Mathematics teaching and learning in the late 1970s in Portugal: intentions and implementations

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Abstract

By the late 1970s, on the wake of the 1974 democratic revolution, the Portuguese school system has grown very rapidly and is undergoing major changes in its structure. At the same time, modern mathematics reform for middle and upper grades of the secondary school gradually adopted from the late 1960s is confronted with the practicalities of its implementation: textbooks, teacher education, didactical strategies, evaluation, etc.

A series of 16 reports from the Ministry of Education written between 1977 and 1981 under the leadership of Swedish specialists aimed at characterizing the situation and some reports addressed specifically mathematics teaching and learning. This paper will study the implementation of mathematics curricula in Portugal in the late 1970s, in particular the programs, the mathematics classes, students' learning.

This work studies school culture of the late 1970s in Portugal, especially as it relates to the conditions of teaching and learning of mathematics in secondary schools as they appear in a coherent set of technical reports from the *Project for the Evaluation of Unified Secondary Schooling* published by the Ministry of Education between 1977 and 1981.

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One of the consequences of the revolution of 1974 that ended the Portuguese dictatorial regime was the elimination of the distinction between two educational tracks and the creation of the *Unified Secondary Schooling* beginning at 7th grade (*7º ano unificado*) in 1975. By 1974, there was a formal centralized education system based on legislation in the late 40s, and several several sub-systems, all of them "experimental". Portuguese students entered school with six years of age and went through four years of primary schooling followed by two years at preparatory schools after which they could pursue studies either at Liceus oriented to the universities, or at technical schools oriented to the work market or polytechnic institutes. The elimination of this distinction in 1975 was viewed as a means to balance educational opportunities for all students.

The *Project for the Evaluation of Unified Secondary Schooling*

In May 1976, when the first year of the unification reform was nearing the end, the Ministry of Education commissioned the evaluation of its implementation (Freitas, 1979). In the following years this assessment was extended to the next two years of unified schooling (the 8th and 9th grades and the Portuguese Ministry of Education established a cooperation agreement with Sweden. From January 1977, a major project designed to evaluate this process of unification of the two cycles, the *Project for the Evaluation of Unified Secondary Schooling* began[2]. By 1979 the Project had monitored the generalization of the unified schooling until the 9th grade and later published a total of 16 reports, 15 of which refer to empirical research by collecting data from students, teachers, parents, members of Executive Councils, and businesses. Various methods are used in particular questionnaires, tests, interviews, among others, and in parallel with analysis of large samples of students and teachers data, particular schools.

As mentioned, the evaluation of the Unified Schooling starts in 1975/76 with a comprehensive study of its implementation. In the second year the Project began to think on the need to assess the appropriateness of curricula to the age level of students as well as their degree of apprehension of the new curricula. In this second phase the discipline of mathematics was the subject of specific studies (it was, in fact, the only subject worthy of an in-depth study), mainly because it is, in the words of researchers, a discipline "easy to handle, both with regard to the hierarchy of objectives in content as the preparation of tests "(Catela & Kilborn,

[2] The term *Project* will be used in the text.

1979, p. 2[3]). Thus, from the 16 studies produced five refer specifically to teaching and learning of mathematics between 1975 and 1979 (Catela, 1978a, 1980; Catela & Kilborn, 1979; Leal & Fägerlind, 1981; Leal & Kilborn, 1981). Other reports, though not focused on mathematics, also contain data on this subject.

Under the Luso-Swedish agreement, Wiggo Kilborn, "consultant and advisor of the activities inherent to the study of Portuguese unified secondary school mathematics curriculum" (Catela & Kilborn, 1979, p. 3), and Ingemar Fägerlin participated in the implementation of these studies focused on mathematics and worked in collaboration with Maria Emília Catela, science teacher and later with Leonor Cunha Leal, teacher of mathematics, both participating full time in this project.

Mathematics was thus the subject of intensive investigations focusing on the conditions of teaching and learning and these studies assume greater importance today, as they provide an opportunity of studying the conditions in which the reform of modern mathematics was applied, in particular on the functioning of mathematics classes, some dimensions of the mathematics curriculum and, finally, some elements on the learning of mathematics in particular of the typical themes of modern mathematics.

Conditions for teaching mathematics classes

Although not containing descriptions of mathematics classes, the Evaluation Project gives some partial information on how they were conducted during the late 1970s. From the responses to a questionnaire sent to the Executive Councils of public and private schools where the 7th year was taught in 1975/76, we see that mathematics classes began late in many schools (Freitas, 1979, p. 12). Only 17% of the schools had some mathematics classes starting in the beginning of the school year[4] (n = 307). 20% of schools had mathematics classes starting in January, and there was still a significant number of schools with classes (13%) starting in May. This panorama is similar in other disciplines.

In 1976/77 this problem was further investigated with a sample of 29 schools from the northeastern interior and 48 from the Lisbon district (the sample accounts for 80% of schools with grade 7) (Mendonça, 1980). The Executive

[3] I performed all translations.

[4] Beginning of October 1975.

Councils indicated delays in the beginning of classes in mathematics and other disciplines, although apparently there has been a small improvement. Failures of placement of teachers are appointed by the Executive Councils as one of the reasons for this situation. In October, on the district of Lisbon, only half (56%) of the schools had started mathematics classes (31% only started in November). In the northeast the outlook is even worse, with only 14% of schools started classes in October (31% in November and 24% in December). Note that in 7% of sampled schools in the district of Lisbon and 21% in the northeast there were no mathematics classes for 7th graders throughout the school year (p. 18). In the following year the situation was similar (Godinho, 1980; Fägerlind & Leal, 1981) but may have improved in 1978/79. One study reveals that about 90% of Mathematics classes had begun in October (Coutinho, 1981) and another (Leal & Fägerlind, 1981) noted that mathematics classes for some schools only began in February of 1979.

The system for teacher placement was complex and centralized, but the main reason for this situation relates to the shortage of teachers which implied the absence of a stable teaching staff in schools. In the case of mathematics, professionals from other fields (engineering, university students, and even high-school students from the schools themselves) were performing teaching duties in schools. The results of the policy of limiting the access to teacher training from the 1950s (Matos, 2009) are now patent. In the academic year 1975/76 most of the schools (163 of 307) indicated that the number of teachers of mathematics was insufficient (Freitas, 1979, pp. 16, 18). Only 24% of the teachers who taught 7th grade schools had educational qualification (p. 17) and two thirds were under 31 years (p. 21). For teachers of mathematics only 43% had a degree (not necessarily for teaching mathematics). In short, many teachers are missing and among those few had the necessary scientific and pedagogical skills.

The program of modern mathematics

Modern Mathematics started in Portugal from the middle 1960s (Matos, 1989, 2009). From 1965 TV lessons of modern mathematics for grades 5th and 6th were nationally broadcasted and the cycle for the same grades had, since the its beginnings in 1968, a program following the precepts of modern mathematics. The 7th through 9th grades either in Liceus or in the technical schools had programs of modern mathematics since 1971 and grades 10th and 11th of Liceus were in transition from Classical to Modern Mathematics.

After the decision to unify the Liceus and technical schools tracks in 1975, new mathematics programs were designed by Leonor Filipe, Alfredo Osório dos Anjos, and Francelino Gomes (Catela & Kilborn, 1979). The first two came from Liceus and the third from a technical school. The last two had an extensive experience in teacher education. These new programs were very close to the contents of the now extinct course from the Liceus. Essentially, the programs of Liceus were extended to the technical schools.

In 1977, to counter widespread students' failure and the impossibility for many teachers to teach all the required topics, new mathematics programs are published. Table 1 presents the topics of the mandatory mathematics programs of 1977.

Table 1. Mathematics programs of 1977, grades 7th through 9th.

7th grade	1) Language. 2) Rational numbers and their operations. 3) Equations of the first degree and problem solving. 4) Binary relations. 5) Applications and functions. 6) Geometric transformations. 7) Equal triangles.
8th grade	1) Problems — Equations and systems of equations. 2) Set of real numbers (including the Pythagorean Theorem). 3) Homoteties in the plane. 4) Similarities in the plan.
9th grade	1) Order relations in R (including inequalities). 2) Geometry of the plan. 3) Extension of the concept of power. 4) Radicals. 5) Problems and equations of the second degree. 6) Similarities of the plan (from the 8th grade). 7) Trigonometry (from the 8th grade). 8) Space geometry.

The new 9th grade program includes a repetition of 8th grade topics (essentially geometry). *Minimal Programs* — a smaller list of topics that all teachers should teach — were also proposed. Teachers rapidly interpreted that the topics not included in these minimal programs would be "optional". In 1980, even these shorter programs were replaced with new programs, almost all of them explicitly providing for the review of issues in previous years.

Modern mathematics was justified in the discourse of Portuguese educators for psychological reasons and for what was perceived as its closeness to the development of mathematics as a science (Matos, 2009). The match between the three parent bourbakist structures and Piagetian intelligence operative structures reinforced the merits of the new ideas. It was hoped that the new programs were psychologically simpler and mathematically more robust than previous ones.

Apparently what happened was the opposite. The successive reduction program reveals either a generalized implicit resistance to the application of new programs from the teachers, or the practical impossibility of its implementation. The consequence was that whole parts of the programs would no longer be taught in Portuguese schools. The main victim was geometry systematically left to the end of the school year and subsequently dropped. But perhaps more importantly, the adoption of a terminology strange to students, and especially to the teachers, made it more difficult to teach and learn mathematics. Only in 1989 new programs will depart from modern mathematics and adopt a problem solving approach to mathematics (Matos, 2011).

Project's appreciation of the mathematics curricula

The Project assessed mathematics curriculum and as for the prescribed curriculum, reports are generally very critical:

> By analyzing the curricula for the 7th, 8th and 9th grades it appears that they are curricula typical of the first generation of modern mathematics; they show, moreover, a keener interest in teaching pure mathematics that will make the students good mathematicians. (Catela & Kilborn, 1979, p. 41)

The reports mention several times the bad results the new approaches produced in foreign countries. It is possible to note here the strong influence of the opinions of Swedish experts Ingemar Fägerlin and Wiggo Kilborn. Adopting simultaneously a pedagogical and a critical tone the reports explain that in other countries the initial enthusiasm was replaced by a disenchantment caused by learning difficulties, motivation, and major limitations of the students in solving even simple problems. These opinions are supported by references to the three ICME held recently (in 1976) and to international studies.

> To what extent is modern mathematics necessary, both in Portugal and elsewhere, to the child's education or profession in the future. The answer to this question is also one of the main reasons many countries have changed their [modern] mathematics curricula, using other alternatives according to the capabilities and needs of children. (Catela & Kilborn, 1979, p 42).

The curriculum presented to teachers through textbooks was also appreciated by the Project. Limitation to unique textbooks for each grade adopted in 1948 and revoked in the beginning of the 1970s considerably restricted the range of available textbooks. Although the regime of a unique book was gradually falling into disuse, only from 1976 an alternative collection of textbooks becomes available. However, the reports do not mention it, and treat the first collection of books as if they were the only *de facto* books.

Shortages in the use and access to textbooks are reported (Catela & Kilborn, 1979). For example, although in the year 1975/76 new programs had been published, new manuals did not immediately accompany them. Even in 1977/78 the project documents some problems accessing books. In general, students could only acquire the book in December, at the end of the 1st school period.

The content of these books draws strong criticism from the evaluators (Catela & Kilborn, 1979). Firstly, they argue that books are almost replicas of the programs, which can pose problems for students and teachers. They indicate that programs use concepts and sequences very different from regular teaching, so that the structure of "pure" logic of the books does not facilitate their understanding by teachers who are not confortable with meanings and assumptions on which these concepts and sequences are based. The structure of the books is also questioned. Students are often required to go through five or more pages of text before having the opportunity to solve tasks.

A second aspect is addressed. For the authors of the report, the Portuguese books, unlike those of other countries, do not accommodate an individualized learning, because they do not contain diagnostic tests allowing the verification of knowledge by the students. On the other hand, there are very few exercises at the end of each section that allow students to assess understanding before moving on. It also points out the linear structure of the books. Students only have a single contact with each topic, there are no ways to compensate later for a more hasty study.

> For example, students have only contact the Pythagorean theorem once and only once, instead of going in small steps in order to pass from one step to the next only when the first is understood. (Catela & Kilborn, 1979, p. 45).

Learning outcomes

Mathematics learning was studied by the Project. The design used longitudinal testing (three comparable tests, five questions each) through the years 1977/78 and 1978/79. Each question should encompass three different levels of difficulty and so each had three items and it was expected that 75% of students responded correctly to the first, 50% to the second and 25% the third. It was therefore expected that students obtain an average score of 50% in each test question. Almost every question privileged knowledge of algorithmic or algebraic nature, in which the greater complexity corresponds to replacing integers by fractions or by additions and subtractions multiplications and divisions, or incorporate powers.

The first two tests, MI with topics from 7th grade and M-II with 8th grade contents, were prepared by Maria Emília Catela, Wiggo Kilborn and the authors of the new programs (Catela and Kilborn, 1979). The tests were previously verified: the test for the 7th grade was passed on the 8th and the test for the 8th grade in the 9th. The results of the students were very weak. However it was decided to keep MI and change some details of M-II (which now became M-II/2). The third test, M-III, with contents of the 9th year, was apparently prepared by the same team, and tested on students in 10th grade and after some minor changes became test M-III/2 (Catela, 1980).

For example, question 2 from MI, passed in 7th and 8th grades, was:

> 2. Compute the value of each of the expressions:
>
> 2.1. $\dfrac{1-3x}{x-2}$ for $x=-3$.
>
> 2.2. $\dfrac{2-xy}{x+y}$ for $\begin{matrix} x=2 \\ y=-1 \end{matrix}$.
>
> 2.3. x^3-3x^2 for $x=-\dfrac{1}{2}$.

The average score of this question was 3.9% among 7th graders and 18.7% among the 8th (Catela and Kilborn, 1979) well off the expected percentage of 50% for each grade.

The design had some methodological flaws, mainly related to sampling and the instruments chosen. The results were comprehensively examined elsewhere (Ponte, Matos, & Abrantes, 1998). Overall results are extremely low. The performances are much lower than expected, even for the contents of the 7th grade tested on the 8th. On average, students in the 7th grade get a rating of 13%, and the 8th achieve 24% in the contents of 7th grade and 25% in the 8th. In average, students in 9th grade score 29% (Catela, 1980).

Discriminating by topic, it appears that 7th grade results are particularly low (about 5%) in matters involving algebraic expressions and solving equations and the 8th in solving equations of the second degree. The higher scores appear in numeric expressions for 7th grade (27%), and in operations with polynomials (39%) and solving systems of equations (36%) for the 8th, still far from the average of 50% expected by the authors programs. No distinct difference is apparent between the results of the group in the district of Lisbon and the Northeast. As for the 9th grade, the results are also very low. Scores for similarity of triangles (10%), second-degree equations (15%) and powers (29%) are the most difficult topics.

Why these results? Another report (Catela, 1980) reflects on the poor performance of students pointing the excessive difficulty of the tests. Referring to the initial tests that were tried, she states

> These trial tests were constructed by the authors of the programs which are also teachers. The level of the tests MI and M-II (1st [trial] version) shows, therefore, the notion that [these] teachers have of the level of their own classes, and it was ultimately proved that this notion contains higher expectations than the actual knowledge of students. (Catela, 1980, p. 24)

Searching for explanations for this mismatch, the author concludes:

> One reason for the high expectation on the part of authors [of the programs] (and perhaps teachers in general) may be the fact that modern mathematics, whose concepts have guided the current programs is considered easier for students than conventional mathematics. In fact, when mathematics was introduced in secondary programs in several countries there has been no investigation of how students would accept it in terms of learning, and it was

> immediately assumed that Modern Mathematics was actually simpler. However, experience has shown otherwise. (Catela, 1980, pp. 24-5)

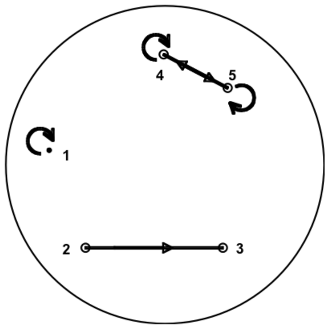
The Project includes another study that is not directly related to the introduction of the unified school (Leal & Kilborn, 1981). This work had the purpose of assessing basic mathematics computing in secondary school. It used a test with several variations of the four arithmetic operations and was applied in 1978/79. This study follows the international movement known as "back to basics", assessing the "basic knowledge for elementary mathematical calculation, such knowledge provided to students by the end of fourth grade students" (p. 24), a movement that appears to please particularly Kilborn.

The results of this last work have also been studied in Ponte, Matos, and Abrantes (1998), and show major flaws of 4th graders in subtraction, multiplication and division. There was also a decrease of correct responses in the 6th grade students. Students repeating the 7th generally had the lowest scores. When compared with similar studies in Sweden and Norway, the results of Portuguese students are superior in the subtraction and similar with respect to multiplication and division of whole numbers.

Learning modern mathematics

The answers to the questions of the tests involving modern mathematics would be of particular importance since they would allow us to have some insight about the quality of learning of these new topics. Unfortunately, there is only one such question, question 5 of M-I that deals with binary relationships and conditions. The question and its three items that the researchers expected had increasing complexity were:

> 5.1. Given the sets
>
> $A = \{ 0, 1, 3 \}$ and $B = \{ -1, 1, 2 \}$ and the condition $x + y < 1$, indicate the ordered pairs of the relationship defined from A to B.
>
> 5.2. Consider the binary relationship defined in the set $\{ 1, 2, 3, 4, 5 \}$ and represented by the diagram. Indicate the missing pairs for the relation to be reflexive.



5.3. Represent in extension the classes in which the set { 11, 18, 21, 28, 37, 31 } is divided by the relationship defined by the condition "if it has the same unit figure of y".[5]

From the available copies of the report it is difficult to read the number of correct answers per class for each item. But in one 7th grade class, which has all numerals legible, the following percentages of correct answers were obtained: 5.1 - 0%, 5.2 - 0%, 5.3 - 6.7 %. It is also possible to know the average percentage of correct answers per class, as computed by the Project. For example, the mean percentage of correct answers for the previous class was 2.2% which corresponds to the average of three percentages (Catela and Kilborn, 1979, p. 21). For purposes of this article, these percentages means were grouped at 10% intervals and the data obtained are presented in table 2.

Table 2. Number of classes with correct answers to question 5 of test MI per grade.

Average percentage of correct answers	Number of classes	%
7th grade		
0 a 10 %	16	64
10 a 20 %	4	16
20 a 30 %	3	12
30 a 40 %	2	8
40 a 50 %	0	0

[5] The answers were: 5.1: (0, -1), (1, -1); 5.2: (2, 2), 3,3); 5.3: { 11, 21, 31 }, { 18, 28 }, { 37 }.

Total	25	
8th grade		
0 a 10 %	12	55
10 a 20 %	8	36
20 a 30 %	1	5
30 a 40 %	1	5
40 a 50 %	0	0
Total	22	

Performance in this question about binary relations is very weak. For example, sixteen 7th grade classes (64%) and twelve 8th grade classes (55%) had an average percentage of correct answers less that 10%. According to the expectations of the authors of the test, the average percentages should be 50%. It is, however, from 8.8% in 7th grade and 9.7% in the 8th, which, at the same time correspond to the lowest percentage of correct responses of the test.

The aggregation of the correct answers to the three items just give us a partial view. Looking at the disaggregated maps (Catela & Kilborn, 1979, pp. 21, 22), we detect many classes in which few or no students (0 or 1) answered correctly, and some (few) classes with slightly better results but still very afar from the expected results. In the Northeast region, in particular, the classes showed a high number of null responses (33 in 57, compared to 28 in 84 in the region of Lisbon). The best results (over 30% average of correct answers) are obtained in three groups of schools in the city of Lisbon which had teachers well acquainted with the reform.

What are the reasons behind such poor results? The MI test was developed in cooperation with the very authors of the program and that there was a pre-testing. We should also reject the hypothesis that students had difficulties computing these items. These difficulties were clearly present in the items related to algebra or fractional numbers, but the answer question 5 only requires a correct linguistic interpretation.

The poor performance of students on the issue of binary relations seems to be due to two factors: shortcomings in the teaching process and intrinsic difficulty. In other words, it is very likely that, not having learned binary relations during their initial scientific formation, teachers tended not to teach this subject or teach it in inappropriately. But even the classes taught by teachers conversant with the new ideas, as those from Lisbon, performed poorly. And even these teachers seemed not being able to teach in such a way that learning would remain as students went from the 7th to the 8th grade, contrary to what happens in other “classical” items. Binary relations seem indeed to have an intrinsic difficulty of its own. This is stated in the report that concludes that this topic is not appropriate for these students.

These results are coherent with a previous study about the performance of 6th grade students on topics of modern mathematics in the national examinations of 1972 (Matos, 2005). There it seemed clear that the introduction of terminology characteristic of modern mathematics, although superficially seized by students, kept them from answering some mathematically simple questions. This study also suggests problems with teacher preparation and the inadequacy of certain topics to the mental age of the students.

Conclusions

When these reports were conducted, the debate in Portugal about the teaching and learning of mathematics was very limited (Matos, 2011). The regime that ended in 1974 limited for nearly half a century the democratic functioning of organizations in which social forces could meet. No associations of teachers were allowed and the Portuguese Mathematical Society had a reduced activity since the expulsion of many of his lieders in the late 1940s. As a consequence, this reports had no effect on actual teaching of mathematics.

Together with this the picture of the spread of mathematics educators have the vision given by the reports of a rapidly changing educational system. On the one hand, the sustained growth of the school population since the 50 was not accompanied by the creation of infrastructures and an adequate training of human resources. Therefore we have simultaneously overcrowded schools and very young teachers, many of them (the majority in some areas of the country) without adequate scientific and pedagogical training.

The landscape of learning mathematics is disappointing. Schools changing sharply (integrating Liceus and technical schools, rapid expansion of the school network), classes starting with delays of months, entire sections of the program that are not taught, general shortage of teachers with adequate training, new programs, mathematics approached by a radically different philosophy appreciating the more formal aspects of mathematics combine to produce poor quality learning. Although many of these problems are now outdated, this is good not to forget the past to better understand the current situation.

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