univ. of Education, Taiwan	C65. A comparative study of illustrations in the old and new middle school math textbooks **Xiaomei Liu**, Capital Normal Univ.; **Chunxia Qi**, Beijing Normal Univ., China
	A16. Textbook use in England: Mining OFSTED reports for views on mathematics textbooks **Christian Bokhove**, **Keith Jones**, Univ. of Southampton, UK	B20. Contemporary study of 5th grade textbooks – tasks on whole numbers and their compliance with mathematics Olympiad content **Ingrida Veilande**, Latvian Maritime Academy, Latvia	F83. How do textbooks incorporate graphing calculators? **Carlos Carvalho**, Escola Secundária Lima de Freitas/UIED FCT UNL; **José Matos**, New Univ. of Lisbon, Portugal	D99. Teacher guides as instruments for teaching maths – A case study **Sebastian Rezat**, Univ. of Paderborn, Germany	
13.00–14.00	**Break/Lunch** **(Building 40, Garden Court)**				

ICMT2014 Conference Programme Day 1 (Afternoon), 29 July 2014

14.00–15.00	Parallel Session 2.1 Room: 58/1007 Chair: T. Fujita	Parallel Session 2.2 Room: 58/1009 Chair: T. Miyakawa	Parallel Session 2.3 Room: 58/1065 Chair: C. Morgan	Symposium 1 Room: 58/1023 Chair: R. Even	Maths in the Science Curriculum Symposium Room: 58/1067 Chair: A. Christodoulou
	B50. Open approach in Japanese textbooks: Case of the teaching of geometry in lower secondary schools. **Taro Fujita**, Univ. of Exeter, UK; **Yutaka Kondo**, Nara Univ. of Educ., Japan; **Susumu Kunimune**, Shizuoka Univ., Japan; **Keith Jones**, Univ. of Southampton, UK	C19. Functions of proof: A comparative analysis of French and Japanese national curricula and textbooks **Takeshi Miyakawa**, Joetsu Univ. of Education, Japan	E15. The creation of mathematics in school textbooks: Palestine and England as example **Jehad Alshwaikh**, Birzeit Univ., Palestine; **Candia Morgan**, Institute of Education, Univ. of London, UK	E98. Teachers editing textbooks: Transforming conventional connections among teachers, curriculum developers, mathematicians, and researchers **Ruhama Even**, **Michal Ayalon**, Weizmann Institute of Science, Israel; E100. Teachers editing textbooks: Changes suggested by teachers to the math textbook they use in class **Shai Olsher**, **Ruhama Even**, Weizmann Institute of Science, Israel Discussants: **Charalambos Y. Charalambous**, University of Cyprus, Cyprus; **Edriss Titi**, Weizmann Institute of Science, Israel	G110. Mathematics within bioscience undergraduate and postgraduate UK higher education **Jenny Koenig**, Univ. of Cambridge, UK
	B23. Mathematics textbook analysis: Supporting the implementation of a new mathematics curriculum **Lisa O'Keeffe**, Univ. of Bedfordshire, UK	C28. A cross-cultural analysis of the voice of curriculum materials **Janine Remillard**, Univ. of Pennsylvania, USA; **Hendrik Van Steenbrugge**, Mälardalen Univ., Sweden; **Tomas Bergqvist**, Umeå Univ., Sweden	E5. Development of curriculum units as basic course for calculus **Yuang-Tswong Lue**, Taipei Chengshih Univ. of Science and Technology, Taiwan		
15.00–16.00	Parallel Session 3.1 Room: 58/1007 Chair: F. Leung	Parallel Session 3.2 Room: 58/1009 Chair: R. S. Rabelo	Parallel Session 3.3 Room: 58/1065 Chair: P. Teixeira		G111. Mathematics: the language of physics and engineering **Peter Main**, Institute of Physics, UK
	B62. Concept of probability: discursive analysis of Japanese secondary school textbooks **Koji Otaki**, Hiroshima Univ., Japan	C32. Dewey and mathematics textbooks **Rafaela Silva Rabelo**, Univ. de São Paulo, Brazil	F38. Building new teaching tools in mathematics: teacher and technology resources **Paula Teixeira, Mária Almeida, António Domingos, José Matos,** New Univ. of Lisbon, Portugal		
	B74. Reading geometrically: The negotiation of expected meaning of diagrams in maths textbooks. **Leslie Dietiker**, Boston Univ., USA; **Aaron Brakoniecki**, Michigan State Univ., USA	C29. A comparative analysis of national curricula relating to fractions in England and Taiwan **Hui-Chuan Li**, Univ. of Cambridge; **Yan-Shing Chang**, King's College, London, UK	F51. Modern Descriptive Geometry Supported by 3D Computer Modeling **Petra Surynkova**, Charles Univ. in Prague, Czech		

ICMT2014 Conference Programme Day 1 (Afternoon), 29 July 2014

16.00–16.30	**Coffee Break** (Building 40, Garden Court)				
16.30–17.30	Symposium 2 Room: 58/1007 Chair: Z. Usiskin	Parallel Session 4.1 Room: 58/1009 Chair: D. Yang	Parallel Session 4.2 Room: 58/1065 Chair: B. Pepin	Parallel Session 4.3 Room: 58/1023 Chair: I. Mok	Maths in the Science Curriculum Symposium Room: 58/1067 Chair: A. Christodoulou (Continued)
	C53. Lessons learned from three decades of textbook research **Denisse Thompson**, Univ. of South Florida**;** **Sharon Senk**, Michigan State Univ., USA	C31. A comparison of function in middle school textbooks among Finland, Singapore and Taiwan **Der-Ching Yang**, **Yung-Chi Lin**, National Chiayi Univ., Taiwan	B35. Choosing textbooks without looking at the textbooks – the role of the other's interpretations **Rúbia Barcelos Amaral**, São Paulo State Univ., Brazil, & Univ. of Algarve , Portugal; **C. Miguel Ribeiro**, **Juliana Samora Godoy**, São Paulo State Univ., Brazil	F2. How technology use is being reflected in junior secondary mathematics textbooks in Hong Kong? **Ida Ah Chee Mok**, Univ. of Hong Kong, Hong Kong	G112. Chemistry and Maths: A symbiotic relationship? **David Read**, Univ. of Southampton, UK
		B30. Assessing a new Indonesian secondary mathematics textbook: How does it promote authentic learning? **Mailizar Mailizar**, Syiah Kuala Univ., Indonesia**,** and Univ. of Southampton, UK; **Lianghuo Fan**, Univ. of Southampton, UK	A73. Reflections on trends in maths education in Brazil set in the context of textbooks for teaching maths **Maria Margarete Do Rosário Farias**, State Santa Cruz Univ., **Andriceli Richit**, State Sao Paulo Univ., **Rejane Waiandt Schuwartz Faria**, Intitution Bradesco Foundation, Brazil	F80**.** In-service teachers education and e-textbook development: an integrated approach **Victor Giraldo**, **Letícia Rangel**, Univ. Federal do Rio de Janeiro, **Cydara Ripoll**, Univ. Federal do Rio Grande do Sul; **Francisco Mattos**, Univ. do Estado do Rio de Janeiro, Brazil	
17.30–19.00	**Happy Hour** **(Building 40, Garden Court)**				

ICMT2014 Conference Programme Day 2 (Morning), 30 July 2014

Time					
08.00–09.00	**Registration** (Day Chair: **Keith Jones**; Foyer, Level 1, Building 58 (Murray Building))				
09.00–10.00	**Plenary Session 2** Speaker: Prof. Michal Yerushalmy, University of Haifa, Israel ***Challenging the Authoritarian Role of Textbooks*** (Chair: Dr. Christian Bokhove; Lecture Theatre, 58/1067)				
10.00–10.30	**Coffee Break** (Building 40, Garden Court)				
	Parallel Session 5.1 Room: 58/1007 Chair: B. O'Sullivan	Parallel Session 5.2 Room: 58/1009 Chair: G. Kim	Parallel Session 5.3 Room: 58/1023 Chair: J. Edwards	Symposium 3 Room: 58/1067 Chair: G. Howson	Parallel Session 5.4 Room: 58/1065 Chair: Y. Wang
10.30–11.30	A84. Change comes slowly: Using textbook tasks to measure curriculum implementation in Ireland **Brendan O'Sullivan**, St. Patrick's College, Ireland	B60. Textbook analysis: examining how Korean secondary mathematics textbooks support students' mathematical thinking and learning **Gooyeon Kim**, Sogang Univ., South Korea	D118. Pedagogical and curricular decision-making as personalised textbook development **Julie-Ann Edwards, Ian Campton,** Univ. of Southampton, UK	B71. US math textbooks in the common core era **William H Schmidt**, **Richard T Houang**, Michigan State Univ., USA	C47. Understanding of linear function: A comparison of selected mathematics textbooks from England and Shanghai **Yuqian Wang**, **Patrick Barmby**, **David Bolden**, Durham Univ., UK
	A95. What official documents tell us about textbook use in times of curricular change: The case of the "new math movement" in Portugal **Cristolinda Costa**, Univ. do Algarve/UIED; **José Matos**, Univ. Nova de Lisboa/UIED	B85. Model Method in Singapore primary mathematics textbooks **Tek Hong Kho**, **Shu Mei Yeo**, Ministry of Education, Singapore; **Lianghuo Fan**, Univ. of Southampton, UK	D10. If not textbooks, then what? English mathematics teachers' use of alternative curricular resources **Helen Siedel**, **Andreas Stylianides**, Univ. of Cambridge, UK		E55. Mathematics textbook research and development for the promotion of independent learning and inquiry learning **Fei Zhang**, **Xiujuan Zhu**, Jiangsu Second Normal Univ., China
11.30–13.00	**Exhibition** (Building 40, Garden Court)	**Poster Session*** (Building 40, Garden Court)			
13.00–14.00				**Break/Lunch** (Building 40, Garden Court)	

ICMT2014 Conference Programme Day 2 (Afternoon), 30 July 2014

	Parallel Session 6.1 Room: 58/1007 Chair: C. C. Ripoll	Parallel Session 6.2 Room: 58/1009 Chair: M. Almeida	Parallel Session 6.3 Room: 58/1023 Chair: B. Grevholm	Symposium 4 Room: 58/1067 Chair: M. Grace	Workshop 2 Room: 58/1065 Chair: P. Charlton
14.00–15.00	C54. An international comparison of mathematical textbooks **Cydara Cavedon Ripoll**, Univ. Federal do Rio Grande do Sul, Brazil	C37. Mathematics textbooks, in Portugal: The unique textbook **Mária Almeida**, **Paula Teixeira**, Ag. Escolas dos Casquilhos–UIED, **António Domingos, José Matos**, Faculdade de Ciências e Tecnologia da UNL–UIED, Portugal	B63. Possible misconceptions from Japanese mathematics textbooks with particular reference to the function concept. **Yusuke Uegatani**, Hiroshima Univ., Japan	F3.Transition to college mathematics and statistics: a problem-based, technology-rich capstone course for non-STEM students **Christian Hirsch**, Western Michigan Univ., USA	F18. Authoring your own creative, electronic book for mathematics **Christian Bokhove**, **Keith Jones**, Univ. of Southampton, UK; **Patricia Charlton**, **Manolis Mavrikis**, **Eirini Geraniou**, London Institute of Education, UK
	C56. A comparative study of statistics in junior high schools based on maths textbooks of China, US and Australia **Jianbo Wang**, **Yiming Cao**, Beijing Normal Univ., China	E69. Mathematics textbook development and learning under difficult circumstances in schools in Nigeria **R Abiodun Ogunkunle**, Univ. Of Port Harcourt, Nigeria	B75. Integrated education at the primary school in Lithuania. **Saulius Žukas**, Baltos Lankos Publishing House; **Ričardas Kudžma**, Vilnius University, Lithuania		
	Parallel Session 7.1 Room: 58/1007 Chair: L. Melo	Parallel Session 7.2 Room: 58/1009 Chair: K. Hemmi	Symposium 5 Room: 58/1023 Chair: Y. Wang	Parallel Session 7.3 Room: 58/1067 Chair: T. Rowland	
15.00–16.00	A24. Scientific mathematics and school mathematics: knowledge, conceptions and beliefs of teachers and mathematicians during the development of an E-Textbook. **Lucas Melo**, **Victor Giraldo, Letícia Rangel**, Universidade Federal do Rio de Janeiro, Brazil	C34. Crossing the boundaries: Swedish teachers' interplay with Finnish curriculum materials **Kirsti Hemmi**, **Heidi Krzywacki**, Mälardalen Univ., Sweden	E120. The New Century Primary Mathematics Textbook Series: Textbooks with specific consideration to characteristics of children's thinking. **Huinu Wei**, **Fengbo He**, Editorial Board of New Century Primary Maths Textbooks, Beijing Normal University Press, China	B22. Modes of reasoning in Israeli 7th grade mathematics textbook explanations **Boaz Silverman**, **Ruhama Even**, Weizmann Institute of Science, Israel	
	A4. Pre-service and in-service teachers' preference when selecting mathematics textbooks **Hana Moraova**, Charles Univ., Czech	(Reserved for special purpose)		C45. A comparison of two grade 7 mathematics textbooks. **Jaguthsing Dindyal**, National Institute of Education, Singapore	

ICMT2014 Conference Programme Day 2 (Afternoon), 30 July 2014

16.00–16.30	**Coffee Break** (Building 40, Garden Court)
16.30–18.00	**Plenary Session 3 (Panel Discussion)** Speakers: Prof. Kenneth Ruthven (Panel Chair; University of Cambridge, UK) Prof. Jere Confrey (North Carolina State University, USA) Mr. John Ling (Former School Mathematics Project, UK) Prof. Binyan Xu (East China Normal University, China) ***Back to the Future of Textbooks in Mathematics Teaching*** (Chair: Dr. Charis Voutsina; Lecture Theatre, 58/1067)
18.30–20.30	**Conference Dinner**** (Building 40, Garden Court)

***Contributions accepted for poster session:**

F7. The use of technology in textbooks: A grade-7 example from Hong Kong. **Ida Ah Chee Mok, King-Woon Yau**, The Univ. of Hong Kong

C9. The study of geo. contents in the middle grade maths textbooks in Singapore, Taiwan, & USA. **Der-Ching Yang**, Nat'l Chiayi Univ., Taiwan

A25. Situational authenticities in lower secondary school mathematics problems: Reasons for calculation and origin of quantitative information. **Lisa O'Keeffe**, Univ. of Bedfordshire, UK; **Josip Slisko**, Benemérita Univ. Autonoma de Puebla, Mexico

B26. Analysis of integral and differential calculus textbooks and mathematical modelling activities in the light of the didactic transposition theory. **Lourdes Maria Werle De Almeida**, Univ. Estadual de Londrina; **Kassiana Surjus**, PUC, Brazil

B41. The characteristics of new mathematics textbooks for junior secondary school in China: A case study. **Fu Ma**, Nanjing Normal Univ.; **Chunxia Qi**, Beijing Normal Univ.; **Xiaomei Liu**, Beijing Capital Normal Univ., China

E58. An introduction to mathematics textbooks policies in China. **Huiying Zhang**, Shijiazhuang Research Institute of Education Science, China

D61. Korean students' use of mathematics textbook. **Na Young Kwon**, Inha Univ.; **Gooyeon Kim**, Sogang Univ., South Korea

C82. The broken-tree problem: Formulations in Mexican middle-school textbooks and students' constructions of the corresponding situation model. **Josip Slisko**, **José Antonio Juárez López**, Benemérita Univ. Autónoma de Puebla, Mexico

D92. Math knowledge and skills higher educ. programs expect of high school graduates. **Cengiz Alacaci**, Istanbul Medeniyet Univ.; **Gulumser Ozalp**, Gaziantep C. Foundation Private Sch; **Mehmet Basaran**, SANKO Private Sch; **İlker Kalender**, Ihsan Dogramaci Bilkent U., Turkey

B93. Forewarned is forearmed: A mathematics textbook. **Peter McWilliam**, The College of The Bahamas, Bahamas

G94. Differential and integral calculus in textbooks: An analysis from the point of view of digital technologies. **Andriceli Richit**, State Sao Paulo Univ.; **Adriana Richit**, Federal Univ. of Fronteira Sul; **Maria Margarete Do Rosário Farias**, State Santa Cruz Univ., Brazil

D102. Enhancing a teacher's fundamental interaction with the textbook through a school-based mathematics teacher research group activity in Shanghai. **Liping Ding, Svein Arne Sikko**, Sør-Trøndelag University College, Norway

F119. The potential of handwriting recognition for interactive mathematics textbooks. **Mandy Lo**, University of Southampton, UK.

Appendix B: ICMT-2014 conference participant list

Title	Name	Affiliation	Country/ Region
Dr	Cengiz Alacaci	Istanbul Medeniyet University	Turkey
Miss	Manahel Alafeq	University of Southampton	UK
Dr	Rúbia Amaral	UNESP-Universidade Estadual Paulista	Brazil
Prof	Mária Almeida	UIED-Universidade Nova de Lisboa	Portugal
Dr	Michal Ayalon	Weizmann Institute of Science	Israel
Dr	Tomas Bergqvist	Umeå University	Sweden
Dr	Christian Bokhove	University of Southampton	UK
Mr	Jeremy Burke	King's College London	UK
Dr	Jenny Byrne	University of Southampton	UK
Mr	Ian Campton	University of Southampton	UK
Dr	Carlos Carvalho	UIED FCT UNL	Portugal
Dr	Charalambos Charalambous	University of Cyprus	Cyprus
Dr	Patricia Charlton	Institute of Education London	UK
Ms	Xiaomei Chen	Jilin No. 1 Experimental Primary School	China
Dr	Andri Christodoulou	University of Southampton	UK
Prof	Jere Confrey	North Carolina State University	USA
Ms	Jill Cornish	Oxford University Press	UK
Mrs	Cristolinda Costa	University of the Algarve	Portugal
Dr	Leslie Dietiker	Boston University	USA
Dr	Jaguthsing Dindyal	National Institute of Education	Singapore
Dr	Liping Ding	HiST, Sør-Trøndelag University College	Norway
Dr	Olivera Djokic	University of Belgrade	Serbia
Prof	António Domingos	Universidade Nova de Lisboa	Portugal
Dr	Julie-Ann Edwards	University of Southampton	UK
Prof	Ruhama Even	Weizmann Institute of Science	Israel
Prof	Lianghuo Fan	University of Southampton	UK
Dr	Taro Fujita	University of Exeter	UK
Dr	Jianliang Gao	University of Southampton	UK
Ms	Caro Garrett	University of Southampton	UK
Dr	Victor Giraldo	Universidade Federal do Rio de Janeiro	Brazil
Dr	Dubravka Glasnović Gracin	University of Zagreb	Croatia
Dr	Haiyan Gong	East China Normal University Press	China
Prof	Marcus Grace	University of Southampton	UK
Prof	Barbro Grevholm	University of Agder	Norway
Dr	Janice Griffiths	University of Southampton	UK
Prof	Jiling Gu	Nanjing Normal University	China
Miss	Li Hao	University of Southampton	UK
Mr	Lifeng He	Zhejiang Education Publishing House	China
Prof	Fengbo He	Jilin Provincial Institute of Education	China
Dr	Kirsti Hemmi	Mälardalen University	Sweden
Prof	Christian Hirsch	Western Michigan University	USA
Mr	Foo Him Ho	Singapore Ministry of Education	Singapore

Ms	Huiying Hou	Beijing New Century Jiuzhang Education & Technology Institute	China
Dr	Richard Houang	Michigan State University	USA
Prof	Geoffrey Howson	University of Southampton	UK
Ms	Qiong Hua	Zhejiang Education Publishing House	China
Ms	Lihua Huang	Beijing No. 2 Experimental School Chaoyang Campus	China
Mrs	Rosalyn Hyde	University of Southampton	UK
Ms	Jo Issa	Oxford University Press	UK
Dr	Dustin Jones	Sam Houston State University	USA
A/Prof	Keith Jones	University of Southampton	UK
Mr	Ilyas Karadeniz	University of South Florida	USA
Dr	Jan Kaspar	Charles University in Prague	Czech
Dr	Sibel Kazak	University of Exeter	UK
Prof	Anthony Kelly	University of Southampton	UK
Dr	Tek Hong Kho	Mathematics Education Consultant	Singapore
Prof	Jeremy Kilpatrick	University of Georgia	USA
Dr	Gooyeon Kim	Sogang University	South Korea
Dr	Elizabeth Kimber	Cambridge Mathematics Education Project	UK
Dr	Jennifer Koenig	University of Cambridge	UK
Prof	Yutaka Kondo	Nara University of Education	Japan
Dr	Heidi Krzywacki	Mälardalen University	Sweden
Mr	Ricardas Kudzma	Vilnius University	Lithuania
Prof	Susumu Kunimune	Shizuoka University	Japan
Prof	Na Young Kwon	Inha University	South Korea
Mrs	Moneoang Leshota	University of the Witwatersrand	South Africa
Prof	Frederick Leung	University of Hong Kong	Hong Kong
Ms	Hui-Chuan Li	University of Cambridge	UK
Prof	Pi-Jen Lin	National Hsinchu University of Education	Taiwan
Dr	Yung-Chi Lin	National Chiayi University	Taiwan
Mr	John Ling	School Mathematics Project (SMP)	UK
Ms	Anna Littlewood	Cambridge University Press	UK
Prof	Jian Liu	Beijing Normal University	China
Ms	Keqin Liu	Beijing Zhong Guan Cun No. 3 Primary School	China
Prof	Xiaomei Liu	Beijing Capital Normal University	China
Miss	Liyuan Liu	University of Southampton	UK
Miss	Mandy Lo	University of Southampton	UK
Miss	Mei Yoke Loh	Ministry of Education	Singapore
Mrs	Sue Lowndes	Oxford University Press	UK
Dr	Yuang-Tswong Lue	Taipei Chengshih University of Science and Technology	Taiwan
Prof	Fu Ma	Nanjing Normal University	China
Prof	Constantino José Machado de Sousa	Centro de Educaçao Elementar e Secundaria e Universitaria Professor Paulo Freire	Brazil
Ms	Heather Mahy	Cambridge University Press	UK
Mr	Mailizar Mailizar	Syiah Kuala Univ./Univ. of Southampton	Indonesia/UK
Prof	Peter Main	Institute of Physics	UK

Mr	David Mantovani	Cambridge University Press	UK
Mr	José Matos	UIED-Universidade Nova de Lisboa	Portugal
Mr	Peter McWilliam	The College of The Bahamas	Bahamas
Ms	Zhenzen Miao	University of Southampton	UK
Mr	Takeshi Miyakawa	Joetsu University of Education	Japan
Dr	Ida Ah Chee Mok	University of Hong Kong	Hong Kong
Dr	Hana Moraova	Charles University	Czech
Prof	Candia Morgan	Institute of Education London	UK
Mrs	Debbie Morgan	National Centre for Excellence in the Teaching of Mathematics (NCETM)	UK
Prof	Daniel Muijs	University of Southampton	UK
Prof	Eizo Nagasaki	Shizuoka University	Japan
Mr	Ming Ni	East China Normal University Press	China
Prof	Graham Niblo	University of Southampton	UK
Prof	Hiro Ninomiya	Saitama university	Japan
Dr	Keiichi Nishimura	Tokyo Gakugei University	Japan
Prof	Jarmila Novotna	Charles University	Czech
Dr	Abiodun Ogunkunle	University of Port Harcourt	Nigeria
Ms	Lisa O'Keeffe	University of Bedfordshire	UK
Mr	Shai Olsher	Weizmann Institute of Science	Israel
Mr	Brendan O'Sullivan	St. Patrick's College, Drumcondra, Dublin	Ireland
Mr	Koji Otaki	Hiroshima University	Japan
Prof	Birgit Pepin	HiST, Sør-Trøndelag University College	Norway
Ms	Ping Lu	Ningbo University	China
Mr	Stuart Price	University of Southampton	UK
Prof	Chunxia Qi	Beijing Normal University	China
Mr	Shouwang Qian	Beijing New Century Jiuzhang Education & Technology Institute	China
Miss	Rafaela Silva Rabelo	University of Sao Paulo	Brazil
Dr	David Read	University of Southampton	UK
Dr	Janine Remillard	University of Pennsylvania	USA
Prof	Sebastian Rezat	University of Paderborn	Germany
Mr	Andriceli Richit	Sao Paulo State University	Brazil
Mrs	Cydara Ripoll	Universidade Federal do Rio Grande do Sul	Brazil
Prof	Tim Rowland	Cambridge and UEA	UK
Prof	Kenneth Ruthven	University of Cambridge	UK
Dr	Christopher Sangwin	Loughborough University	UK
Dr	Carl Saxton	Cambridge University Press	UK
Prof	William Schmidt	Michigan State University	USA
Mrs	Corina Seal	Institute of Education London	UK
Prof	Sharon Senk	Michigan State University	USA
Prof	Bingxing Shi	Beijing Institute of Education	China
Dr	Helen Siedel	University of Cambridge	UK
Dr	Svein Sikko	HiST, Sør-Trøndelag University College	Norway
Mr	Boaz Silverman	Weizmann Institute of Science	Israel
Prof	Josip Slisko	Benemerita Universidad Autonoma de Puebla	Mexico
Ms	Jinghong Sun	Beijing Haidian Teachers Training College	China

Dr	Petra Surynkova	Charles University in Prague	Czech
Dr	Akihiko Takahashi	Tokyo Gakugei University	Japan
Prof	Paula Teixeira	UIED-New Lisbon University	Portugal
Prof	Mike Thomas	University of Auckland	New Zealand
Prof	Denisse Thompson	University of South Florida	USA
Prof	Edriss Titi	Weizmann Institute of Science	Israel
Mr	Yusuke Uegatani	Hiroshima University	Japan
Mrs	Karen Usiskin	Pearson Education	USA
Prof	Zalman Usiskin	University of Chicago	USA
Prof	Marja van den Heuvel-Panhuizen	Utrecht University	Netherlands
Mr	Stefaan van Malderen	Plantyn Publishing	Belgium
Dr	Hendrik van Steenbrugge	Mälardalen University	Sweden
Mr	Marc van Zanten	Utrecht University	Netherlands
Ms	Ingrida Veilande	Latvian Maritime Academy	Latvia
Dr	Chronoula Voutsina	University of Southampton	UK
Dr	Jianbo Wang	Beijing Normal University Publishing Group	China
Miss	Yuqian (Linda) Wang	Durham University	UK
Mr	Yong Wang	BNUP Editorial Board of New Century Textbooks	China
Mr	Karl Warsi	Oxford University Press	UK
Ms	Huinu Wei	Henan Experimental Primary School	China
Dr	Lourdes Maria Werle de Almeida	Universidade Estadual de Londrina	Brazil
Ms	Joanna Williamson	University of Southampton	UK
Mr	Ka Lok Wong	University of Hong Kong	Hong Kong
Mr	Oon Hua Wong	Ministry of Education	Singapore
Prof	Binyan Xu	East China Normal University	China
Prof	Der-Ching Yang	National Chiayi University	Taiwan
Ms	Yanmei Yang	Beijing Academy of Educational Sciences	China
Miss	Jinyu Yang	University of Southampton	UK
Prof	Michal Yerushalmy	University of Haifa	Israel
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Subject index

Note: *this subject index is not intended to be exhaustive. Rather, it directs to papers in the proceedings for which a term in the index is a key word.*

Mathematics and Science Education Research Centre

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ICMT-2014, the International Conference on Mathematics Textbook Research and Development, brought together mathematics educators, textbook researchers and developers, and policy makers from different parts of the world to share their research results, development experiences and reform ideas, and discuss issues and directions concerning mathematics textbook research and development.

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